CONTROL STRATEGIES FOR INTERACTIVE WATER POLLUTANTS

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ABSTRACT
Excess nutrient loads to coastal waters lead to increased production of algae that, when decaying, cause oxygen depletion in bottom sediments and, at length, major changes in marine ecosystems. In this paper, a cost minimization model is investigated, where nitrogen and phosphorus are assumed to interact with respect to algae production. The analysis shows that policies should focus on reductions in a single nutrient if cost functions are linear or the elasticity of substitution is small. If the elasticity of substitution is large and marginal abatement costs are increasing, it can be cost-effective to reduce both nutrients. The model is applied to the Baltic Proper, which is the largest of the seven major basins in the Baltic Sea. Results indicate that larger abatement efforts should be directed towards phosphorus, in spite of nitrogen being the major limiting nutrient today. This contrasts with the Baltic-wide agreement to undertake proportional reductions in nitrogen and phosphorus.

Keywords: algae, cost-effectiveness, CES production function, elasticity of substitution, nitrogen, phosphorus.

INTRODUCTION
Excess nutrient loads lead to eutrophication of both coastal waters and the open sea. One of the major symptoms of eutrophication is increased production of algae. When decaying, the algae cause oxygen depletion in bottom sediments and, ultimately, changes in whole marine ecosystems (Rabalais, in press). The need for a reduction in nutrient loads is confirmed in the North Sea and the Baltic Sea conventions, where it is stipulated that nitrogen and phosphorus loads should both be reduced by half (Ebbeson, 1996). Such equal reduction rates are not likely to be cost-effective if the target is to reduce algae production. The reason is that nitrogen and phosphorus both are necessary for algae to grow.

The purpose of this paper is to calculate cost-effective solutions to reductions in algae production, when nitrogen and phosphorus interact with regard to algae production. The role of the substitutability between nutrients for the allocation of nitrogen and phosphorus emissions is analyzed. The model is applied to the Baltic Proper, which is the largest of the seven major basins in the Baltic Sea.

The theoretical model in this paper draws on work on multiple pollutants by Beavis and Walker (1979) and Ungern-Sternberg (1987). This paper extends on those by investigating the role of the elasticity of substitution between pollutants for the cost-effective allocation of emissions of different pollutants. In terms of previous research on the Baltic Sea, this work draws on Gren, Elofsson and Jannke (1997) and Elofsson (1999), and expands on those by introducing nutrient interaction. Important limitations of the analysis are the static and deterministic perspective and comparatively little data as regards nutrient interaction and aggregate algae production. The paper is organized as follows; first a background description of the links between nutrients and algae production is presented, then the theoretical model is developed. Thereafter, the empirical example is presented and finally, the paper ends with conclusions.

THE RELATIONSHIP BETWEEN NUTRIENTS AND ALGAE PRODUCTION
The production of algae depends on the availability of nitrogen and phosphorus. Algae take up nitrogen and phosphorus in fixed proportions and for most algae species and other aquatic organisms, the nutrient uptake is about 7 kilo N per kilo of P (Tyrrell, 1999; Wulff, 2000). This relationship is known as the Redfield ratio. This ratio is typically found in both organisms and in concentrations of inorganic nutrients in most of the world’s oceans. However, in coastal regions and regional seas the concentration ratio can be far from the Redfield ratio (see e.g. Gabric and Bell, 1993; Wulff, 2000). In such cases, the nutrient that is available in smaller amounts limits algae production, and smaller alterations in the concentration of this nutrient will increase or decrease algae production, while changes in the other nutrient have little or no effect.

It is generally hard for natural scientists to determine the limiting nutrient for large aquifers and coastal waters. The reason is the complex physical, chemical and biological interactions involved (Gabric and Bell, 1993; Tyrrell, 1999). Large aquifers are heterogeneous and may consist of several, heterogeneous sub-basins that differ with regard to nutrient limitation depending on their physical and bio-chemical properties and even within a single basin, nutrient concentrations may vary (Wulff, 2000; Elmgren, 2001). Nutrient limitation may fluctuate over the season in the short run, and the processes that govern the long-run dynamic adjustment of nutrient concentrations are not fully known (Tyrrell, 1999; Wulff, 2000; Elmgren, 2001). Also, different algae species take up nutrients in different proportions (see e.g. Magnusson et al., 1994; Coffaro and Sfriso, 1997).
THE MODEL

In order to understand the role of nutrient interaction for a cost-effective abatement policy, a simple model is developed in the following. Consider first an aquifer that is negatively affected by nutrient emissions from human activities in the surrounding watershed. The watershed is divided into \( i = 1, \ldots, n \) regions, and the aquifer itself consists of \( j = 1, \ldots, k \) basins.

There are two different nutrients emitted to the aquatic environment, nitrogen and phosphorus. The nutrient is denoted by a subscript \( r \), with \( r = N, P \), where \( N \) stands for nitrogen and \( P \) for phosphorus. The emissions from land-based sources in region \( i \) are denoted \( e_{ri} \), and the land-based load to basin \( j \) is \( \sum_i \alpha_{rij} e_{ri} \), where \( \alpha_{rij} \) is the fraction of emissions from region \( i \) emitted directly to basin \( j \). The total load to a basin is, however, also determined by marine transports of nutrients from other basins. Let \( \alpha_{rjk} \) be a marine transport coefficient that defines how total loads to basin \( j \) are affected by total loads to basin \( k \). The total load, \( L_{rj} \), is then defined by:

\[
L_{rj} = \sum_k \gamma_{rjk} L_{rj} + \sum_i \alpha_{rij} e_{ri}.
\]

The nutrients in the aquifer interact with regard to the production of algae. It is assumed that isoquants, in terms of nitrogen and phosphorus, are convex to the origin and hence that the corresponding algae production function is quasi-concave. This assumption is motivated by nitrogen and phosphorus both being necessary for algae production. It is also assumed that the aggregate algae production can be described by a smooth, differentiable function. This can be reasonable when an aquifer is heterogeneous and there are differences among algae species (cf. e.g. Berck and Helfand (1990) for a discussion of the properties of the agricultural production function).

In production theory, the curvature of an isoquant can be measured by the elasticity of substitution between inputs (see e.g. Debertin, 1986). The elasticity of substitution can therefore also be considered as a measure of nutrient limitation of algae production. If the elasticity of substitution is close to zero, and the ratio of nutrients in seawater is far from the Redfield ratio, then one nutrient is clearly more limiting for algae production than the other. However, if the elasticity of substitution is large or the ratio of nutrients is close to the Redfield ratio, then changes in either of the nutrients may affect algae production. In the following, it is assumed that algae production, \( D_j \), is defined by a CES-function (see e.g. Chiang, 1984):

\[
D_j = A_s \left[ \beta_j \left( L_{nj} \right)^{\rho_j} + \left( 1 - \beta_j \right) \left( L_{pj} \right)^{\rho_j} \right]^{\frac{1}{\rho_j}},
\]

where \( A_s, \beta_j \) and \( \rho_j \) are parameters. The elasticity of substitution, \( \rho_j \), is defined by \( \rho_j = \frac{1}{1 + \rho_j} \). If \( \rho_j \) is close to zero, isoquants are nearly right-angled and if \( \rho_j \) is close to one, isoquants have a smooth curvature and are convex towards the origin. For ease of calculation, other characteristics of the production function are highly simplified. In particular, the above specification of the production function implies constant returns to scale and a linear relationship between the loads of a nutrient and the concentration of the same nutrient in the sea.

It is assumed that there is an international environmental agent who wants to reach an environmental target for algae production, \( D^* \), in one of the basins, say basin \( B \), at minimum cost. Furthermore, it is assumed that there are cost functions for nutrient emissions, denoted \( c_{ri} \left( e_{ri} - e_{ri}^0 \right) \), where \( e_{ri}^0 \) is the initial emissions. The cost functions are assumed to be decreasing and convex in emissions. Suppressing subscripts, the environmental constraint for basin \( B \) can be expressed as

\[
A \left[ \beta \left( L_{nb} \right)^{\rho} + (1 - \beta) \left( L_{pb} \right)^{\rho} \right]^{\frac{1}{\rho}} \leq D^*.
\]

The decision problem of the environmental agent can then be written as:

\[
\min \sum\sum c_{ri} \left( e_{ri}^0 - e_{ri} \right)
\]

subject to (1)-(3) and \( 0 \leq e_{ri} \leq e_{ri}^0 \).

The cost-efficient allocation of emissions can be determined from the solution to (4). As the algae production constraint is quasi-convex, the Kuhn-Tucker conditions in (5)-(7) are necessary but not sufficient for a global solution to the environmental problem:
\[
\frac{\partial L}{\partial e_{ni}} = \frac{\partial c_{ni}}{\partial e_{ni}} + \lambda \frac{\beta}{A^p} \left( \frac{D}{L_{NB}} \right)^{1+p} \left[ \sum_k \gamma_{nk} \frac{\partial L_{nk}}{\partial e_{ni}} + \alpha_{ni} \right] \geq 0, \quad e_{ni} \geq 0, \quad e_{ni} \frac{\partial L}{\partial e_{ni}} = 0
\] (5)

\[
\frac{\partial L}{\partial e_{pi}} = \frac{\partial c_{pi}}{\partial e_{pi}} + \lambda \left(1 - \beta\right) \frac{D}{A^p} \left( \frac{D}{L_{PB}} \right)^{1+p} \left[ \sum_k \gamma_{pk} \frac{\partial L_{pk}}{\partial e_{pi}} + \alpha_{pi} \right] \geq 0, \quad e_{pi} \geq 0, \quad e_{pi} \frac{\partial L}{\partial e_{pi}} = 0
\] (6)

\[
\frac{\partial L}{\partial \lambda} = D^* - A \left[ \beta \left( \frac{L_{NB}}{p} \right) + \left(1 - \beta\right) \left( \frac{L_{PB}}{p} \right)^{-\rho} \right] \lambda \geq 0, \quad \lambda \frac{\partial L}{\partial \lambda} = 0.
\] (7)

Under the following three circumstances there will be a corner solution where only emissions of one nutrient are reduced: if costs are linear in emissions for both nitrogen and phosphorus, the cost-effective solution is always to reduce only one of the pollutants when isoquants are convex (see Ungern-Sternberg) if the elasticity of substitution between nitrogen and phosphorus is zero, a corner solution will be cost-effective independently of whether costs are linear or convex in emissions if algae production isoquants are more convex than isocost curves (in terms of nitrogen and phosphorus), a corner solution is cost-effective. Even if there is a corner solution, it does not have to be the same nutrient that is mainly controlling algae production in both the initial situation and in the cost-effective solution. Imagine for example a situation where nitrogen is, initially, the limiting nutrient, and there is a large surplus of phosphorus in the water. Assume that phosphorus load reductions are costless but nitrogen load reductions can only be made at a cost. Then it would be cost-effective to reduce phosphorus loads to zero, in spite of nitrogen limiting algae production initially.

In the case of a unique interior solution, where both nitrogen and phosphorus emissions are reduced, the cost-efficient allocation of two emission sources, say nitrogen in region \(i\) and phosphorus in region \(h\) it holds that, in optimum,

\[
\frac{\partial L}{\partial e_{ni}} = \frac{\partial c_{ni}}{\partial e_{ni}} + \lambda \frac{\beta}{A^p} \left( \frac{L_{PB}}{L_{NB}} \right)^{1+p} \left[ \sum_k \gamma_{nk} \frac{\partial L_{nk}}{\partial e_{ni}} + \alpha_{ni} \right] \geq 0, \quad e_{ni} \geq 0, \quad \lambda \frac{\partial L}{\partial \lambda} = 0.
\] (8)

The left-hand side of the expression in (8) is the ratio of marginal costs of abatement. This ratio is a measure of the rate, at which reductions in emissions from one of the sources can be replaced by increases in emissions from the other source while keeping the total cost constant. The right hand side is the marginal rate of technical substitution for the two emission sources. This is the rate at which reductions in emissions from one of the sources can be replaced by increases in emissions from the other source while keeping algae production constant.

**Proposition.** If the elasticity of substitution between nutrients is small, it is cost-effective to concentrate most of the abatement efforts on a single nutrient. However, the nutrient to focus on need not be the one that is limiting in the initial situation. If the elasticity of substitution is high, then it is cost-effective to reduce the emissions of both nutrients.

**Proof.** From (8) one can see that if \(\frac{L_{PB}}{L_{NB}} < 1\) then \(\frac{\partial c_{ni}}{\partial e_{ni}}\) is decreasing when the elasticity of substitution decreases. This implies that if phosphorus loads are smaller than nitrogen loads in optimum, a smaller elasticity of substitution implies relatively larger reductions in phosphorus loads compared to the reductions in nitrogen loads. Conversely, a higher elasticity of substitution implies a larger marginal cost ratio \(\frac{\partial c_{ni}}{\partial e_{ni}}\), such that more nitrogen reductions should be undertaken in optimum compared to phosphorus reductions. Thus, the cost-effective loads of nitrogen and phosphorus will be more similar with a higher elasticity of substitution, provided that isocost-curves are more convex than the isoquants for algae production, as noted above. An analysis of the case where \(\frac{L_{PB}}{L_{NB}} > 1\) yields the same conclusions as when \(\frac{L_{PB}}{L_{NB}} < 1\).

The absolute change in the physical amounts of nitrogen and phosphorus are determined by the relative costs, and so is the choice of nutrient to focus on. A decrease in the marginal cost ratio in (8) is not equivalent with phosphorus emissions being smaller in optimum and nitrogen emissions being higher. The emissions of both nutrients may fall or increase, but it will be cost-effective to undertake relatively more expensive reductions of phosphorus compared to reductions of nitrogen.
EMPIRICAL EXAMPLE

In the following, an empirical example is presented that illustrates the above model. The application is made to the Baltic Proper, that suffers from significant eutrophication problems (Turner et al., 1999). In the open Baltic Proper nitrogen is the major limiting nutrient in spite of the large nitrogen inputs compared to phosphorus inputs (Wulff, 2000). Still, changes in phosphorus emissions may affect algae production: simulations by Wulff (2000) indicate that the Baltic Proper may change into a phosphorus-limited state if phosphorus loads are, approximately, halved. Also, nitrogen-fixing algae are favored by the higher phosphorus availability in the open sea during summer (Wulff, 2000; Elmgren, 2001) and a reduction in the load of phosphorus is likely to reduce the frequency of toxic blooms of nitrogen-fixing algae in the late summer (Turner et al., 1999). Policy-makers thus have to take into account both that the system might turn into a phosphorus-limited state and that reductions in the load of phosphorus may reduce the production of particularly harmful algae.

Empirical model

CES production functions have been calculated for three different values of the elasticity of substitution: 0.3, 0.5 and 0.8. The true elasticity may well be different from either of those three. For each value of the elasticity of substitution, the parameters $a$ and $d$ in equation (3) are computed under the following assumptions:

1. The concentration of a nutrient is linear in the loads of the same nutrient.
2. There are constant returns to scale in algae production. It is assumed that the algae production target, $D^*$, is reached if loads of both nutrients are reduced by 50%. The 50% reduction in loads is chosen as a level that would be enough to reach the target because it is the reduction rate agreed upon in connection with Baltic Sea Convention (HELCOM, 1998).
3. If phosphorus loads to the Baltic Proper are reduced by 50%, while nitrogen loads remain at the initial level, nutrients are balanced in the Sea. Balanced nutrients imply that the marginal rate of technical substitution between nitrogen and phosphorus loads, $MRTS_{PN}$, is equal to the Redfield ratio, i.e. it is required that $MRTS_{PN} = \frac{1}{\beta} \left( \left( \frac{L_{NB}^0}{0.5L_{PB}^0} \right)^{\alpha} \right) = -7$, where $L_{PB}^0$ and $L_{NB}^0$ are the initial loads to Baltic Proper. This assumption implies that Baltic Proper would change to a mainly phosphorus-limited state when phosphorus loads are reduced by more than 50%, as suggested by Wulff.

The resulting isoquants for the three different production functions are illustrated in Figure. 1. The elasticity of substitution for each isoquant, 0.3, 0.5 or 0.8, is indicated in the Figure. Each of the isoquants corresponds to the algae production target $D^*$, but they differ with regard to assumptions about nutrient substitutability. Thus, if one for example believes that the elasticity of substitution equals 0.5, the environmental target is reached at any point along the isoquant denoted by “0.5”, etc. The initial loads, $L_{PB}^0$ and $L_{NB}^0$, are denoted on the axes in the figure. If both loads are reduced by 50%, to $0.5L_{PB}^0$ and $0.5L_{NB}^0$, $D^*$ will be reached independently of the choice of production function, as stipulated by assumption 2 above. Note also that the isoquant with a 0.3 elasticity of substitution intersects the two other isoquants. Therefore, one cannot say à priori that a single one of these production functions has to be associated with the lowest cost.

Data on costs for reducing emissions of nitrogen and phosphorus in the nine countries that surround the Baltic Sea and methods for calculation of loads to the seven different basins in the Baltic Sea can be found in Elofsson (2002).

RESULTS

The results have been computed using GAMS software and CONOPT solver (Brooke, Kendrick and Meeraus, 1996). The total minimum cost for reaching the algae production target $D^*$ is computed for each of the above production functions. Interior local optima have been computed with the corner solutions where only one nutrient is reduced, to control for the possibility that the local optimum is a local maximum instead of a local minimum. Also, different initial values have been used to control for the possibility of more than one interior solution.

For the scenarios investigated, a higher elasticity of substitution is associated with a higher minimum cost and vice versa, see Table 1. Reducing algae production by half in Baltic Proper would cost 10.5 Billion SEK per year if the elasticity of substitution were 0.8. With a 0.5 elasticity of substitution, total cost would be reduced to 9.7 Billion SEK, and with a 0.3 elasticity of substitution, total cost would be 8.0 Billion SEK. A smaller elasticity of substitution implies that it is easier to suppress algae production through a reduction in a single nutrient, in this case phosphorus. The surplus of nitrogen will remain idle, as nitrogen and phosphorus are needed in (nearly) fixed proportions. If you do the same phosphorus reduction, but the elasticity of substitution is large, then the remaining algae production is larger as nitrogen can, to larger extent, replace the phosphorus taken away.
Fig. 1. Isoquants for $D^*$ for different algae production functions. The elasticity of substitution is indicated for each production function (0.3, 0.5 and 0.8, respectively).

Table 1. Total abatement cost, cost-effective total loads and marginal costs of emission reduction.

<table>
<thead>
<tr>
<th></th>
<th>Total cost (Billion SEK)</th>
<th>N load (percent of initial load)</th>
<th>P load (percent of initial load)</th>
<th>Marginal cost (SEK/kg N)</th>
<th>Marginal cost (SEK/kg P)</th>
<th>Marginal cost ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0.8$</td>
<td>10.5</td>
<td>50.2</td>
<td>48.9</td>
<td>206</td>
<td>954</td>
<td>5</td>
</tr>
<tr>
<td>$\sigma = 0.5$</td>
<td>9.7</td>
<td>89.9</td>
<td>10.3</td>
<td>23</td>
<td>4605</td>
<td>202</td>
</tr>
<tr>
<td>$\sigma = 0.3$</td>
<td>8.0</td>
<td>95.6</td>
<td>14.5</td>
<td>7</td>
<td>3945</td>
<td>571</td>
</tr>
</tbody>
</table>

For all scenarios, the optimum is an interior one in the sense that both nitrogen and phosphorus loads are reduced. This is explained by cost functions rising rapidly for high levels of emission reductions. In the table above, the cost-effective total loads of nutrients to Baltic Proper are shown for the three targets with different elasticities of substitution. In all cases, phosphorus load reductions are relatively larger than the nitrogen load reductions. For the case with a 0.8 elasticity of substitution, the cost-effective allocation of nitrogen and phosphorus loads implies close to a 50% reduction in both nutrients. For a 0.5 elasticity of substitution, nitrogen loads are much higher, but phosphorus loads are much smaller than in the 0.8 case. Finally, in the case with a 0.3 elasticity of substitution, the loads of both nutrients are higher than in the 0.5 case. Both with 0.5 and 0.3 elasticity, there is a strong focus on phosphorus load reductions in relative terms.

The marginal cost ratio, expressed as the marginal cost for phosphorus reductions divided by the marginal cost for nitrogen reductions, increases when the elasticity of substitution falls, as was concluded in the theoretical analysis. Already with a 0.8 elasticity of substitution, marginal phosphorus reduction cost is five times as high as marginal nitrogen reduction costs. With a 0.3 elasticity of substitution, the marginal cost for phosphorus reductions is more than 570 times as high as for nitrogen reductions.

CONCLUSIONS
This paper shows the importance of including both nitrogen and phosphorus in economic models regarding measures against eutrophication. The theoretical analysis shows that if the elasticity of substitution between nutrients is small, emission reductions should, to larger extent, be focussed on one of the nutrients. The reason is that if the load of one of the nutrients is reduced substantially, the other nutrient cannot be taken up by growing algae and therefore, cannot cause any harm. The choice of nutrient to focus on is determined by the relative marginal costs of emission reductions and the relative marginal impact on algae production. The nutrient where the largest reduction is made in the cost-effective solution does not have to be the nutrient that is limiting for algae production in the initial situation. If, on the other hand, marginal costs rise rapidly with the level of abatement and the elasticity of substitution is high, then it can be cost-effective to reduce both nutrients. The reason is that with a high elasticity of substitution, a reduction in one nutrient will not reduce algae production effectively, because the other nutrient will still contribute to algal growth. This situation seems more likely to appear for larger aquifers because of the large emission reductions that may be necessary to improve water quality and because of the larger heterogeneity of the aquifers themselves. The analysis is applied to cost-effective reductions in
algae production in the Baltic Proper. The empirical model suggests that a stronger focus on phosphorus reductions compared to nitrogen reductions may be advocated from a cost-effectiveness point of view. This contrasts with the current Baltic-wide policy with equal reduction rates for nitrogen and phosphorus.

REFERENCES


