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Nonlinearity as an Explanation of the Forward Exchange Rate Anomaly

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Abstract

This paper shows that nonlinearity can provide an explanation for the forward exchange rate anomaly (Fama, 1984). Using sterling-Canadian dollar data, and modelling nonlinearity of unspecified form by means of a random field, we find strong evidence of time-wise nonlinearity and, significantly, obtain parameter estimates that conform with theory to a high degree of precision: the anomaly disappears.

Keywords: Forward exchange rate anomaly; nonlinearity; random field regression.

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1 Introduction

The fact that the forward exchange rate does not provide an unbiased forecast of the future spot exchange rate has generated an extensive literature since the seminal paper of Fama (1984); see the surveys of Hodrick (1987) and Engel (1996). Some recent work, such as that by Gil-Alana (2002), has suggested that this conflict with theory may be due to fractional integration of the forward rate; see also Baillie and Bollerslev (2000) and Maynard and Phillips (2001). However, the findings of Maynard (2006), using tests robust to persistence in conditioning variables, suggest that a substantial economic puzzle remains. In light of this, another approach that focuses on nonlinearity as a way of explaining the anomaly is gathering interest.

Nonlinearity may arise in exchange rate data for several economic reasons, including transactions costs, central bank interventions and the existence of limits to speculation; see Taylor (2006). Markov-switching models have been used in an attempt to handle such nonlinearity (Engel and Hamilton, 1990) but smooth transition autoregression is now more popular; see, for example, Sarno et al. (2004) and Baillie and Kiliç (2005). The aim of this paper is to report on the use of random field regression (Hamilton, 2001), which does not require the specification of a particular nonlinear functional form nor the choice of any transition variable. The outline of the paper is as follows: Section 2 summarizes the models and methodology used; Section 3 describes the data and results; and Section 4 concludes that in the case considered, the anomaly disappears.

2 Models and methodology

The equations commonly used to test the forward rate unbiasedness (FRU) hypothesis are

$$\Delta_k s_{t+k} = \alpha_1 + \beta_1(f_{t,k} - s_t) + u_{1,t+k} \quad (1)$$

and

$$s_{t+k} = \alpha_2 + \beta_2 f_{t,k} + u_{2,t+k}, \quad (2)$$

where s_t and $f_{t,k}$ are the (log) spot and forward exchange rates, respectively, at time t , k is the length of the forward contract, Δ_k is the k -period change, and the $u_{i,t+k}$, $i = 1, 2$, are hypothesized white noise disturbance terms. Theoretically, $\alpha_i = 0$ and $\beta_i = 1$, $i = 1, 2$, but numerous empirical studies, based on a wide variety of exchange rates and time periods, have generally failed to corroborate this.

The approach here is to allow for the possibility of nonlinearity over time by re-specifying (1) and (2) as

$$\Delta_k s_{t+k} = \alpha_1 + \beta_1(f_{t,k} - s_t) + \gamma_1 t + u_{1,t+k} \quad (3)$$

and

$$s_{t+k} = \alpha_2 + \beta_2 f_{t,k} + \gamma_2 t + u_{2,t+k}, \quad (4)$$

and the disturbances as

$$u_{i,t+k} = \lambda_i m(\bar{\mathbf{x}}_{i,t}) + \epsilon_{i,t+k}, \quad (5)$$

where, for $i = 1, 2$, $m(\bar{\mathbf{x}}_{i,t})$ is a random field intended to capture any nonlinearity, λ_i is a parameter that measures the “weight” of the nonlinearity in the specification, $\bar{\mathbf{x}}_{i,t} = \mathbf{x}_{i,t} \odot \mathbf{g}_i$ is the Hadamard product of the 2-vector of explanatory variables, $\mathbf{x}_{i,t}$, and an associated vector of parameters, \mathbf{g}_i , and $\epsilon_{i,t+k} \sim N(0, \sigma^2)$. The individual elements of \mathbf{g}_i , g_{i1} and g_{i2} , indicate the extent to which each explanatory variable contributes to any nonlinearity, g_{i2} being associated with the time variable for both $i = 1$ and $i = 2$. As before, $\alpha_i = 0$ and $\beta_i = 1$ under FRU theory, and $\gamma_i = 0$, $i = 1, 2$, is also expected.

Adopting a Gaussian random field and the Hamilton (2001) specification of its variance-covariance matrix, (3) and (4) may be estimated by maximum likelihood using standard algorithms. A simple check for nonlinearity in each equation is also available by testing the null hypothesis $H_0 : \lambda_i = 0$, $i = 1, 2$, using the Lagrange multiplier principle. Details of the estimation and testing procedures are given by Hamilton (2001), and summarized in Bond et al. (2005).

3 Data and results

Following Gil-Alana (2002), the case of the Canadian dollar was chosen to explore the role of nonlinearity in the forward rate anomaly. Weekly series of sterling-Canadian dollar rates for the period December 1994 to June 2005 were used, the source being *Thomson Financial Datastream*. The computations were done using the Hamilton (2001) GAUSS code and the algorithm-switching optimization procedure proposed by Bond et al. (2005).

The results from the random field regression analysis are given in Table 1, where the figures for ζ are estimates of $\frac{\lambda}{\sigma}$.

Table 1: Random field estimates

	linear				nonlinear			
Equation (3)	c	$f_{t,k} - s_t$	t	σ	ζ	$f_{t,k} - s_t$	t	
	2.741 (6.090)	1.305 (0.578)	-0.246 (2.144)	0.128 (0.967)	84.867 (639.699)	-0.014 (0.036)	12.893 (0.599)	
Equation (4)	c	$f_{t,k}$	t	σ	ζ	$f_{t,k}$	t	
	-0.001 (0.002)	1.004 (0.004)	0.0002 (0.0003)	0.001 (0.00002)	1.974 (0.245)	-0.0001 (0.064)	1.169 (0.102)	

Note: c denotes the dummy unity associated with the intercepts in the equations. Estimated standard errors are given in parentheses.

Estimating the models without constant terms produced very similar results to those in Table 1. There is overwhelming evidence of nonlinearity, with the Hamilton Lagrange multiplier test statistics for the two equations being 381.46 and 5925.76, far in excess of the 5 per cent χ_1^2 critical value of 3.84. Moreover, the estimates show that the nonlinearity is associated with the time variable, which has a statistically significant coefficient in the nonlinear component of both models. The high significance of the σ and ζ estimates in the exchange rates equation, (4), and the contrasting lack of significance of these estimates in the premium equation, (3), may reasonably be assumed to stem from what, in the time series literature, is known as the ‘‘pile up’’ phenomenon; see Hamilton (2005).

The most significant aspect of these results is that when nonlinearity is modelled with a random field, the intercept in the exchange rates equation is not significantly

different from zero and the slope coefficient is estimated, with great precision, to be unity, in accordance with exchange rate theory. Similarly, in the exchange premium equation the intercept and slope are not significantly different from zero and unity, respectively, though as the estimated standard errors are larger in this case, the result is not quite as striking. The coefficient γ_i is also estimated to be statistically insignificant in both equations, as expected.

4 Concluding remarks

In this paper we have investigated nonlinearity as a possible explanation of the well-known foreign exchange rate anomaly. Using the Hamilton (2001) approach to nonlinear inference and sterling-Canadian dollar data, we find strong evidence of time-dependent nonlinearity and that when the nonlinearity is modelled by means of a random field, exchange rate theory is confirmed and the anomaly removed. This result provides an alternative explanation of the rejection of the hypothesis of unbiasedness of the forward rate as a predictor of the future spot rate to that of fractional integration of the forward rate proposed by Gil-Alana (2002), and unit roots as suggested in earlier work such as that by Crowder (1995) and Hai et al. (1997). It therefore adds weight to the earlier work on the relevance of nonlinearity or parameter instability to the forward anomaly debate, and may have some contribution to make to the puzzle that Maynard (2006) suggests remains.

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