Signal Accuracy
and
Informational Cascades

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Abstract

We extend the Bikhchandani, Hirshleifer and Welch (1992) informational cascade framework to allow for asymmetric signal accuracy. Simulations demonstrate that even small departures from symmetry may lead to non-monotonic effects of signal accuracy on the likelihood of an inefficient cascade.

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I. Introduction

Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992), henceforth BHW, show that it may be optimal for a rational agent to herd, i.e. to follow the actions of his predecessors when his own private signal suggests the opposite. If early movers’ signals are incorrect, followers will be mislead, yielding an inefficient informational cascade. This paper studies the effect of signal accuracy on the probability of an inefficient cascade.

Analysis of factors that affect the likelihood of an inefficient cascade may be of interest in helping to reduce the probability of such events. For instance, the 1933 Securities Act and 1934 Securities Exchange Act were enacted to prevent crashes like Black Thursday in 1929. They require reporting of financial information concerning traded securities to improve signal accuracy. Depending on the market, signal accuracy may be affected by factors such as accounting standards, technological advancement in information dissemination and advertising.

In BHW, signals have symmetric accuracy in both states. An increase in signal accuracy leads to a decrease in the probability of inefficient herding since early movers are more likely to take the correct action. We extend the BHW framework to allow for asymmetric signal accuracy. The signal need not have the same accuracy in high and low states. For instance, a good candidate may come to a job interview on time with 95% probability and a bad candidate may be on time with 85% probability. As long as the probabilities are different, promptness may be a useful signal of candidate quality. The symmetric case restricts the probability of the bad candidate sending the correct signal (being late) to 95%. We show that even small departures from symmetry may lead to
non-monotonic results. An increase in signal accuracy may result in a higher likelihood of an inefficient cascade.

II. Symmetric Accuracy

The following is equivalent to the BHW framework: Each risk-neutral agent chooses between two investment projects. The risky project yields either 1 or 0 and the safe project yields \( \frac{1}{2} \). The payoff matrix is:

<table>
<thead>
<tr>
<th></th>
<th>Risky</th>
<th>Safe</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>1</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>Low</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

Prob(High State)=0.5

Each agent receives a private, conditionally independent signal about the value of the risky project, either \( h \) or \( \ell \). The signal is correct with probability \( p \). The sequence of moves is predetermined and agents observe the actions of those ahead of them. Agents follow Bayes’ Rule. When indifferent an agent randomizes with even probabilities.

Equivalently, there are two urns; H and L. Each urn has some balls marked \( h \) and some marked \( \ell \). Urn L has a higher percentage of balls marked \( \ell \) than urn H. The percentile of correct balls in each urn, \( p_h \) and \( p_{\ell} \), is symmetric. Nature draws one urn with even probabilities. All agents privately draw one ball with replacement from the same urn. The agent’s problem is to determine which urn the ball comes from.

BHW show that at some point public information overwhelms the informational content of a single signal. If most early agents happened to receive signal \( \ell \), all newcomers may choose L even when the state is H. An L cascade when the true state is H is called an inefficient negative cascade. An H cascade when the true state is L is an inefficient positive cascade. The inefficient cascade probability is given by the probability of these weighted by the \textit{ex ante} probabilities of states H and L.
When signal accuracy is symmetric the recursive nature of the framework allows closed-form solutions for the probabilities. When we depart from symmetry the recursive nature breaks down. All simulations use 10 million runs per data point. In all cases 99% confidence intervals are less than the width of the symbols used to represent data points.

Figure 1 summarizes our replication of BHW’s results. An increase in signal accuracy always leads to a decrease in the probability of inefficient herding.
III. Asymmetric Accuracy

The BHW framework is symmetric because the signal has the same accuracy in both states and the probability of each state is even. However even small departures from symmetry of signal accuracy across states may lead to violation of the monotonicity result.

BHW’s Result 1 is closely related. It shows that, in a symmetric setting, all agents after the second are better off when the first agent’s signal accuracy (expertise) is slightly decreased. This results in more information for later individuals. Here all agents have the same expertise but right around the point of symmetry of signal accuracy across states, we will have a similar story. Building on the intuition gained from this we analyze cases where the probability of inefficient cascades is non-monotonic in signal accuracy.

Asymmetric signal accuracy translates into asymmetric percentile of correct balls in each urn. For the signals to be informative $p_h l-p_h$. Without loss of generality take $p_h p_h>1$. Increasing either $p_r$ or $p_h$ increases the informativeness of the signal in the Blackwell sense.

Figure 2 reports simulation results right around the point of symmetry for signal accuracy in state H fixed at 70% and varying the accuracy in state L. Fixing $p_h$ at different levels does not change the spirit of the results. The payoff matrix and probability of state H are as in Table 1.

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See Anderson and Holt (1997) for experiments using asymmetric accuracies.
The probability of an inefficient cascade is monotonic. However, the probability of an inefficient positive cascade (“Prob. H when L”) is not. It decreases with an improvement in signal accuracy until the point of symmetry. Then it jumps from 0.12 to 0.38. The probability of an inefficient negative cascade jumps down.

When $p_h$ and $p_r$ are symmetric, if the first and second signals are different the second simply cancels the first since they have equal accuracy. When there is asymmetry, however slight, signal $h$ and signal $r$ do not cancel each other out because they have different weights in the updating process. Therefore herding can start earlier.
An agent receiving signal $h$ updates his belief that the state is high from 0.5 to
\[
\text{Prob}(H|h) = \frac{p_h}{1 + p_h - p_r}.
\]
An agent receiving signal $\ell$ updates his belief that the state is low from 0.5 to
\[
\text{Prob}(L|\ell) = \frac{p_r}{1 + p_r - p_h}.
\]
As long as $p_r < p_h$, $\text{Prob}(L|\ell) > \text{Prob}(H|h)$.

• When $p_r$ is just below $p_h$, the second agent always herds when the first agent
  chooses $L$. Hence the probability of an inefficient $L$ cascade is high.
• When $p_r$ is just above $p_h$, the second agent always herds when the first agent
  chooses $H$. Hence the probability of an inefficient $H$ cascade is high.

And at the point of symmetry the inefficient negative and positive cascade probabilities
are equal. Therefore the inefficient positive cascade probability jumps up and the
inefficient negative cascade probability jumps down.

While our primary purpose is to analyze the probability of an inefficient cascade,
in many markets a planner may place greater weight on inefficient negative cascades than
on inefficient positive cascades due to externalities from the market to society at large;
Bank panics, capital flight and market crashes may have drastic external consequences.
In the IPO market, however, companies may simply try to increase the probability of a
positive cascade.

3.2. Inefficient Cascade Probability

The inefficient cascade probability may be non-monotonic in signal accuracy with
uneven ex ante probabilities. Here is a payoff matrix with a riskier project but the same
expected value:

<table>
<thead>
<tr>
<th></th>
<th>Risky</th>
<th>Safe</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>2</td>
<td>1/2</td>
</tr>
<tr>
<td>Low</td>
<td>0</td>
<td>1/2</td>
</tr>
</tbody>
</table>

\[
\text{Prob}(\text{High State}) = 0.25
\]

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$^3$An agent receiving signal $h$ updates his belief that the state is high from 0.5 to
\[
\text{Prob}(H|h) = \frac{p_h}{1 + p_h - p_r}.
\]
An agent receiving signal $\ell$ updates his belief that the state is low from 0.5 to
\[
\text{Prob}(L|\ell) = \frac{p_r}{1 + p_r - p_h}.
\]
As long as $p_r < p_h$, $\text{Prob}(L|\ell) > \text{Prob}(H|h)$. 

- 6 -
$p_h = p_l$ yields the same monotonicity result as in BHW. Now fix the signal accuracy in state H, but vary the signal accuracy in state L. Figure 3 summarizes the simulation results for $p_h = 0.7$. Fixing $p_h$ at other levels does not change the qualitative results. The probability of an inefficient cascade is non-monotonic in signal accuracy. It jumps up at three levels: At 0.505, at 0.7 (the point of symmetry), and at 0.9275.

![Figure 3](image.png)

The jumps are due to the binary nature of the problem. The agent decides whether to follow his own signal or not. As signal accuracy improves, the expected value of each of these options changes continuously, but the agent’s decision switches between them in a discrete jump.
The point of symmetry has the same incentives as before. However, since the *ex ante* probability of L is now 0.75, the positive cascade probability has a higher weight in the *ex ante* inefficient cascade probability.

Examine the jump at 0.505. When \( p_r = 0.5 \), just below 0.505, consider the sequence of actions: H,L,H,H. At this level of signal accuracy these actions reflect the private signals. Having observed this sequence, it is optimal for an agent to follow his own signal. But when \( p_r = 0.51 \), just above 0.505, having observed the same sequence (at this level of accuracy the actions still reflect the private signals) it is optimal for the agent to herd to H. Therefore, just past \( p_r = 0.505 \) the probability of an inefficient H cascade jumps up. And the probability of an inefficient L cascade jumps down. The weighted average, the probability of an inefficient cascade, jumps up from 0.32 to 0.335.

Many alternative sequences of signals could occur before herding starts. The discontinuities in the probabilities arise at points where small changes in parameters switch agents in some sequence from one action to the other. The size of the discontinuity is related to the likelihood of that sequence.

3.3. Changing Both Signal Accuracies

This exercise is in the same spirit as in BHW where a single parameter represents the signal accuracy in both states. Figure 4 gives results for payoff Table 2, starting with asymmetric signal accuracies \( p_h = 0.5 \) and \( p_r = 0.8 \). Then both accuracies are changed together. The effect of changes in signal accuracies on the probability of an inefficient cascade is non-monotonic.
IV. An Example

Consider an example from the labor market. The safe alternative is to hire an adjunct professor with payoff \( \frac{1}{2} \). The risky alternative is to hire a tenure-track professor with payoff of either 2 or 0 (Table 2). The candidate for the tenure-track position presents himself in private office meetings to each hiring committee member. Committee members then vote sequentially.\(^4\)

A good candidate has a higher probability of successfully presenting himself than a bad candidate. Now imagine that schools stop training bad candidates for presentation skills. This leads to a decline in the probability of bad candidates successfully presenting themselves (signal accuracy in the Low state increases).

There are two forces at work. Observing a good presentation by a bad candidate is now less likely. But if the first voter happens to have seen a good presentation, herding may start early since all followers would put more weight on that good report. This tends to increase in the probability of hiring a bad candidate. The second effect may overwhelm the first depending on the initial levels of signal accuracy (Figure 3).
References