WEFLARE VERSUS MARKET ACCESS:
THE IMPLICATIONS OF TARIFF STRUCTURE FOR TARIFF REFORM*

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Abstract

We show that the effects of tariff changes on welfare and import volume can be fully characterised by their effects on the generalised mean and variance of the tariff distribution. Using these tools, we derive new results for welfare- and market-access-improving tariff changes, which imply two "cones of liberalisation" in price space. Because welfare is negatively but import volume positively related to the generalised variance, the cones do not intersect, which poses a dilemma for trade policy reform. Finally, we show that generalised and trade-weighted moments are mutually proportional when the trade expenditure function is CES.

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Welfare and trade volume increase together in a small open economy when only one good is subject to a tariff and that tariff is reduced. Tariff reduction thus serves both domestic and international goals: on the one hand it raises home welfare; on the other hand it increases foreign access to domestic markets as required by multilateral trade obligations under the WTO.1 However, in the empirically relevant case where there are many tariff-ridden goods, the analytic relationship between changes in tariffs, welfare and trade volume forms a difficult tangle due to cross effects. The theory of the second best noted long ago that cutting a single tariff need not raise welfare, and it is easy to see that it need not improve market access either. What tariff changes do improve welfare? Market access? What is the relationship between the two?

Computation can in principle provide answers to these questions. But computations of the effects of multiple tariff changes from any applied general equilibrium model will always be suspect because of uncertainty about the parameters and specification of the ‘true' model of the economy. The theory of trade policy reform is a promising alternative which seeks to specify directions of change which can raise welfare or improve market access under plausibly general conditions. Unfortunately, however, progress in this research program has been relatively limited thus far (See Bruno, 1972; Foster and Sonnenschein, 1970; Hatta, 1977; Diewert, Turunen-Red and Woodland, 1989). There are but two results, the uniform radial reduction result (reduce all tariffs by the same proportion) and the "concertina rule" (reduce the highest tariff rate). Each

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1 Bagwell and Staiger (1999) note that reciprocity, which they interpret as trade policy "concessions" that yield equal increases in market access and so keep world prices constant, is one of the foundational principles of GATT.
characterizes a single welfare-improving path in tariff space, and neither is consistent with trade reforms typically proposed or implemented in negotiations or in applied policy-making advice such as that dispensed by international institutions. Moreover, as usually stated, the concertina rule requires implausibly strong restrictions on the general equilibrium substitution effects matrix. As for the effects of tariff reform on market access, recent work by Ju and Krishna (2000) provides some useful insights but much remains to be done.

This paper realizes the promise of the theory of trade reform research program. It provides general characterizations of cones of welfare-improving and market-access-improving trade reforms. Under fairly mild and plausible restrictions, tariff change paths within these cones guarantee improvements in welfare or increases in market access for small open economies. The directions of change within the cones encompass many of the practical tariff-cut formulae of multilateral negotiations. They also contain dispersion-changing directions of change which provide the first formal justification for the World Bank's emphasis on reducing dispersion of tariff structure.

The cones of liberalization are closely related to new concepts of the generalized mean and variance of tariff schedules. Standard atheoretic measures of mean and variance of tariffs are often used as indexes by the World Bank and other analysts because they appear intuitively to be linked to welfare. There is something right about the intuition, but the weights for the

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2 Anderson (1995) introduced generalised tariff moments (though defined in terms of world prices rather than, as here, in terms of domestic prices) and used them to elucidate the properties of the Trade Restrictiveness Index (TRI) of Anderson and Neary (1996). Feenstra (1995, p. 1562) also note the importance of tariff dispersion. These papers, like ours, consider the moments of the distribution of tariffs in a deterministic multi-good framework, whereas Francois and Martin (2004) explore the implications for market access of the moments of a single tariff’s distribution in a stochastic framework.
appropriate generalized mean and variance are based on marginal substitution effects rather than on average trade shares.\textsuperscript{3} In a special CES case, we show that the generalized mean and variance reduce to the trade-weighted mean and a simple function of the trade-weighted variance.

In surprising contrast to the one-good case, welfare increase and market access increase are substantially in conflict. The two cones of liberalization are disjoint except on a single path along which tariff changes preserve relative domestic prices, so the economy is identical to one in which only a single composite commodity is subject to tariffs. This highlights the inadequacy of the one-good framework: the coincidence between the sufficient conditions for welfare improvements and market-access increases falls apart once we move to the realistic case where two or more goods are subject to tariffs. A key reason for this contrast is that reductions in dispersion are good for welfare but bad for market access.

Section 1 introduces the economic model and the generalised moments of tariffs. Sections 2 and 3 show how they lead to significant extensions of the radial reduction and concertina reform results for welfare-improving tariff reforms. Sections 4 and 5 cover the same ground from the perspective of market access. Section 6 shows how changes in the generalised moments relate to changes in the true index numbers of trade policy, the Trade Restrictiveness Index (TRI) and the Mercantilist Trade Restrictiveness Index (MTRI), introduced by Anderson and Neary (1996, 2003). Finally, Section 7 relates generalised to trade-weighted moments in the CES case.

\textsuperscript{3} Kee, Nicita and Olarreaga (2004) highlight the importance of taking substitution effects into account when tariffs are non-uniform. They show that, because of the high variance of the U.S. tariff schedule, the simple and trade-weighted average tariffs underestimate the true average tariff (the TRI) for the U.S. by more than any other country in their sample. (The atheoretic average tariffs for the U.S. are about 4\%, whereas the TRI uniform tariff is approximately 15\%.)
1. Generalised Tariff Moments

The economic model we use is standard in the trade reform literature. A competitive small open economy produces and consumes $n$ imported goods which are subject to a vector of specific tariffs $t$. There may be many other goods in the economy, some of which are exported, but it is convenient to aggregate them into a single composite commodity whose price is set equal to one by choice of numeraire. We ignore distributional and political-economy issues in order to focus on the efficiency effects of changes in tariffs. Hence the behaviour of the private sector can be summarised in terms of the trade expenditure function, which equals the difference between an aggregate expenditure function and the GDP function:

$$E(\pi, u) = e(\pi, u) - g(\pi)$$

(1)

Here $u$ is domestic aggregate welfare and $\pi$ denotes the vector of domestic prices for the $n$ goods, which equal world prices $\pi^*$ plus tariffs. The GDP function also depends on technology and factor endowments, but these are assumed constant throughout and so need not be specified explicitly.

The key property of the trade expenditure function is that its derivatives with respect to prices are the general-equilibrium net import demand functions:

$$m(p, u) = E_\pi(\pi, u) - e_\pi(\pi, u) - g_\pi(\pi)$$

(2)

This follows from Shephard’s and Hotelling’s Lemmas, which imply that the price derivatives

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4 See Neary (1995) and the references given there.

5 The algebra is consistent with the case where some of the $n$ goods are exported and/or are subject to trade subsidies rather than trade taxes. The model can easily be extended to incorporate non-traded goods with endogenous prices, provided they are not subject to domestic taxes.
of \( e \) and \( g \) equal domestic demand and supply functions respectively. The matrix of second
derivatives \( E_{nn} \) is therefore the substitution matrix for non-numeraire goods, which we assume
is negative definite. As for the cross-derivatives with respect to utility, they are proportional to
the Marshallian income derivatives of demand, \( E_{nu} = e \cdot x' \), where \( e \) is the marginal cost of
utility.

In order to characterise changes in the distribution of tariffs, we introduce generalised tariff
moments. These are equal to weighted moments, where the weights are the elements of the
substitution matrix \( E_{nn} \) normalised by domestic prices.\(^6\) For compatibility with these, it turns out
to be easiest to work with both the level and the change in tariffs also deflated by domestic
prices. Hence, define the ad valorem tariff rate on good \( i \) as \( T_i \equiv t_i/\pi_i = (\pi_i - \pi_i^*)/\pi_i \). Note that tariff
rates defined in this way must lie between zero and one: \( 0 \leq T_i < 1 \). They are related to tariff
rates defined with respect to world prices \( (\tau_i \equiv t_i/\pi_i^*) \) by: \( T_i = \tau_i/(1+\tau_i) \). Next, to express the vector
of tariff rates in matrix notation, let \( \mathbf{x} \) denote a diagonal matrix with the elements of the vector
\( x \) on the principal diagonal. Then we can write the vector of tariff rates \( T \) as:

\[
T = \pi^{-1} t = 1 - \pi^{-1} \pi^*
\]

where \( t \) denotes an \( n \)-by-1 vector of ones. Furthermore the derivations are enormously simplified
if we define the change in tariff rates as the changes in specific tariffs relative to domestic prices:
\( dT_i \equiv dt_i/\pi_i \), or in matrix notation:

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\(^6\) This is more convenient than normalising by world prices as in Anderson (1995), since it
allows us to make use of the homogeneity restrictions on import demand.
\[ dT = \pi^{-1} dt \]  

Note that \( dt \) equals \( d\pi \) since world prices are fixed, and so the vector of changes in \( T, dT, \) equals the vector of tariff-induced proportional changes in domestic prices, \( d\pi/\pi. \)

Next, define the matrix of substitution effects normalised by domestic prices as:

\[ S = -\bar{s}^{-1} \pi E_{\pi\pi}, \quad \text{where:} \quad \bar{s} = -\pi' E_{\pi\pi} \pi > 0. \]  

By construction \( S \) is a symmetric \( n \)-by-\( n \) positive definite matrix all of whose elements sum to one: \( \iota'S\iota = 1 \). We can use \( S_{ij} \) to denote both the individual elements of \( S \) and (when either \( i \) or \( j \) is zero) the corresponding cross-price effects with the numeraire good:

\[ S_{ij} = -\bar{s}^{-1} \pi_i E_{ij} \pi_j, \quad i, j = 0, 1, \ldots, n. \]  

Note the sign convention: the normalised own-price effects \( S_{ii} \) are positive for all \( i \), while the normalised cross-price effects \( S_{ij} \) are negative if and only if goods \( i \) and \( j \) are general-equilibrium net substitutes. The standard homogeneity restrictions on the \( E_{\pi\pi} \) matrix imply corresponding restrictions on the \( S \) matrix:

\[ E_{0j} + \sum_{i=1}^{n} \pi_i E_{ij} = 0 \quad \iff \quad S_{0j} + \sum_{i=1}^{n} S_{ij} = 0, \quad \forall \; j = 0, 1, \ldots, n. \]  

Thus the elements of column \( j \) of \( S \) sum to \(-S_{0j}\), which are the normalised cross-price effects between the numeraire good and good \( j \). (From symmetry the elements of row \( i \) of \( S \) sum to \(-S_{ii}\).)

After these preliminaries, we can now define two generalised moments of the tariff structure. The first is the generalised average tariff:
\[ \bar{T} = \tau'^{ST}, \]  

(8)

This equals a weighted average of the individual tariff rates, where the weights are the row (or column) sums of \( S \):

\[ \bar{T} = \sum_{j=1}^{n} \omega_j T_j \quad \text{where:} \quad \omega_j = \sum_{i=1}^{n} S_{ij} - S_{0j} \]  

(9)

(The last equality follows from (7).) The weights must sum to one, since \( \sum_{j} \omega_j = \sum_{i} \sum_{j} S_{ij} = \tau' S \tau = 1 \). However, they need not lie in the \([0,1]\) interval. It follows that \( \bar{T} \) itself need not lie in the unit interval. Two conditions which are sufficient to ensure that it does are immediate. First, \( \bar{T} \) must lie in the unit interval if tariffs are uniform. In that case \( T = \tau \beta \) (\( 0 < \beta < 1 \)), so \( \bar{T} = \tau' S \tau \beta = \beta \).

Second, recalling that \( S \) is defined to be positive definite, the weight on a given tariff rate is more likely to be positive the higher the own-substitution effect for that good and the more it is a complement rather than a substitute for other tariff-constrained goods. Equation (7) implies a more succinct condition: the weight attached to the tariff rate on good \( j \) in the expression for \( \bar{T} \) is positive if and only if that good is a substitute for the numeraire. Hence, if all goods are substitutes for the numeraire, \( \bar{T} \) must be positive and less than one. For later reference we state these results formally:

\textit{Lemma 1:} Sufficient conditions for the generalised average tariff to be positive and less than one are: (a) that all tariff rates are the same; or (b) that all goods subject to tariffs are general-equilibrium substitutes for the numeraire good.

Clearly, the conditions in Lemma 1 are over-strong. \( \bar{T} \) can only be negative if tariffs are
disproportionately higher on goods which are relatively strong complements for the numeraire good, and it can only be greater than one if tariffs are disproportionately higher on goods which are relatively strong substitutes for the numeraire good.

The second generalised moment we introduce is the generalised variance of tariffs:

\[ V = (T-\bar{T})'S(T-\bar{T}) = T'ST - \bar{T}^2. \]  

Unlike the generalised average tariff, \( V \) is unambiguously positive in sign, since it is a quadratic form in the positive definite matrix \( S \). Finally, we define the changes in the two generalised moments as follows:\(^7\)

\[ d\bar{T} = \nu'SdT \quad \text{and} \quad dV = 2(T'SdT - \bar{T}d\bar{T}). \]

As we will see, these changes in generalised moments provide an invaluable intermediate step when we come to assess the effects of changes in actual tariffs on welfare and import volume.

2. Welfare and Trade Policy Reform

We begin with the effects of tariff changes on welfare. The equilibrium condition for our small open economy is that the trade expenditure function (which equals the excess of expenditure over GNP at domestic prices) should equal tariff revenue:

\[ E(\pi, u) = t'E_\pi(\pi, u) \]

Totally differentiating this gives the basic equation linking changes in welfare to changes in tariffs:

\(^7\) These definitions deliberately ignore tariff-induced changes in \( \pi \) and \( E_{\pi \pi} \). The change in the variance of tariffs can be interpreted as twice the (generalised) covariance between initial tariff rates and their changes: \( dV = 2(T-\bar{T})'S(dT-\bar{T}d\bar{T}) = 2Cov(T,dT). \)
Here $\mu \equiv (1-t'x_1)^{-1}$ is the scalar tariff multiplier, or the "shadow price of foreign exchange", which we assume to be positive. (For justification of this, see for example Neary (1995).) We can rewrite (13) in terms of the normalised substitution matrix by using (3) and (4) to replace specific by ad valorem tariffs:

$$\mu^{-1} e_d \mu = t'E_{\pi\pi} \mu.$$

Since both $\mu$ and $\bar{s}$ are positive, the sign of the change in welfare is given by the right-hand side of (14). Using (11) we can express this in terms of the changes in the generalised tariff moments:

$$\langle \mu \bar{s} \rangle^{-1} e_d \mu = -T'SdT.$$

Hence we have shown that the change in welfare is related in a particularly simple way to the changes in the generalised moments of the tariff structure. We can summarise this result as follows:

**Proposition 1**: The effects on welfare of an arbitrary small change in tariffs are fully described by their effects on the generalised mean and variance of tariffs. An increase in the generalised mean lowers welfare if and only if its initial value is positive, while an increase in the generalised variance always reduces welfare.

It is not too surprising that welfare is a decreasing function of the generalised mean tariff in a wide range of circumstances. (Recall Lemma 1 and the associated discussion.) More important
is the result that it is always decreasing in the generalised variance. This provides a rationalisation for the common practice of viewing changes in the variance of tariffs as harmful.

A corollary of Proposition 1 is that a generalised-mean-preserving spread, in the sense of an increase in $V$ with no change in $\bar{T}$, must lower welfare. We will return to this theme in Section 3 below.

Next we want to see how the standard results on trade policy reform can be expressed in terms of the generalised tariff moments. We consider the radial tariff reduction result in this section and the concertina reform result in the next. If all specific tariffs are reduced by a given proportion, $dt_i = -t_i d\alpha$ ($d\alpha > 0$), then all ad valorem tariffs fall by the same proportion: $dT_i = -T_i d\alpha$, or in vector form $dT = -Td\alpha$. Substituting from this, the expressions for the changes in generalised moments in (11) become:

$$d\bar{T} = -\sqrt{ST}d\alpha - \bar{T}d\alpha \quad \text{and} \quad dV = -2(T'ST - \bar{T}^2)d\alpha = -2Vd\alpha.$$ (16)

Summarising:

**Proposition 2:** An equiproporionate reduction in all tariffs reduces the absolute value of the generalised average tariff by the same proportion, and lowers the generalised tariff variance by twice as much.

Note that if $\bar{T}$ is negative then a uniform reduction in all tariffs paradoxically raises the generalised average tariff, moving it closer to zero. Notwithstanding this, the generalised variance falls sufficiently to ensure that welfare rises. From (14), the change in welfare is proportional to $T'STd\alpha$, which is always positive. This is of course just the standard radial
We can also derive a new result for the case of a uniform absolute reduction in the tariff rates $T$. In this case $dT_i$ is the same for all goods, so $dT_i = -d\alpha$, or $dT = -d\alpha$. The contrast between uniform proportionate and uniform absolute changes is illustrated in the first two rows of Table 1. The former changes all tariffs in the same proportion however they are measured (whether in nominal units, or relative to either home or world prices). By contrast, a uniform absolute reduction in the tariff rates $T$ leads to a fall in specific tariffs $t$ in proportion to domestic prices: $dT = \iota d\alpha$ implies that $dt = \pi d\alpha$, so domestic relative prices are unchanged. Table 1 also shows that this change has very simple effects on the generalised moments: the generalised mean definitely falls while the variance does not change. Hence welfare rises if and only if the generalised mean tariff is positive.

Figure 1 compares the effects on domestic prices of a uniform proportionate and a uniform absolute reduction in tariffs in the two-good case. Starting at point $A$, a uniform proportionate reduction in tariffs moves the equilibrium along a ray towards the free-trade point $F$. By contrast, a uniform absolute reduction in tariffs moves the equilibrium along a ray towards the origin $O$, as indicated by the vector $AB$. We know from the standard radial reduction result that a move towards $F$ always raises welfare, and we have just seen that a move towards $O$ raises welfare provided $\overline{T}$ is positive. Hence the shaded area between the two rays is a "cone of liberalisation": a movement from $A$ towards any point in this region must raise welfare if $\overline{T}$ is positive. (Similarly, all points in the unshaded area above $A$ and between the two rays have lower welfare than $A$ if $\overline{T}$ is positive.) Such movements can be characterised by a convex combination of uniform proportionate and uniform absolute tariff reductions:
The effects of such a change on the generalised moments are given by:

\[ dT = -[\beta T + (1 - \beta)] \, d\alpha, \quad 0 \leq \beta \leq 1. \]

(17)

The effects of such a change on welfare equals:

\[ \frac{d\bar{T}}{d\alpha} = -[\beta + 1 - \beta] \, d\alpha \quad \text{and} \quad dV = -2\beta \, V \, d\alpha, \]

(18)

while its effect on welfare equals:

\[ (\mu \bar{s})^{-1} e_u \, du = [\beta T' T + (1 - \beta)] \, d\alpha. \]

(19)

The implications of these results can be summarised as follows:

**Proposition 3**: Any convex combination of a uniform proportionate and a uniform absolute reduction in tariff rates as given by (17): (a) lowers the generalised average tariff if and only if \( T > -1/(1 - \beta) \); (b) always lowers the generalised variance of tariffs, strictly so for \( \beta > 0 \); and (c) raises welfare if and only if \( \bar{T} > -\beta T'T/(1 - \beta) \).

This result substantially increases the scope of what is known about the welfare effects of across-the-board cuts in tariffs. It is particularly important since successive trade rounds under the GATT and the WTO have considered many different types of tariff-cutting formula. (Laird and Yeats (1987), Panagariya (2002) and Francois and Martin (2003) review the different approaches under consideration in the current and previous rounds.) Many of these formulae are special cases of (17), and so Proposition 3 guarantees that they increase welfare given reasonable restrictions on the degree of complementarity between the tariff-constrained goods and the numeraire.

Before leaving this topic, it is important to recall that the new results presented here refer to
a uniform absolute reduction in $T$, the tariff rates measured with respect to *domestic* prices. This is not the same as a uniform absolute reduction in $\tau$, the tariff rates measured with respect to *world* prices. Moreover, the latter change is not guaranteed to raise welfare under reasonable conditions. The effects of this change are shown in the third row of Table 1. It reduces domestic prices in proportion to *world* prices ($d\pi = -\pi^* d\alpha$), and so corresponds to a movement away from $A$ in Figure 1 along the dotted line $AC$ which is parallel to $OF$. This lowers the generalised mean tariff provided $T$ is less than one (which from Proposition 1 is ensured if all tariff-constrained goods are substitutes for the numeraire). However, it raises the generalised variance. The net effect on welfare is proportional to $T - T'$, or $(\pi - T)'ST$, which cannot be unambiguously signed even when all goods are substitutes. Hence this type of tariff reform is not helpful from a welfare perspective, though we will see in Section 4 that it is very important from the perspective of import volume.

3. Welfare Effects of Tariff Reforms which Reduce Dispersion

Consider next the concertina reform result. Suppose without loss of generality that the highest tariff rate is on good 1. This implies an identical ranking whether tariff rates are defined relative to world prices, $\tau_i > \tau$, $\forall i \neq 1$, or relative to domestic prices, $T_i > T$, $\forall i \neq 1$. The fact that only the tariff on good 1 is changed can be expressed by setting $dT_1 = -d\alpha$, and by writing the vector of tariff changes as follows:
We use $\varepsilon_i$ to denote an $n$-by-1 vector with one in the $i$’th entry and zeroes elsewhere. With a concertina-type reform specified in this way, the change in the generalised average tariff can be expressed as follows:

$$
\begin{align*}
\frac{dT}{\alpha} & = - e_i \cdot d\alpha \\
& = - e_i, \quad \text{where:} \quad e_i = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
\end{align*}
$$

(20)

(21)

(22)

To show that this is negative with substitutability, expand the expression in parentheses as follows:

$$
T' \varepsilon_1 - \bar{T}' \varepsilon_1 = \sum_i T_i S_i - \bar{T} \omega_i
$$

(23)

The expression in parentheses in (23) equals the tariff rate on good 1 less a weighted average of $n$ other tariff rates, where from (7) the weights sum to one:
By assumption, all the other \(n-1\) individual tariff rates are less than \(T_i\); if all goods are substitutes for good 1, then, from (9), \(\bar{T}\) itself is less than \(T_i\); and if all goods are substitutes for good 1, then, from (24), all the weights are positive. Hence substitutability is sufficient to ensure that the generalised variance is reduced by a concertina reform. Combining (21) and (22) we get the effect on welfare:

\[
(\mu S)^{-1}e_a du = T^i S_{e_i d\alpha} = S_{11}(T_1 - \sum_2^n T_i \omega_{il}) d\alpha,
\]

which must be positive if all tariff-constrained goods are substitutes for good 1. Using generalised moments thus throws a new perspective on why a concertina reform raises welfare. In addition, the results found here for its effects on the generalised moments will prove useful when we consider the effects of tariffs on market access in Section 5.

We can get a new result by considering a tariff reform which affects the dispersion of tariffs in a different way. Consider an equiproportionate reduction in the gap between all tariff rates and an arbitrary uniform tariff rate, denoted by \(\beta\):\(^8\)

\(^8\) This bears a superficial resemblance to a result of Fukushima (1979). However, the results are different. Fukushima shows that a welfare change of the form \(d\tau = -(\tau - \gamma) d\alpha\) raises welfare for \(\gamma = \beta/(1-\beta)\). However, a key difference is that Fukushima includes in the vector \(\tau\) the distortions on all traded goods; i.e., unlike our model his does not allow for a numeraire traded good which is not subject to tariffs. As a result the level of welfare is the same in his model for all levels of \(\beta\), since a uniform tariff rate on all traded goods does not distort relative prices from their free-trade values, and so it yields no revenue and imposes no welfare cost. Fukushima’s result is thus a restatement of the uniform proportional reduction result. See Neary (1998) for further discussion.
\[ dT = -(T - \beta) d\alpha \]  \hspace{1cm} (26)

This is a generalisation of the uniform radial reduction rule, for which \( \beta \) is zero. Substituting from (26) into (11), the changes in the generalised moments become:

\[ d\bar{T} = -(\bar{T} - \beta) d\alpha \quad \text{and} \quad dV = -2V d\alpha. \]  \hspace{1cm} (27)

(These and other effects of (26) are given in full in the last row of Table 1.) Thus a tariff reform as in (26) reduces the generalised mean if and only if \( \beta \) is less than \( \bar{T} \) and unambiguously reduces the generalised variance. The implications for welfare can be found by substituting into (15):

\[ (\mu s)^{-1} e_{\alpha} du = [\bar{T}(\bar{T} - \beta) + V] d\alpha. \]  \hspace{1cm} (28)

Hence we have a new result for trade policy reform:

\[ \text{Proposition 4: A sufficient condition for welfare to rise following an equiproportionate reduction in the gap between all tariff rates and an arbitrary uniform rate } \beta, \text{ as in (26), is that } \beta \text{ is no further from zero than the generalised mean tariff } \bar{T}. \]

Note that this result holds whether \( \bar{T} \) is positive or negative. A corollary is that, if \( \beta \) equals the generalised average tariff \( \bar{T} \), then a rise in welfare is guaranteed. This makes intuitive sense, since, in this case, the reform is a generalised-mean-preserving contraction of the tariff distribution: \( d\bar{T} = 0 \) and \( dV = -2V d\alpha. \)

This type of tariff reform can be illustrated in Figure 2, which repeats the essential features of Figure 1. Any reform of the type given in (26) can be represented by a movement along a straight line from the initial point \( A \) towards a point on the ray from the origin through \( F \).
location of the point depends on \( \beta \). For example, if \( \beta \) is zero the point coincides with \( F \); if \( \beta \) equals \( T_1 \) the point is given by \( D \), which is horizontally aligned with \( A \); while if \( \beta \) exceeds \( T_1 \) the point lies above \( D \). A corollary of Proposition 4 is that, provided \( \overline{T} \) is positive, then the tariff reform rule (26) is welfare-improving for all \( \beta \) such that \( 0 \leq \beta \leq \overline{T} \). Let \( E \) denote the point at which \( \beta \) equals \( \overline{T} \). (In the two-good case, \( \overline{T} \) exceeds \( T_1 \), and so \( E \) lies above \( D \), provided good 2 is a substitute for the numeraire, since \( \overline{T} - T_1 = \omega_2(T_2 - T_1) \).) Hence, the region \( EAF \) can be added to the region \( FAB \) already derived in the last section to give an expanded cone of welfare-increasing liberalisation, indicated by the full shaded area.


We turn next to see how changes in tariff moments can be used to summarise the effects of tariff changes on the volume of imports. For reasons to be discussed, we measure this at world prices: \( M \equiv \pi^* m \). We shall see later that this seemingly innocuous convention has important implications. The change in import demand can be expressed in terms of income and substitution effects as follows:

\[
dM = \pi^*.dm = \pi^*.\left( E_{\pi \pi} d\pi + x_e d\mu \right).
\] (29)

However, we know from (13) that, in general equilibrium, the income effect is itself related to tariff changes via the substitution matrix \( E_{\pi \pi} \). Substituting into (29), we can write the full effect of changes in tariffs on import volume as follows: \(^9\)

\(^9\) This is equation (15) in Ju and Krishna (2000).
Here $M_b$ is the marginal propensity to spend on importables, defined as follows:

$$dM = (\pi^* + M_b t) E_{\pi} dt.$$  \hfill (30)

Hence $M_b$ must lie between zero and one provided only that both the tariff-constrained goods as a whole and the unconstrained good (i.e., the numeraire) are normal in demand, a very mild restriction.

The next step is to express equation (30) in terms of the generalised moments. Note first that the coefficient in parentheses can be written as follows:

$$M_b = \mu \frac{\pi^* \cdot x_l}{\pi^* \cdot x_l + x_{0l}}.$$  \hfill (31)

where $x_{0l}$ is the income derivative of demand for the numeraire good. A similar series of derivations to those in Section 2 now allows us to write the change in import volume as follows:

$$-T' SdT$$

This can be expressed in terms of changes in generalised moments in two alternative ways. First, recalling from (14) that $-T' SdT$ is proportional to the change in utility, we can rewrite the expression as follows:

$$dM + (1 - M_b) \mu^{-1} e_d du = -\bar{s} d\bar{T}.$$  \hfill (34)

Recalling from (14) that $-T' SdT$ is proportional to the change in utility, we can rewrite the expression as follows:

$$dM + (1 - M_b) \mu^{-1} e_d du = -\bar{s} d\bar{T}.$$  \hfill (33)

This yields a generalisation of Proposition 4 in Ju and Krishna:
Proposition 5: Both welfare and the value of imports cannot fall following a reduction in
tariffs, defined as a fall in $\bar{T}$.

When combined with the result in (21) that a reduction in an arbitrary tariff lowers $\bar{T}$ if and only if the good in question is a substitute for the numeraire good, we have a corollary of Proposition 5: if all goods are net substitutes for the numeraire, then any reduction in tariffs must lower $\bar{T}$, as a result of which either $M$ or $u$ or both must rise.

The second way to reexpress (33) is, as in Section 2, to substitute from (11) to write the change in import volume in terms of changes in $\bar{T}$ and $V$ rather than $\bar{T}$ and $u$:

$$\tilde{s}^{-1}dM = -\left[1 - (1-M_b)\bar{T}\right]d\bar{T} + \sqrt{2}(1-M_b)dV.$$  \hspace{1cm} (35)

This shows that, as with the change in welfare in equation (15), the change in import volume is fully determined by the changes in the two generalised moments. However, a key difference is that the volume of imports is increasing in the generalised variance, for a given level of the generalised mean. Hence we can state a result which parallels Proposition 1:

Proposition 6: The effects on import volume of an arbitrary small change in tariffs are
fully described by their effects on the generalised mean and variance of tariffs. A
sufficient condition for an increase in $\bar{T}$ to lower import volume is that both $M_b$ and
$\bar{T}$ are less than one. A sufficient condition for an increase in $V$ to raise import volume
is that $M_b$ is less than one.

The fact that import volume is increasing in $V$ is an important and surprising finding, which has
a major influence on the results to be presented below.

It is worth noting that the positive relation between import volume and tariff variance does \textit{not} hold if import volume is measured in domestic prices, which we denote by \( \tilde{M} \equiv \pi \prime m \). A series of derivations similar to those which led to (35) above now yields:

\[
d\tilde{M} = m \prime d\pi + \pi \prime dm \\
= m \prime dt + (\pi + \tilde{M}_p) \{ E_{\pi\pi} dt \\
= (\pi \prime m) d\tau^a - \bar{S} (1 + \tilde{M}_p \bar{T}) d\bar{T} \\
= (\pi \prime m) d\tau^a - \bar{S} (1 + \tilde{M}_p \bar{T}) d\bar{T} - \partial_2 \tilde{M}_p dV
\]  

(36)

This shows that the change in \( \tilde{M} \) consists of two components. The first, \( m \prime d\pi \), is a valuation effect, which is proportional to the change in the trade-weighted average tariff \( \tau^a \) (defined as \( m \prime t / m \prime \pi \)). The second, \( \pi \prime dm \), is the change in import volume at constant domestic prices. The latter in turn is \textit{negatively} related to both \( \bar{T} \) and \( V \). Indeed, it is affected by changes in \( \bar{T} \) and \( V \) in a very similar manner to the level of welfare, as a comparison between (15) and (36) makes clear.\textsuperscript{10} In particular, radial reduction and variance reduction rules for tariff reform (including the concertina rule) can be shown to increase \( \tilde{M} \) in a manner similar to those derived for welfare in Sections 2 and 3. Unfortunately, however, these results are not of great interest. As far as trade negotiations are concerned, market access matters primarily from the perspective of \textit{exporters} to the economy under consideration. Hence it is import volume at world rather than

\textsuperscript{10} A minor difference from the earlier case is that the marginal propensity to spend on importables in (36), \( \tilde{M}_p \equiv \mu \tau \prime x_p \), is measured at domestic rather than world prices. Unlike \( M_b \), this could be greater than one if tariffs are sufficiently high, even if all goods are normal in demand. The difference between the two marginal propensities depends directly on the size of the tariff multiplier: \( M_b - \tilde{M}_p = \mu - 1 \).
at domestic prices which is the main focus of interest, and we concentrate on it from now on.\footnote{A different reason for being interested in import volume at domestic prices is that the difference between it and import volume at world prices equals tariff revenue: $R = \ell' \hat{m} = \hat{M} - M$. Hence the change in tariff revenue is: $dR = d\hat{M} - dM$. Substituting from (30) and (36) yields a particularly simple expression: $dR = m' dt + e du$. This is important in discussions of trade policy reform subject to a tariff revenue constraint, where it leads to a version of the Ramsey Rule for tariff rates, an application we do not pursue further here. See for example Falvey (1994).}

5. Tariff Changes and Market Access

We are now ready to consider the effects of various types of tariff reform on the volume of imports as measured by $M$. We begin with a uniform radial reduction in tariffs. It is clear from (30) that if only one good is subject to tariffs then import volume rises monotonically as the tariff is reduced towards zero. However, it does not follow that import volume must rise following a uniform radial reduction when many goods are subject to tariffs. The reason is that, as we have already seen in (16), both $\bar{T}$ and $V$ fall in this case. Since, from (35), falls in $\bar{T}$ and $V$ have opposite effects on import volume, we would therefore expect the overall effect to be ambiguous. To see this explicitly, substitute from (16) into (35), so the change in import volume becomes:

$$\bar{s}^{-1}dM = \left[\bar{T} - (1 - M_B)T'S_T\right]d\alpha \quad (37)$$

Rewriting this in terms of changes in the generalised moments:

$$\bar{s}^{-1}dM = \left[\{1 - (1 - M_B)\bar{T}\} \bar{T} - (1 - M_B)V\right]d\alpha \quad (38)$$

Even when $\bar{T}$ lies between zero and one, the expression on the right-hand side can be either positive or negative. Figure 3, drawn in \{V,T\} space, illustrates. Higher curves in the figure correspond to higher values of $M_B$, each curve showing the threshold value of $V$ as a function of
\( \bar{T} \), above which a uniform radial reduction in tariffs lowers welfare. The implications of the figure can be summarised as follows:

**Proposition 7:** A uniform radial reduction in tariffs is more likely to raise import volume the lower the generalised variance of tariffs, \( V \), and the higher the marginal propensity to spend on importables, \( M_b \).

Ju and Krishna (2000) consider the special case where all tariff rates are the same. This implies that \( V \) is zero, and so corresponds to the horizontal axis (when \( \bar{T} \) lies in the unit interval) in Figure 3. Proposition 7 shows that their result can be considerably relaxed, especially for high values of \( M_b \).

While the effects on import volume of a uniform radial reduction are ambiguous, we can get much sharper results for other kinds of uniform tariff reductions. First, by inspecting (33), it is clear that import volume must rise (without any restrictions on substitutability) if tariff rates are changed according to the following rule:

\[
\frac{dT}{d\alpha} = - [1 - (1 - M_b) \bar{T}] d\alpha.
\]  \hspace{1cm} (39)

Substituting this into (33) yields a quadratic form in \( S \), which must be positive. The rule for reducing tariff rates given by (39) implies, from (4), reducing domestic prices in proportion to a weighted average of domestic and world prices, where \( M_b \) is the weight attached to domestic prices:
\[ \frac{d\pi}{dt} - \pi dT = -[\pi^* + M_b t] d\alpha \]
\[ = -[M_b \pi^* + (1-M_b)\pi^*] d\alpha. \] (40)

We can state this formally as follows:¹²

**Proposition 8:** Import volume must rise if tariffs are reduced in a manner which reduces domestic prices in proportion to a weighted average of domestic and world prices, where \( M_b \) is the weight attached to domestic prices.

We saw in Sections 2 and 3 that a radial reduction in tariffs is in the interior of a cone of welfare-increasing tariff reforms, and has the special feature that it ensures a welfare gain irrespective of the pattern of substitutability between goods. A similar set of results hold for import volume. The tariff change in Proposition 8 does not require substitutability, and lies in the interior of a cone of import-volume-increasing tariff changes. To determine the extent of this cone, we turn next to the effects on import volume of uniform absolute reductions in tariff rates. Consider first a uniform absolute reduction in tariff rates measured with respect to domestic prices, \( T \). From Table 1 we know that this leaves the generalised variance unchanged and lowers the generalised mean, and so it raises import volume provided the coefficient of \( d\bar{T} \) in (35) is positive:

¹² Ju and Krishna present this result in a rather different form: see the first part of their Proposition 1.
As for a uniform absolute reduction in tariff rates measured with respect to world prices, $\tau$, this must also raise import volume under similar conditions, since we know from Table 1 that it lowers the generalised mean and increases the generalised variance. The full expression is:

$$\bar{s}^{-1}dM = \left[1-(1-M_b)\bar{T}\right]d\alpha.$$  \hspace{1cm} (41)

Finally, consider a convex combination of these two types of uniform absolute reduction (where $\beta$ is the weight attached to the uniform absolute reduction in $T$):

$$\bar{s}^{-1}dM = \left[1-(1-M_b)\bar{T}\right]\left[1-\bar{T}\right] + (1-M_b)V]d\alpha.$$ \hspace{1cm} (42)

This is clearly positive for a wide range of parameters. Of course, when $\beta$ equals $M_b$, it is definitely positive and reduces to the case given in Proposition 8. For other values of $\beta$ we can summarise these results as follows:

Proposition 9: Sufficient conditions for any convex combination of uniform absolute reductions in $T$ and $\tau$ to raise import volume are that both $M_b$ and $\bar{T}$ are less than one.

The resulting cone of import-volume-increasing tariff changes is represented in Figure 4 by the shaded area between the rays $AB$ and $AC$ (which are repeated from Figure 1).

Consider next the changes in tariffs which explicitly reduce variance. In Section 3 we were
able to show that these often raise welfare. However, in the present context, such changes are likely to reduce import volume, since it is positively related to the generalised variance. Consider first a concertina reform, or, more generally, a reduction in the tariff on good 1 only. The changes in generalised moments in this case are given by equations (21) and (22). Substituting from these into the expression for changes in import volume in (35), a similar series of derivations which led to (23) now gives:

\[ \bar{s}^{-1} dM = - \left[ (1-M_b) T^i T^j S e_i - \bar{T}^i \bar{T}^j S e_i \right] d\alpha \]

\[ = - (1-M_b) S \left[ T_1 - \sum_{i=2} T_i \omega_{ij} - \frac{\omega_{01}}{1-M_b} \right] d\alpha. \]  

(45)

The expression in parentheses can be unambiguously signed in some special cases. In many such cases it is positive, implying that a rise in \( T_1 \) (i.e., a negative value for \( d\alpha \)) raises import volume. For completeness we state these conditions formally.\(^{13}\)

**Proposition 10:** Assume \( M_b \) is positive and less than one. Then, a reduction in the highest tariff will raise import volume if that good is a substitute for all other tariff-constrained goods and unrelated to or a complement for the numeraire good \( (\omega_{01} \leq 0) \). A reduction in the lowest tariff will lower import volume if that good is a substitute for all other goods, including the numeraire.

Finally, consider the effects of a variance-reducing tariff reform as in (26):

\(^{13}\) This extends Propositions 2 and 3 of Ju and Krishna.
\[ \tilde{s}^{-1} dM = \left[ (\tilde{T} - \beta) \left\{ 1 - (1-M_b) \tilde{T} \right\} - (1-M_b) V \right] d\alpha. \] (46)

Not surprisingly, this kind of reform may lower import volume. It must do so if \( \beta \) is greater than the generalised mean, assuming that \( M_b \) is less than one and \( \tilde{T} \) lies between zero and one. In particular, if \( \beta \) equals \( \tilde{T} \), the case of a generalised-mean-preserving contraction of the tariff distribution, then import volume definitely falls. Turning this result around, we can state sufficient conditions for a variance-increasing tariff reform to raise import volume:

**Proposition 11:** Sufficient conditions for a variance-increasing tariff reform (i.e.,

\[ dT = (T - \beta) d\alpha \]

to raise import volume are that \( M_b \) is less than one, \( \tilde{T} \) lies between zero and one, and \( \beta \) is greater than or equal to \( \tilde{T} \).

This result allows us to expand further the cone of import-volume-increasing tariff changes in Figure 4. When \( \beta \) equals \( \tilde{T} \), the tariff change is represented by the ray \( AG \) (opposite in direction to \( AE \) in Figure 2 and, like it, drawn under the assumption that \( \tilde{T} \) is positive). When \( \beta \) equals one, the tariff change becomes \( dT = -(1-T) d\alpha \), which is equivalent to a uniform absolute reduction in \( \tau \). (Recall the third row in Table 1.) It is therefore represented by the ray \( AC \) as in Figure 1. From Proposition 11, all movements from \( A \) between these two extremes, i.e., all tariff changes in the cone \( CAG \), raise import volume provided \( \tilde{T} \) lies between zero and one. But we have already seen in Proposition 9 that all changes in the cone \( BAC \) also raise import volume. Hence, combining the two results, the full cone of import-volume-increasing tariff changes is denoted by the shaded area \( BAG \).

Comparing Figures 3 and 5, the cones of liberalisation which raise welfare and import volume
are non-intersecting. It should be emphasised that each cone shows only those regions in which an unambiguous increase in the target of interest is guaranteed, given the mild assumptions that $\tilde{T}$ and (in the case of import volume) $M_b$ are between zero and one. They thus correspond to types of tariff reform which can be recommended to attain either target subject to minimal informational requirements. If additional information is available then it may be possible to identify further tariff changes which can attain the desired goal. For example, Proposition 7 gives additional assumptions which ensure that import volume rises following a uniform radial reduction in tariffs, which we know will always raise welfare. Nevertheless, the conclusion is inescapable that there is likely to be a conflict between the objectives of raising welfare and increasing market access. The only tariff change which is guaranteed to achieve both goals, with no assumptions other than that $\tilde{T}$ lies between zero and one, is a uniform absolute reduction in $T$, which reduces domestic prices proportionally.

6. Changes in the TRI and MTRI and Measures of Tariff Dispersion

Next, we want to relate changes in the TRI and the MTRI to changes in the distribution of tariffs, as summarised by changes in the generalised tariff moments. The generalised moments can be thought of as a pair of index numbers which together provide a complete characterisation of the structure of tariffs and which could in principle be used for many purposes. By contrast, both the TRI and MTRI provide a single index which is relevant for one purpose: measuring the restrictiveness of tariffs from the perspective of welfare or import volume respectively. Both approaches have their uses, and it is of interest to explore the links between them.

Begin with the TRI uniform tariff. From Anderson and Neary (1996), this is defined as the
uniform tariff which yields balance of trade equilibrium at the same level of utility as the initial (and presumably non-uniform) tariff vector. Balance of trade equilibrium can be characterised in terms of a new function, the balance of trade function, which equals the deviation of (12) from equilibrium:

\[ B(\pi, u) = E(\pi, u) - t^i E_u(\pi, u) \]  \hspace{1cm} (47)

Hence the TRI uniform tariff can be defined as:

\[ \tau^\Delta(\pi, u): \quad B[(1 + \tau^\Delta)\pi^*, u] - B(\pi, u). \]  \hspace{1cm} (48)

This provides a true index number which gives a scalar representation of the vector of tariffs which generate the domestic prices \( \pi \). Suppose now that tariffs change. Totally differentiating (48), and making use of the fact that the right-hand side of (48) equals zero since the initial situation is an equilibrium, the proportional change in the TRI uniform tariff can be written as follows:

\[ \frac{d\tau^\Delta}{1 + \tau^\Delta} = \psi^\Delta \frac{B_\pi \cdot d\pi}{B_{\pi \cdot \pi}}. \]  \hspace{1cm} (49)

Here, \( \psi^\Delta \) is a correction factor which is needed because the derivatives of the balance of trade function \( B \) are evaluated at two different points:

\[ \psi^\Delta = \frac{B_u^\Delta \cdot B_{\pi \cdot \pi}}{B_u^\Delta \cdot B_{\pi \cdot \pi^\Delta}} \]  \hspace{1cm} (50)

The derivatives \( B_\pi \) and \( B_u \) are evaluated at the initial tariff-distorted price vector \( \pi \); while the derivatives \( B^\Delta_\pi \) and \( B^\Delta_u \) are evaluated at the uniform-tariff-equivalent price vector \( \pi^\Delta = (1 + \tau^\Delta)\pi^* \).
For small tariffs and for many functional forms (even with large tariffs) this correction factor is unity, and it is reasonable to assume that it will be close to unity in many applications.

Returning to (49), we need to express the derivatives of the balance of trade function in terms of the normalised substitution matrix $S$. From equations (5) and (47) above, this can be done as follows:

$$B_\pi' = -t'\pi' = -T'\pi E_\pi = sT'S\pi^{-1}. \quad (51)$$

Substituting into (49), the proportional change in the TRI uniform tariff becomes:

$$\frac{d\tau}{1+\tau} = \psi T'\pi DT \frac{T}{\pi}. \quad (52)$$

This has a nice interpretation. From (14), any change in tariffs has a welfare cost which is proportional to $-T'SdT$, while from (15) a unit change in the generalised mean tariff has a welfare cost which is proportional to $-\pi$. The resulting change in the TRI uniform tariff is proportional to the ratio of these two welfare costs. Finally, using (15) again, the change in the TRI uniform tariff can be written as a function of changes in the two generalised moments:

$$\frac{d\tau}{1+\tau} = \psi \left( d\pi + \frac{\nu dV}{\pi} \right). \quad (53)$$

Thus $\tau$ is increasing in $\pi$ and (provided $\pi$ is positive) in $V$.

Next, consider the MTRI. From Anderson and Neary (2003), this is defined as the uniform tariff which would yield the same level of imports (at world prices) as the initial tariffs:\footnote{Whereas the TRI is evaluated at a given level of aggregate welfare, the MTRI is evaluated at a given level of the trade balance. See Anderson and Neary (2003) for further details.}
Totally differentiating, the proportional change in the MTRI uniform tariff can be written as follows:

\[
\frac{d\tau^\mu}{1 + \tau^\mu} = \frac{M_\pi \cdot d\pi}{M_\pi^\mu \cdot \pi^\mu} = \psi^\mu \frac{M_\pi \cdot d\pi}{M_\pi \cdot \pi}. \tag{55}
\]

As before, \(\psi^\mu\) is a correction factor which is needed because the derivatives of the import volume function are evaluated at two different points:

\[
\psi^\mu = \frac{M_\pi \cdot \pi}{M_\pi^\mu \cdot \pi^\mu}. \tag{56}
\]

\(M_\pi\) is evaluated at the initial tariff-distorted price vector \(\pi\), and \(M_\pi^\mu\) is evaluated at the uniform-tariff-equivalent price vector \(\pi^\mu \equiv (1 + \tau^\mu)\pi^*\). The correction factor \(\psi^\mu\) differs from \(\psi^\delta\) in (50), though both are likely to be close to unity.

To express (55) in terms of changes in generalised tariff moments, we make use of (30) and (32) above:

\[
M_\pi^I = (\pi^* + M_{bT})E_{\pi\pi} = [1 - (1 - M_b)T]\pi^*E_{\pi\pi} = -\overline{S} [1 - (1 - M_b)T]S_\pi^{-1}. \tag{57}
\]

Substituting into (55) gives:

\[
\frac{d\tau^\mu}{1 + \tau^\mu} = \psi^\mu \frac{[1 - (1 - M_b)T]SdT}{1 - (1 - M_b)\overline{T}}. \tag{58}
\]

As with the change in the TRI uniform tariff, we can use (33) and (35) to interpret this. The change in the MTRI uniform tariff is proportional to the ratio of the change in imports arising
from the actual change in tariffs to the change in imports arising from a unit change in the generalised mean tariff $\bar{T}$. Finally, substituting from (35) we obtain:

$$\frac{d\tau^\mu}{1 + \tau^\mu} = \psi^\mu \left[ d\bar{T} - \frac{\psi_2(1-M_b)dV}{1 - (1-M_b)\bar{T}} \right].$$ \hspace{1cm} (59)

Thus the MTRI uniform tariff is increasing in the generalised average tariff but decreasing in the generalised variance.

We can summarise the results of this section so far as follows:

**Proposition 12**: Both the TRI and the MTRI uniform tariffs are increasing in the generalised mean of the tariff schedule. The TRI uniform tariff is also increasing in the generalised variance of tariffs (provided $\bar{T}$ is positive) whereas the MTRI uniform tariff is decreasing in $V$ (provided both $M_b$ and $(1-M_b)\bar{T}$ are less than one).

Next, consider the difference between the changes in the two indexes. Direct calculation shows:

$$\frac{d\tau^\Delta}{1 + \tau^\Delta} - \frac{d\tau^\mu}{1 + \tau^\mu} = \frac{\psi^\Delta - \psi^\mu}{\psi^\Delta} \frac{d\tau^\Delta}{1 + \tau^\Delta} + \psi^\Delta \frac{\psi_2dV}{T[1 - (1-M_b)\bar{T}]}.$$ \hspace{1cm} (60)

Hence we may conclude:

**Proposition 13**: Assume that both $\bar{T}$ and $(1-M_b)\bar{T}$ are positive and that the difference between $\psi^\Delta$ and $\psi^\mu$ can be ignored. Then, a change in tariffs raises the TRI uniform tariff
by more than the MTRI uniform tariff if and only if the generalised variance of tariffs rises.

The full relationship between changes in the TRI and MTRI uniform tariffs on the one hand and changes in the generalised tariff moments on the other is illustrated in Figure 5, drawn in the space of \((dV, d\overline{T})\). From equations (53) and (59) and Proposition 12, both tariff indices increase together in Region II and fall together in Region IV. In the other regions (denoted I and III) they move in opposite directions, with \(\tau^a\) changing in the same direction as \(V\) and \(\tau^u\) changing in the opposite direction. A higher initial value of the generalised mean tariff \(\overline{T}\) reduces the algebraic slope of both loci, causing the four regimes to pivot in a clockwise direction. A higher marginal propensity to spend on imports \(M_b\) leaves the slope of the iso-\(\tau^a\) locus unchanged but increases that of the iso-\(\tau^u\) locus, causing regions II and IV, in which the two uniform tariff indices move together, to expand at the expense of the ambiguous regions I and III.15 Note finally that a rise in \(\tau^a\) is equivalent to a rise in welfare and a rise in \(\tau^u\) is equivalent to a rise in import volume. Hence Figure 5 gives a complete characterisation of the effects on welfare and import volume of arbitrary changes in the generalised tariff moments.

7. Relating Generalised to Observable Moments

So far we have shown how changes in welfare, market access and trade restrictiveness can be expressed in terms of the generalised mean and variance of the tariff schedule. These two

\[15 \text{ From (53) and (59) the slope of the iso-}\tau^a\text{ locus is } -2\overline{T}, \text{ while the slope of the iso-}\tau^u\text{ locus is } 2[1-(1-M_b)T]/(1-M_b).\]
generalised moments serve in effect as sufficient statistics for the whole $n$-by-one vector of tariff rates. Of course, the generalised moments are not independent of the structure of the economy: on the contrary, they are defined in terms of the general-equilibrium substitution matrix. But there is clearly a huge economy of information from the fact that everything that is relevant to small changes in welfare and market access can be summarised in terms of changes in the two moments. However, there is no guarantee that the generalised moments are closely related to the standard moments which can be calculated using only information on the tariff schedule and the levels of imports. In this section, we show that the generalised moments coincide with the standard moments in a special but important case, where preferences take a homothetic constant-elasticity-of-substitution (CES) form, and where imports are imperfect substitutes for home-produced goods. The latter assumption follows Armington (1969) and is made in the vast majority of CGE models. Hence our result greatly enhances the usefulness of the generalised moments and the results based on them presented in previous sections.\footnote{Anderson (1995) showed that generalised moments defined with respect to world prices coincide with standard moments when the trade expenditure function is Cobb-Douglas.}

The trade expenditure function in this case can be written as follows:

$$E(\pi_0, \pi, u) = \left[ e(\pi_0, \pi, u) - g(\pi_0) \right] - u \Pi(\pi_0, \pi) - g(\pi_0). \quad (61)$$

Here we have made explicit the dependence of both the expenditure function $e$ and the GDP function $g$ on the prices of goods not subject to tariffs, which we can aggregate into a composite "numeraire" good with price $\pi_0$. Expenditure is linear in $u$ because preferences are homothetic. It depends on $\pi$, the prices of the $n$ goods subject to tariffs, in a separable manner, mediated through $\Pi$, which is a constant-elasticity-of-substitution aggregate price index defined over both
As for GDP, it is independent of \( \pi \) because of the assumption of non-competing imports, though it does depend on \( \pi_0 \).

We proceed to explore the implications of the specification in (61) for the elements of the normalised substitution matrix \( S \). Differentiating (61) with respect to \( \pi_i \) gives the Hicksian demand functions for all goods, which are also the import demand functions for goods 1 to \( n \):

\[
E_i = u\Pi_i = u\beta_i \left( \frac{\pi_i}{\Pi} \right)^{-\alpha} \quad i=0,1,...,n. \tag{63}
\]

Multiplying by prices and summing over all \( i \) gives total expenditure at domestic prices:

\[
e = \sum_{i=0}^{n} \pi_i E_i = u\Pi \left( \sum_{i=0}^{n} \beta_i \pi_i^{1-\alpha} \right)^{1-\alpha} = u\Pi. \tag{64}
\]

From this we can derive the share of good \( i \) in the value of imports:

\[
\phi_i = \frac{\pi_i m_i}{\pi m} = \frac{\pi_i E_i}{M} = \frac{\beta_i \left( \frac{\pi_i}{\Pi} \right)^{1-\alpha}}{\nu} \tag{65},
\]

where \( \nu \) is the share of imports in aggregate expenditure:

\[
\nu = \frac{M}{e} = \frac{\sum_{i=0}^{n} \pi_i E_i}{\sum_{i=0}^{n} \pi_i E_i} \tag{66}.
\]

Differentiating (63) now yields:
\[ \pi_i E_{ij} \pi_j = -\sigma M \phi_i (\delta_{ij} - \nu \phi_j), \quad (67) \]

where \( \delta_{ij} \) is the Kronecker delta (equal to one when \( i=j \) and zero otherwise). Recalling the definition of the elements of the normalised substitution matrix from (6), we can now express them as follows:

\[ S_{ij} = -\frac{\phi_i (\delta_{ij} - \nu \phi_j)}{1 - \nu}, \quad (68) \]

or in matrix form:

\[ S = -\frac{\phi - \phi v \phi'}{1 - \nu}. \quad (69) \]

Note that the row sums of the \( S \) matrix equal the import shares themselves:

\[ S t = \frac{\phi_1 - \phi v \phi'}{1 - \nu} = \phi, \quad (70) \]

making use of the fact that the import shares sum to one: \( t^t \phi = \phi t = 1 \).

We can now state the main result of this section:

*Proposition 14: When the trade expenditure function takes the form given by (61), the generalised average tariff is equal to the trade-weighted average tariff, and the generalised variance of tariffs is proportional to the trade-weighted variance, both evaluated at domestic prices.*

The proof is immediate. The generalised average tariff becomes:
\[ \bar{T} = \bar{t}'ST = \phi'T = \frac{t.m}{\pi.m}. \]  

which is simply the standard atheoretic trade-weighted average tariff at domestic prices. As for the generalised variance of tariffs, it becomes:

\[
V = (T-\bar{T})'S(T-\bar{T}) = T'ST - \bar{T}^2
- \frac{T' (\Phi - \Phi \phi') T}{1-\nu} - \bar{T}^2
- \frac{T' \phi T - \bar{T}^2}{1-\nu}.
\]  

The numerator of the final expression is simply the atheoretic trade-weighted variance of tariffs:

\[
\bar{V} = (T-\bar{T})' \phi(T-\bar{T}) = T' \phi T - \bar{T}^2.
\]  

This proves the proposition.

Q.E.D.

8. Conclusion

In this paper, we have developed a new approach to characterising the structure of tariff rates. Practical researchers have often attempted to summarise such structure in terms of the mean and variance of tariffs, but, to date, this approach has had no theoretical justification. Empirical measures which only use data on tariff levels and import shares fail to give an adequate picture because they ignore marginal responses. To deal with this problem, we drew on Anderson (1995) to introduce two generalised moments of the tariff structure. The generalisation involves weighting actual tariff rates by the elements of the substitution matrix, so the generalised
moments incorporate information on marginal responses by construction.

The first contribution of this paper was to show that the effects of tariff changes on welfare and import volume can be fully characterised by their effects on the generalised mean and variance of the tariff distribution. We were thus able to use these tools to derive new results for welfare- and market-access-improving tariff changes. These imply significant generalisations of existing results, and can be conveniently illustrated in terms of two "cones of liberalisation" in price space. The cones do not intersect, except along one boundary, because welfare is negatively but import volume positively related to the generalised variance. For practical policy-making, this suggests that negotiations on trade liberalisation face a difficult choice between tariff-cutting formulae which guarantee an improvement in domestic welfare and formulae which ensure an increase in market access (and so are likely to be acceptable to foreign exporters).

Another contribution of the paper was to relate changes in the TRI and MTRI to changes in the two generalised moments. We show that, under very general conditions, small changes in tariffs raise the TRI uniform tariff $\tau^\Delta$ by more than the MTRI uniform tariff $\tau^\mu$ if and only if the generalised variance of tariffs rises. Given the practical interest in measures of tariff dispersion, these techniques seem likely to prove useful in many other contexts.

Just like the TRI and MTRI, the generalised tariff moments depend on the general-equilibrium substitution matrix, which at best is observable with a lot of error. Hence applying them in practice requires care and judgement. Of course, if a computable general equilibrium model has been estimated for the economy, then they can be calculated explicitly. Alternatively, we can try to establish the properties of the generalised moments under special assumptions about the structure of the economy. This is the approach adopted in Section 7, where we showed that, if
the trade expenditure function takes a CES form, the weights reduce to trade weights and the
generalised moments are proportional to observable trade-weighted moments. The CES form is
extremely special of course. Nevertheless this result provides further insight into the role of
inter-commodity substitution in mediating the effects of tariff changes, and also provides a partial
justification for the practical use of trade-weighted tariff moments.
<table>
<thead>
<tr>
<th>Type of Tariff Change</th>
<th>Effect on $dt$ ($= d\pi$)</th>
<th>Effect on $dT$ ($= \pi^{-1} dt$)</th>
<th>Effect on $d\tau$ ($= \pi^* d\tau$)</th>
<th>Effect on $dT$</th>
<th>Effect on $dV$</th>
<th>Effect on $(\mu_s)^{-1} e, du$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform Proportionate Reduction</td>
<td>$-td\alpha$</td>
<td>$-Td\alpha$</td>
<td>$-\tau d\alpha$</td>
<td>$-T\tau d\alpha$</td>
<td>$-2Vd\alpha$</td>
<td>$(\tilde{T}^2 + V)d\alpha$</td>
</tr>
<tr>
<td>Uniform Absolute Reduction in $T$</td>
<td>$-\pi d\alpha$</td>
<td>$-\tau d\alpha$</td>
<td>$-(1+\tau)d\alpha$</td>
<td>$-d\alpha$</td>
<td>$0$</td>
<td>$T\alpha$</td>
</tr>
<tr>
<td>Uniform Absolute Reduction in $\tau$</td>
<td>$-\pi d\alpha$</td>
<td>$-(1-\tau)d\alpha$</td>
<td>$-\tau d\alpha$</td>
<td>$-(1-\tilde{T})d\alpha$</td>
<td>$2Vd\alpha$</td>
<td>$[\tilde{T}(1-\tilde{T})-V]d\alpha$</td>
</tr>
<tr>
<td>Uniform Contraction in $T$</td>
<td>$-(\tau-\pi\beta)d\alpha$</td>
<td>$-(\tau-\beta)d\alpha$</td>
<td>$-\tau(1+\tau)\beta d\alpha$</td>
<td>$-(\tilde{T}-\beta)d\alpha$</td>
<td>$-2Vd\alpha$</td>
<td>$[\tilde{T}(\tilde{T}-\beta)+V]d\alpha$</td>
</tr>
</tbody>
</table>

Table 1: Alternative Tariff Reduction Formulae
References


Figure 1: Uniform Proportionate and Uniform Absolute Changes in Tariffs

\[ d\pi = -\tau d\alpha \]

Figure 2: The Cone of Welfare-Increasing Liberalisation

\[ d\pi = -(1-\tau) d\alpha \]
Figure 3: Threshold Value of \( V \) above which a Uniform Proportionate Reduction in Tariffs Lowers Import Volume

Figure 4: The Cone of Import-Volume-Increasing Liberalisation

\[ \frac{dT}{dt} = (T - T^*) \, \text{d}t \]

\[ \frac{dT}{dt} = -(1 - M_c) T^* \, \text{d}t \]
Figure 5: Changes in the TRI and MTRI
Expressed in Terms of Changes in Generalised Tariff Moments