

The Concepts of Necessary and Sufficient Conditions

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1. Introduction

Everyone is familiar with the concept of a *necessary* condition. For example, we all know that air is *necessary* for (human) life. Without air, there is no (human) life. Similarly, a microscope (or some other instrument) is *necessary* for human beings to see viruses. (Viruses are too small to be seen by the naked eye.)

Similarly, everyone is familiar with the concept of a *sufficient* condition. For example, it suffices (i.e. it is sufficient for) an object's having four sides that it is a square. Or, again, it is sufficient for your having something to drink that you have a glass of Coca-Cola®.

2. Definition of “necessary condition”

Definition: A condition *A* is said to be *necessary* for a condition *B*, if (and only if) the falsity (/nonexistence /non-occurrence) [as the case may be] of *A* guarantees (or brings about) the falsity (/nonexistence /non-occurrence) of *B*.

So common is this notion of *necessary condition* that there are, not surprisingly, a great many ways to express that something is a necessary condition. Here are a number of examples, all - more or less - saying the same thing:

- "Air is necessary for human life."
- "Human beings must have air to live."
- "Without air, human beings die (i.e. do not live)."
- "If a human being is alive, then that human being has air (to breathe)."

In an 'if-then' statement (such as the last example immediately above), the clause that follows the "then" (i.e. the so-called 'consequent') states the necessary condition for the antecedent (i.e. the clause immediately following the "if"). Thus that some *human being has air (to breathe)* is a **necessary condition for that human being's being alive**.

Let's look at some further examples (again, all saying pretty much the same thing). Notice the use of "if-then" (in the sixth [highlighted] example):

- "Having a microscope (or some other instrument) is a necessary condition for (our) seeing viruses."
- "A microscope (or a similar instrument) is needed to see viruses."
- "Human beings must use (have) a microscope to see viruses."
- "Human beings cannot see viruses without a microscope."
- "Anyone who sees viruses has (uses) a microscope."
- "If someone sees viruses, **then** that person uses a microscope." **NOTE**
- "Without a microscope, a person cannot see viruses."
- "If a person does not have (the use of) a microscope, then that person does not see viruses."
- "Whoever lacks a microscope does not see viruses."
- "One must have a microscope to see viruses."

But of course, as we well know, in general a necessary condition is not a *sufficient condition*. All sorts of conditions may be *necessary* for others, but do not - by themselves - *suffice for*, or *guarantee*, those others.

3. Definition of “sufficient condition”

Definition: A condition *A* is said to be *sufficient* for a condition *B*, if (and only if) the truth (/existence /occurrence) [as the case may be] of *A* guarantees (or brings about) the truth (/existence /occurrence) of *B*.

For example, while air is a necessary condition for human life, it is by no means a sufficient condition, i.e. it does not, by itself, i.e. alone, suffice for human life. While someone may have air to breathe, that person will still die if s/he lacks water (for a number of days), has taken poison, is exposed to extremes of cold or heat, etc. There are, in fact, a very great many conditions that are necessary for human life, and no one - or even just a few of them - will suffice for [or guarantee] human life. Or, further, consider the property of having four sides. While *having four sides* is a necessary condition for something's being a square, that single condition is not, by itself, sufficient (to guarantee) something's being a square, i.e. some four-sided things (e.g. trapezoids) are not squares. There are several necessary conditions for something's being a square, and all of these must be satisfied for something's being a square:

- x has (exactly) four sides
- each of x's sides is straight
- x is a closed figure
- x lies in a plane
- each of x's sides is equal in length to each of the others
- each of x's interior angles is equal to the others (they are each right [i.e. 90°] angles)
- the sides of x are joined at their ends

The foregoing is a *complete* set of necessary conditions, i.e. the set comprises a set of **sufficient** condition for x's being square.

Frequently the terminology of "individually necessary" and "jointly sufficient" is used. One might say, for example, "each of the members of the foregoing set is **individually necessary** and, taken all together, they are **jointly sufficient** for x's being a square."

Caution: In this example, we have been able, with ease, to list a set of individually necessary conditions that is also sufficient for something's being a square. However, we must not generalize from this simple example and believe that it is usually, or often, a straightforward task to specify sets of conditions that are individually necessary and jointly sufficient. Sometimes it is much easier to specify (some, or many, of the) necessary conditions even though we are unable to specify a set that is jointly sufficient. Other times, the converse is true: for some cases it will be easier to specify sufficient conditions without our being able to specify individually necessary ones.

4. Necessary conditions that are not sufficient

Example 4.1 - A set of conditions that are individually necessary without being jointly sufficient.

Thomas White, the author of a recent textbook in philosophy, attempted to use as his example the specifying of the necessary and sufficient conditions for hearing music from a Walkman®. Here is the list of necessary conditions that White offered (irrelevant conditions have been here removed, and the list has been renumbered):

1. The Walkman is in good working order.
2. The batteries are good. [[Note 1](#)]
3. The earphones are plugged in.
4. The tape has music on it and is in good working condition.
5. You operate the controls correctly.

-- *Discovering Philosophy*, by Thomas I. White, p. 25
(Englewood Cliffs, NJ: Prentice Hall), 1991.

White then goes on to make a too-hasty claim: "... taken all together they are *sufficient*" (*ibid.*, p. 25). Unfortunately, for his illustrative purposes, the list is by no means sufficient. Here are just a few of the many additional necessary conditions that my own students, in previous years, have offered:

6. The listener must not be deaf.
7. The ambient (surrounding) sound must not drown out the earphones.
8. The listener must be wearing the earphones, or must be close enough to them, to hear the music.
9. There must be nothing blocking the sound in the listener's ears.

10. The tape must be inserted correctly; the door of the Walkman must be closed; and the tape must not be at the end of the reel (more specifically, it must be positioned so that some of the parts of the tape which contain recorded music will pass over the playback heads).
11. The earphones are in good working order.
12. The listener does not die in the time between operating the controls correctly and the music's emerging from the earphones.

Even this expanded list is not complete. I imagine that with little effort, you yourself can come up with still further necessary conditions. Indeed, there does not seem to be any practical limit to the number of necessary conditions.

Conclusion: Sometimes (as in the case of hearing music from a Walkman), it is (far) easier to specify necessary conditions than sufficient ones.

5. Sufficient conditions that are not necessary

Example 5.1 - A set of conditions that are (jointly) sufficient without being individually necessary.

A **sufficient condition** for travelling from Calgary to Vancouver would be your taking an uneventful trip as a passenger on a regularly scheduled air flight. But while that method of getting from the one city to the other would suffice, it is by no means necessary. There are all sorts of *other* conditions that would also suffice for your getting from Calgary to Vancouver.

1. You could take VIA rail; or
2. You could travel by car; or
3. You could hike; or
4. You could ride a bicycle; or
5. You could travel on horseback; or
6. ..., ...

Example 5.2 - A(nother) set of conditions that are (jointly) sufficient without being individually necessary.

If you'll forgive the morbid example, think of all the ways a person might die: having his/her head chopped off; being at 'Ground-Zero' when a nuclear bomb is detonated; tearing a gaping hole in one's space suit while on a 'space-walk' on the Moon; etc., etc. But none of these conditions is a *necessary*

condition for a person's dying. Indeed almost everyone dies without having satisfied one of *these* unusual sufficient conditions.

Conclusion: Sometimes, it is easier to specify sufficient conditions than necessary ones.

6. The concept of converse relations

Here are some examples of *two-place* relations:

- ... is larger than ..
- ... is taller than ...
- ... is above ...
- ... is a parent of ...
- ... is a child of ...
- ... loves ...
- ... employs ...
- ... detests ...

Examples of *three-place* relations:

- ...is between ... and ...
- ...travels to ... by ...
- ...sends ... to ...

Certain two-place relations are *converses* of one another. For example, each member of the following pairs is a converse of the other member:

- ... is a parent of ...
- ... is a child of ...

- ... is taller than ...
- ... is shorter than ...

- ... is above ...
- ... is below ...

- ... loves ...
- ... is loved by ...

Definition: Two (two-place) relations, R_1 and R_2 , are *converses* of one another, if and only if, (1) xR_1y (e.g. Sandra is taller than Louise) guarantees yR_2x (e.g. Louise is shorter than Sandra), **and** (2) yR_2x guarantees xR_1y .

These two-place relations are *not* converses of one another:

- ... is a daughter of ...
- ... is a parent of ...

The 'trouble' with this case is that although "x is a daughter of y" does indeed guarantee "y is a parent of x", the second condition stipulated in the definition for "converse relation" does not hold. For it is false that "y is a parent of x" guarantees "x is a daughter of y".

Or, consider this case:

- ... is not taller than ...
- ... is taller than ...

Again, these two relations are *not* converses of one another. In this instance the first condition of the definition of "converse relation" does not hold. From "x is not taller than y" it does not follow that "y is shorter than x". Perhaps x and y are exactly the same height. If so, "x is not taller than y" will be true, but "y is taller than x" will be false.

7. "Is a necessary condition for" and "is a sufficient condition for" are converse relations

If x is a necessary condition for y, then y is a sufficient condition for x.

And, equivalently,

If y is a sufficient condition for x, then x is a necessary condition for y.

Let's look at some examples that illustrate the claim in the box above:

- "Since having a microscope is necessary for seeing viruses, then seeing viruses guarantees that one has a microscope (i.e. suffices for having a microscope)."
- Similarly, "Since having air to breathe is necessary for human life, it follows that the existence of human life (demonstrates, assures, guarantees, i.e.) suffices for the existence of air."
- "Since every square must have four sides (i.e. since having four sides is a necessary condition for being a square), being a square is a sufficient condition (but not a necessary one) for something's having four sides." Put another way: "All squares (must) have four sides; but not all four-sided things [e.g. trapezoids] are squares."
- "Being a father is a sufficient condition for being male, and being male is a necessary condition for being a father." (But being a father is not a necessary condition for being a male [e.g. James Dean was not a father, but was male]; and being a male is not a sufficient condition for being a father [again the case of James Dean].)

8. Four possible combinations

Pick any two conditions whatsoever. The relationship between the two conditions must be exactly one of the following four possibilities:

1. The first is a necessary, but not a sufficient, condition for the second; or
2. The first is a sufficient, but not a necessary, condition for the second; or
3. The first is both a necessary and a sufficient condition for the second; or
4. The first is neither a necessary nor a sufficient condition for the second.

Examples 8.1 - The first is a necessary, but not a sufficient, condition for the second.

- "Sam's being a male is a necessary, but not a sufficient condition, for being a father."
- "A table's having four sides is a necessary, but not a sufficient, condition for its being square."
- "Being more than 6 feet (183 centimeters) tall is a necessary, but not a sufficient condition for being 6 feet 3 inches (190.5 centimeters) tall."
- "Owning a 1996 Chevrolet Cavalier is a necessary, but not a sufficient condition, for owning a red, 6-cylinder, 1996 Chevrolet Cavalier."
- "Having a ticket in a lottery is a necessary, but not a sufficient condition, for winning that lottery."

Examples 8.2 - The first is a sufficient, but not a necessary, condition for the second.

- "Sam's being a father is a sufficient, but not a necessary condition, for being a male."
- "A table's being square is a sufficient, but not a necessary condition, for its having four sides."
- "Being 6 feet 3 inches tall (190.5 centimeters) tall is a sufficient, but not a necessary condition for being more than 6 feet (180 centimeters) tall."
- "Owning a red, 6-cylinder, 1996 Chevrolet Cavalier is a sufficient, but not a necessary condition, for owning a 1996 Chevrolet Cavalier."
- "Winning a lottery is a sufficient, but not a necessary condition, for having a ticket."

Examples 8.3 - The first is both a necessary and a sufficient condition for the second.

- "Sam's being a father is both a necessary and a sufficient condition for his being a male parent."
- "Frankie's being older than Johnny is both a necessary and a sufficient condition for Johnny's being younger than Frankie."
- "Today's being neither Saturday nor Sunday is both a necessary and a sufficient condition for today's being a weekday."
- [TRICKY, but true.] "X's-being-a-necessary-condition-for-Y is both a necessary and a sufficient condition for Y's-being-a-sufficient-condition-for-X."

Examples 8.4 - The first is neither a necessary nor a sufficient condition for the second.

- "John's loving Pamela is neither a necessary nor a sufficient condition for Pamela's loving John."
- "Having a married brother is neither a necessary nor a sufficient condition for being an aunt."
- "Being the smartest student in a class is neither a necessary nor a sufficient condition for achieving the highest grade in that class."
- "Aziz's playing baseball well is neither a necessary nor a sufficient condition for his having a driver's license."
- "Wanting to succeed is neither a necessary nor a sufficient condition for success."

NOTES

Strictly, although the batteries' being good is usually a necessary condition for a Walkman's working, it is not absolutely necessary. For it is possible to operate a Walkman without batteries, e.g. by using a plug-in AC-to-DC adapter. What **is** necessary, along these lines, is that there be a proper power source for the Walkman.