Monetary shocks with nominal wage stickiness and variable effort

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Abstract

Wallers (1989) model which incorporates an effort augmented production function into a traditional Keynesian analysis of supply and demand shocks is generalised by not restricting the elasticity of substitution between effort and employment to be unity. This significantly changes the results in that unanticipated monetary shocks will affect output and indexing real wages will increase the variation of output in response to supply shocks. Involuntary unemployment is not necessary for demand shocks to affect employment and output in this model.
I. Introduction

Traditional Keynesian analysis assumes nominal wage rigidity in the face of nominal shocks. Firms respond to the lower real wage by hiring more labour in the short run. Waller (1989) incorporates effort (as a function of the wage) into the production function. Efficiency units of labour are the product of effort and employment as in Solow (1979). Monetary shocks are shown to leave output unchanged in the presence of nominal wage rigidity. This result arises because while a rise in the price level reduces the real wage which induces the firm to increase employment, the reduction in the real wage also induces a reduction in effort leaving output unchanged. Also Waller shows that indexing wages will not have a destabilizing effect on output as in models without effort such as Gray (1976).

In this paper I replace the Solow(1979) Cobb Douglas model of efficiency units with a constant elasticity of substitution function. The output effect of monetary shocks will only be zero if the elasticity of substitution is zero. If the elasticity of substitution is less than unity\(^1\) (as argued by Akerlof and Yellen (1986) pp. 14-16 for example) the output effect will be positive. In this case a monetary shock with nominal wage rigidity cause higher output and employment in the short run. Fully informed workers are willing to supply more labour because effort is lower. If it is easy to substitute between effort and employment the output effect may be negative. Holmes and Hutton (2000) get a positive output effect by introducing Monopsony power into the efficiency wage model. A

\(^1\) Akerlof and Yellen (1986) argue that the elasticity of effort with respect to the wage will be less than unity in contrast with Solow’s (1979) model. They argue that workers exerting less effort will damage the
sufficient condition for indexing real wages leading to greater output variability in response to real shocks is if the elasticity of substitution is less than unity.

The model is consistent with efficiency wage models that predict involuntary unemployment but is also consistent with a neo-classical model where the relationship between effort and wages reflects compensating differentials and the labour supply and demand curves intersect in equilibrium.

II. The Model

Competitive firms have the following production function

\[ y = \beta f \left( \frac{W}{P} \alpha + n^{\alpha} \right) \]  \hspace{1cm} (II.1)

Where \( y \) is output, \( W \) the nominal wage, \( P \) the price level, \( e \) and \( n \) is employment. effort \( \beta = 1 + \mu \) is a technology parameter where \( \mu \sim G(0, \sigma^2_{\mu}) \). The elasticity of substitution between effort and employment in producing efficiency units is:

\[ \frac{1}{1 - \alpha} \]  \hspace{1cm} (II.2)

The model is the identical to Waller (1989) except that Waller assumed that \( \alpha = 0 \). The profit function divided by price is:

\[ \beta f \left( \left[ \frac{W}{P} \right]^\alpha + n^{\alpha} \right) - \frac{W}{P} n \]  \hspace{1cm} (II.3)

Profit maximisation yields the following first order conditions for \( W \) and \( N \) respectively:

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firm via the effect on the return to other inputs in a model where labour is not the only input. Faria (2000) develops an efficiency wage model where the Solow condition cannot hold in equilibrium.
The first order conditions imply:

\[ e_n \left( \frac{W}{P} \right)^{\alpha} = \left( \frac{n}{e} \right)^\alpha \] (II.6)

The demand and supply for money are given respectively as:

\[ M^D = kPy \] (II.7)
\[ M^S = \delta M \] (II.8)

Where \( \delta = 1 + \varphi \) and \( \varphi \sim H(0, \sigma_\varphi^2) \)

In Equilibrium we can see that:

\[ kPy = \delta M \] (II.9)

We can totally differentiate equations (II.1), (II.5) and (II.9) to get the following system:

\[ dy - \beta f_n dn - \beta f_{nw} dW - \beta f_{wp} dp - fd\beta = 0 \] (II.10)

Using equations (I.3) and (I.4) this can be rewritten as:

\[ dy - \frac{W}{P} dn - \frac{n}{P} dW + \frac{nW}{P^2} dp - fd\beta = 0 \] (II.11)

\[ \beta f_{nw} dn + (\beta f_{nw} - \frac{1}{P})dW + (\beta f_{np} + \frac{W}{P^2})dP + f_n d\beta = 0 \] (II.12)

\[ \bar{M}d\delta = kPdy + kydp \] (I.13)
The money stock is assumed to be constant apart from the random shock.

We can use these equations to analyse the impact of various shocks.

(1) A Monetary shock with sticky wages

In this case we assume there is no supply shock and there is a random money shock. This means $\beta = 1$ and $d\beta = 0$. Nominal wages are sticky so $dW=0$. Solving $dn$ in terms of $dP$ from equation (II.12) and using the result in (II.11) which in turn can be substituted into (II.13) gives the following expression:

\[
\frac{dy}{d\delta} = \frac{M}{Pk} \left[ -f_{ap} - \frac{W}{p^2} - \frac{n}{p} \cdot f_{mp} \right] \quad \text{(II.14)}
\]

Where:

\[
f_{ap} = -f_{w} W^{\alpha-1} \left[ e_{w}[.] \right]^{\frac{2(1-\alpha)}{\alpha}} n^{\alpha-1} - f_{s} W n^{\alpha-1} e_{w}(1-\alpha)[.]^{\frac{1-2\alpha}{\alpha}} \quad \text{(II.15)}
\]

Using equations (II.3), (II.4) and (II.5) this can be rewritten as:

\[
f_{ap} = -\frac{f_{w}s}{p^2} n^{2(\alpha-1)} [.]^{\frac{2(1-\alpha)}{\alpha}} -(1-\alpha) W \frac{n^\alpha}{p^2} n^\alpha e^\alpha \quad \text{(II.16)}
\]

Also:

\[
f_{mn} = f_{s} n^{2(\alpha-1)} [.]^{\frac{2(1-\alpha)}{\alpha}} + (\alpha - 1)n^{\alpha-2} f_{s}[.]^{\frac{1-\alpha}{\alpha}} + (1-\alpha) f_{s} n^{2(\alpha-1)} [.]^{\frac{1-2\alpha}{\alpha}} \quad \text{(II.17)}
\]

This can be rewritten as:

\[
f_{mn} = f_{s} n^{2(\alpha-1)} [.]^{\frac{2(1-\alpha)}{\alpha}} + \frac{(\alpha - 1)}{n} e^\alpha \frac{W}{p} \quad \text{(II.18)}
\]
Substituting (II.16) and (II.18) into the term in square brackets in the numerator of (II.14) gives the following expression:

$$\beta f_{n,p} + \frac{W}{P^2} + \beta \frac{n}{P} f_{n,m} = \alpha \frac{W}{P^2} \tag{II.19}$$

This implies that:

$$\frac{dy}{d\delta} = \frac{\bar{M}}{Pk} \left[ -\frac{\alpha W}{P^2} + \frac{-\alpha W}{P^2} + f_{n,m} \frac{y}{W} \right] \tag{II.20}$$

Note that it is only if $\alpha = 0$ (the Cobb-Douglas case) that the output effect of a monetary shock is zero. If the elasticity of substitution between effort and employment (II.2) is greater than unity $0 < \alpha < 1$ then (II.20) is unambiguously positive since $f_{n,m} < 0$.

(2) The employment effects of an unanticipated monetary shock

In this case where once again $dW=0$ and $\beta = 1$. Solving equations (II.10) ..(II.11) give the following expression:

$$\frac{dn}{d\delta} = \frac{\bar{M}}{Pk} \left[ f_{n,p} + \frac{W}{P^2} \right] \tag{II.21}$$

Using (II.19) in (II.21) we get:

$$\frac{dn}{d\delta} = \frac{\bar{M}}{Pk} \left[ \frac{\alpha W}{P^2} - \frac{f_{n,m} n}{P} \right] \tag{II.22}$$

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$^2$ $\beta = 1$ in this case but the case when $\beta \neq 1$ will be useful later in the paper.
Since \( f_{mm} < 0 \) then a sufficient condition for the employment effects to be positive is that the elasticity of substitution between employment and effort is greater than or equal to unity (\( \alpha \geq 0 \)).

(2) The effects of a supply shock with nominal wage rigidity.

Waller (1989) analyses the impact of a supply shock under sticky nominal wages and full wage indexation. He shows that the output effects are the same with or without wage indexation. A negative supply shock under sticky wages increases the price level and lowers the real wage. Both effort and labour demand fall. On the other hand if wages are indexed effort is constant and employment falls by the more than in the sticky wages case. Thus Waller (1989) shows that full indexation of wages is not destabilising in the face of supply shocks. This result is dependent on the Solow model as we see below\(^3\).

In this case \( dW=0 \) as before and also \( d\delta = 0 \) and \( \delta = 1 \).

From equation (II.13)

\[-Pdy = ydP \quad \text{(II.23)}\]

Using this in (II.12) we get:

\[dn = \frac{(\beta f_{np} + \frac{W}{P^2}) P}{\beta f_{mn}} dy - \frac{f_n}{\beta f_{mn}} d\beta \quad \text{(II.24)}\]

Using these equations in (II.10) gives:

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\(^3\) Solow (1979) showed that it is only in the Cobb-Douglas case that the real wage will be invariant to the firms output. The implication is that in any non Cobb-Douglas case a supply shock will change the optimal real wage, so that wage indexation will not adjust to the “correct” real wage.
This implies that:

\[
\frac{dy}{d\beta} = \frac{\beta f_{mn} f - f_n}{\beta f_{nn} - \frac{W}{y} (\beta f_{np} + \frac{W}{P^2} + \frac{n}{P} \beta f_{nn})}
\]  

(II.26)

Using (II.19) in the denominator this becomes:

\[
\frac{dy}{d\beta} = \frac{\beta f_{mn} f - f_n}{\beta f_{nn} - \frac{W^2}{P^2 y}}
\]  

(II.27)

If \( \alpha > 0 \) then (II.27) is unambiguously positive.

(4) A supply shock with fully indexed wages.

As in the previous case In this case \( d\delta = 0 \) and \( \delta = 1 \), implying that (II.23) still holds. Because real wages are constant:

\[
dW = \frac{W}{P} dP
\]  

(II.28)

Using (II.28) in (II.10) we get:

\[
\frac{dy}{P} = W dn + f d\beta
\]  

(II.29)

Also recognising that \( \beta f_{pW} = -\beta f_{np} \frac{P}{W} \) and using (II.29) in (II.12) we get:
\[ dn = -\frac{f_n}{\beta f_{nn}} d\beta \quad (II.30) \]

Together with (II.29), equation (II.30) implies that:
\[
\frac{dy}{d\beta} = \frac{\beta f_{nn} f_n - f_n W}{\beta f_{nn}} \quad (II.31)
\]

We see that it is only when \( \alpha = 0 \) (the Cobb-Douglas case) that the output effect of a supply shock is the same under nominal wage rigidity and wage indexation. If \( \alpha > 0 \) the supply shock has a bigger effect on output with wage indexation as in the more traditional models such as Gray (1976).

III. Involuntary Unemployment or market clearing?

In this section I show that while the above analysis is consistent with an efficiency wage model generating involuntary unemployment, it is also consistent with a neo-classical model where labour supply and demand intersect in equilibrium.

Solow (1979) showed that when effort depends on wages that a necessary and sufficient condition for the optimal wage chosen by the firm being invariant with respect to output was that efficiency units of labour were the product of effort and employment (the Cobb-Douglas case referred to above). This is a standard rational for “efficiency wages” providing a justification for involuntary unemployment (see Romer pp 444-5 for example). The argument is that because the real wage is chosen by the firm then even if there is excess supply of labour the firm will not find it profitable to cut wages because of the resulting fall in effort. In this section I show that the analysis in section II. does not depend on acceptance of an efficiency wage story where monitoring or turnover costs, or
some other friction leads to unemployment. We could also interpret the relationship
between effort and wages as a compensating differential for harder work or accepting
poorer working conditions that boost productivity. If this is so we still retain all the
results in section I. While there is no involuntary unemployment.

Say there are $N$ identical workers with the following utility function:

$$ u = w - g(e) \quad (III.1) $$

Where $g(e)$ is a convex function of effort. Following Shapiro and Stiglitz (1984) the wage
that satisfies the no-shirking condition is:

$$ w = g(e)[1 + \frac{b + r}{q}] + w + \frac{\bar{e}}{q} = g(e)A + C \quad (III.2) $$

The arrival rate of exogenous job separations, supervisors and new jobs to unemployed
workers are $b$, $q$ and $a$ respectively. Unemployment benefits are $\bar{w}$ and the market
equilibrium effort in other firms is $\bar{e}$ (in equilibrium $e = \bar{e}$ but only $e$ is a choice variable for
the firm). Walsh (1999) shows that the rent of a job (how much it would be worth to an
unemployed worker) equals $\frac{g(e)}{q}$. If we think of $e$ in (III.2) as a standard of expected
effort chosen by the firm. Rearranging equation (III.2) gives the $e(W/P)$ function used in
equation (II.1). The Shapiro and Stiglitz model will have involuntary unemployment
(although Bulow and Summers (1986) and Walsh (1999) show that inclusion of a
secondary sector with perfect monitoring gives an equilibrium with market clearing in the
labour market).
If we take the case where there is perfect monitoring \((q \to 0)\), equation (III.2) becomes:

\[
w = g(e) + \bar{w} \quad (III.3)
\]

Rearranging (III.3) again gives the \(e(W/P)\) function used in (II.1). In this case workers do not earn rents and the labour supply curve will be horizontal at a wage determined by the level of effort chosen by firms, firms set the real wage equal to the marginal product of labour in accordance with (II.5) and labour supply equals labour demand in equilibrium. It is only when workers earn rents that the effort augmented production function provides a rationale for involuntary unemployment.

IV. Conclusion

In general including effort in the production function leads to responses to shocks which are in line with traditional Keynesian analysis under the assumption of nominal wage rigidity. The addition of the effort function provides a means for these responses to occur without the necessity of appealing to the existence of involuntary unemployment.

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