4

Core Themes in the Teaching and Learning of Mathematics

Introduction

This chapter is the first of five chapters examining the mathematics classroom. The focus here is on the common themes in mathematics lessons, in particular on classroom interactions in terms of dominant pedagogical practices.

Chapter 5 elaborates on the core themes investigated in Chapter 4, presenting a more in-depth analysis of the dynamics of classroom interactions. The subsequent two chapters (6 and 7) examine differences in pedagogical practices in the teaching of mathematics arising from gender, social class and grouping practices. Throughout the discussion, the analysis is confined to the public interaction that took place in the lessons between the teacher and the students.

Overall, the findings suggest a high level of uniformity in terms of how mathematics lessons are organised and presented in second level schools. The majority of the lessons largely comprised teacher demonstration and student practice (Table 4.1). Demonstration of mathematics consisted of a combination of verbal explanations and written demonstration, typically using either a blackboard or whiteboard. In one of the ten schools, Nore (SSG Fr), the teacher used an overhead projector with pre-prepared overheads. In most cases, demonstration was based on exercises or examples from the textbook and included teacher-led question-and-answer sessions and reviews. In Table 4.1, revision appears as a separate category from demonstration. In reality, the revision lessons mostly involved demonstration and the demonstration practices observed were identical to those observed in the other lessons where new material was presented. For clarification purposes, the categories are separated to distinguish new from revision material.

Student practice consisted of students practising the methods demonstrated by the teacher. Typically this work was set by the teacher (in the

Table 4.1: Time allocated to pedagogical activities in mathematics lessons

School	Lesson Stages and Activities							
	Lesson number	Lesson aims	Demonstration (new material)		Demonstration (revision material)	Student practice	Setting homework	Total
			Total	(Of which teacher-explanation represents)				
		%	%	%	%	%	%	%
Barrow (SSG F)	1	1.3	62.8	(11.1)	0	35.9	0	100
	2	1.0	26.4	(6.6)	0	72.6	0	100
Nore (SSG Fr)	1	2.2	88.8	(26.1)	0.8	6.7	1.5	100
	2	1.7	50.8	(23.7)	1.7	45.8	0	100
Suir (SSG Fr D)	1	2.0	0	(10.0)	41.0	54.0	3.0	100
	2	1.8	0	(9.9)	33.3	64.0	0.9	100
Liffey (SSB Fr D)	1	0.8	38.0	(15.5)	4.6	54.3	2.3	100
	2	0	29.7	(3.0)	5.0	58.4	6.9	100
Lee (SSB F)	1	0.8	0	(0)	0	87.3	11.9	100
	2	0	0	(0)	0	73.4	26.6	100
Lagan (SC Fr)	1	0.6	78.8	(6.8)	0	18.5	2.1	100
	2	0	47.1	(3.7)	0	52.2	0.7	100
Errigal (CS Fr)	1	0	62.2	(5.4)	0	33.3	4.5	100
	2	0	17.8	(0)	0	82.2	0	100
Mourne (VCC Fr D)	1	0	0	(0)	0	97.6	2.4	100
	2	2.2	17.0	(3.7)	0	76.4	4.4	100
Blackstairs (VCC Fr D)	1	4.4	41.7	(16.2)	5.2	47.2	1.5	100
	2	1.6	46.8	(13.7)	3.2	46.8	1.6	100
Nephin (VCC Fr)	1	0	0	(11.5)	58.2	40.2	1.6	100
	2	0	2.4	(0)	0	94.0	3.6	100

Key: SSG F: Secondary girls', fee-paying; SSG Fr: Secondary girls', free scheme; SSG Fr D: Secondary girls', free scheme designated disadvantaged; SSB Fr D: Secondary boys', free scheme designated disadvantaged; SSB F: Secondary boys', fee paying; SC Fr: Secondary coed, free scheme; CS Fr: Community school, free scheme; VCC Fr D: Vocational/Community college, free scheme designated disadvantaged; VCC Fr: Vocational/Community college, free scheme (where free scheme represents non fee-paying status).

majority of cases from the textbook) and was completed by the students either during the course of the lesson (student work), or at home (homework). Practice of demonstrated methods was conducted individually by the students in their exercise books. In a number of lessons, some interaction between students was evident; this was on an informal basis and was usually confined to students sitting next to each other and conducted quietly in hushed tones. In the lessons where there was evidence of students conferring with each other, the teacher did not give instructions that the students were to work in this way. Instead, there appeared to be a general understanding that this was an acceptable way of working; peer discussion and co-operation was allowed rather than encouraged. The correction of student work and homework was also included in the student practice phase of the lesson script.

The observed lessons also included the statement of *lesson aims* and a segment concerned with the setting of homework, although these were a minor part of the lesson.

What is evident from Table 4.1 is the concentration of class time on the two interrelated activities of teacher demonstration (including revision) and student practice of demonstrated skills. With the exception of the two classes observed in Lee (SSB F), where just over one quarter of one class, and almost one eighth of another was spent setting homework, well over 90 per cent of class time in all other eighteen classes was spent on demonstration and student practice. The balance of the time devoted between these two activities varied depending on the subject matter and the decision of the teacher in a given context. While there were more classes in which student practice was the prevailing work norm,[1] there was also a significant number of classes in which demonstration by the teacher of particular skills was the norm, or in which there was a relatively even balance between the two sets of activities.

Demonstration

Demonstration of mathematical procedures by the teacher was the dominant pedagogical practice of teachers. Although the written demonstration of the method outweighed the verbal explanation for all of the lessons observed, there was considerable variation in terms of content, duration, pace, amount of teacher questioning, and style of questioning used. To a large degree, the demonstration varied according to the ranking of the class grouping in terms of mathematical achievement, particularly with regard to content and pace.

Two illustrative examples of a demonstration are now presented. The first lesson involves a top stream group from a single sex girls' school while the second lesson involves a bottom track group from a coeducational school. Despite great differences in terms of the mathematical attainments of the students, the pedagogical approach is remarkably similar.

A top stream group in Barrow (SSG F)

The first extract is from the first lesson observed in a fee paying, single sex girls' secondary school (Barrow (SSG F)). It involved a demonstration by a very experienced female teacher to a top-stream group. The students were overwhelmingly from a middle class background and achieved a high mean score (29.8 out of a possible 40) on the TIMSS-based mathematics test.

In the course of this particular lesson the teacher covered the factorisation of quadratic expressions with mixed signs, where the coefficient of x^2 is equal to 1 (phase 1) and then where the coefficient of x^2 is greater than 1 (phase 2). Both phases comprised a combination of demonstration and student practice, involving seven and five examples respectively. In her explanation, the teacher used mathematical language and reminded students that they had employed an identical method in a previous lesson. Although aimed at a high stream class, the lesson comprised mostly drill and practice of a particular method, with little explanation of the concepts involved.

After stating the lesson aims, the teacher demonstrated one method of factoring quadratic expressions with mixed signs, namely the trial and error method. Starting with the simplest example of an expression with the coefficient of x equal to 1, the teacher worked through an example on a line by line basis. The demonstration was interspersed with questioning of the whole class and individual students, as can be seen by the following excerpt.

Demonstration: Illustrative Vignette 1

Time code (minutes:seconds)	**Activity**	**Lesson text**
8:12	Lesson aim	*Teacher:* All right girls, just before half term we were learning to factorise. We were working with quadratic equations – quadratic statements first of all – where the coefficient of x^2 was 1 and where we had all plus signs in the expression – so today we are going to work with quadratic expressions with mixed signs.

8:43	Demonstration begins: Example 1: Solving quadratic equation with mixed signs Tricks	*Teacher goes to the board*: *Teacher:* So for a very simple example, *and writes*: $\mathbf{x^2 - 5x + 6}$ *[Text and Tests 1, p.218 question 4]* *Teacher:* We have certain little tricks for figuring out how we will get this pair of factors that will multiply together to give us this here as our answer (*pointing to* $x^2 - 5x + 6$). The last day we just made 2 little brackets to start with – that was the trial and error method **( ) ( )** *Teacher:* And because this is such a simple example we will try and adopt that approach today.
9:20	Whole class question about method already covered, asking for show of hands	*Teacher:* Can you try and remember what we did with the 2 brackets, to start with – what do we put at the beginning of each bracket? *Hands go up and teacher asks one of these students*
9:25	Catherine answers correctly.	*Catherine:* x
9:25	Teacher explains why this is correct procedure.	*Teacher writes*: **(x) (x)** *Teacher:* The reason we are going to have x at the beginning is that x multiplied by x is going to give me x^2. So the next number that we concentrate on is the +6, the last number – the fact that the sign is plus means that the number we are looking for – in other words the two numbers multiplied together will carry the same sign. Now we have got to say to ourselves – is it going to be 2 pluses or 2 minuses. Now comes the little key. Whatever sign appears in front of the middle number, *[pointing to –5x]* that is the sign that both of the numbers will carry.
9:30	Teacher asks whole class question.	*Teacher:* So what numbers are we going to have here?
9:32	Most of the students answer question, giving correct answer.	*Class:* Minus. *Teacher:* Yes *and writes:* **(x –) (x –)** *Teacher:* And the very last thing that we must think about is – these 2 numbers are restricted in 2 ways – number 1, they must multiply out to give you plus 6 and number 2, they must add together – combine together – to give you minus 5.

10:26	Whole class question asking for show of hands.	*Teacher:* Now who can think of 2 such numbers – whenwe put them together – when they combine together (I hesitate to use the word ‘add’ because people get a bit mixed up) – can you think of 2 numbers whose product is plus 6 and whose sum is minus 5? *Show of hands by students* *Teacher:* OK, Gemma can you think of two?
10:42	Gemma answers incorrectly.	*Gemma:* Plus one and plus five.
10:50	Teacher responds to Gemma but then moves on to another student.	*Teacher:* I don’t think that would work, would it? A plus 1 multiplied by a plus 5 would give me what, when I multiply what would I get? *Gemma:* a plus *Teacher:* Well yeah, it doesn’t matter – a minus 5 multiplied by a minus 1 would give me a plus 5 – yeah OK, but I am looking for a plus 6 am I not?
11:10	Whole class question	*Teacher:* Can you think of anything better? Clare?
11:15	Clare volunteers answer.	*Clare:* 2 and 3
11:17	Teacher accepts answer and then demonstrates why this is correct	*Teacher:* OK, 2 and 3. So let’s multiply them out together to see what we get **2** **3** and what sort of sign – let’s test them out and see.
11:20	Teacher asks whole class question but gets no response. Teacher then answers question and continues with explanation.	*Teacher:* OK, if the numbers are going to be 2 and 3 they must be what sorts of numbers? *No response from class.* *Teacher:* Negative numbers, mustn’t they, minus numbers? When I multiply them let me check **–2** **–3** *Teacher:* Does that give me plus 6? *Class and teacher:* Yes, *in unison*. *Teacher:* And when I put them down and combine them together **(–2) + (–3) =** *Teacher:* Imagine I am walking on the number line.
11:38	Teacher asks Ashling a question.	*Teacher:* What about a minus 2 and a minus 3 – Ashling, when I put them together what do I get?
11:40	Ashling answers question correctly.	*Ashling:* Minus 5.

11:42	Teacher acknowledges Ashling's correct answer with praise, continues with rhetorical whole class question about method.	*Teacher:* Yeah, good girl Ashling! When we are doing these sums one after another we won't go through all of this detail will we? *Class:* No! *Teacher:* We won't put them in our copybook like this but just to start off with we must be careful **(x – 3) (x – 2)** *Teacher:* Now of course if we are really diligent, how can we check to make sure that we are right particularly the day of the exam – we won't just leave the answer and say 'hope for the best' would we – what would we do?
12:08	Teacher asks Niamh how to check if answer is correct.	*Teacher:* What might we do – Niamh?
12:10	Niamh answers question correctly	*Niamh:* multiply the two of them out. *Teacher:* That's right! So let's multiply them quickly – not actually – but doing it (*pointing to her head)* – let's multiply them mentally.
12:30	Teacher directs class to check that this is correct answer by multiplying mentally. Teacher then calls out steps and most of the students join in with her. Teacher then asks whole class if they are beginning to understand.	*Teacher:* x by x is grand – it gives us $x^{2,}$ doesn't it? Minus 3 by minus 2 is okay – it gives us plus 6. Now the combination is what people get stuck on – now the x by the minus 2 is minus 2x and plus x by the minus 3 is? *Class and teacher in unison:* Minus 5x. *Teacher:* Now are you beginning to understand? *Class in unison:* Yes!

Having completed this example, the teacher worked through four more questions (see below) using this methodology, this time using more whole class questioning and less explanation.

Example Number	Description	Problem	Time (mins.)
2	All negative signs	$x^2 – x – 6$	2.75
3	Mixed signs	$x^2 – 10x + 21$	3.00
4	All negative signs	$x^2 – 2x – 63$	2.50
5	Mixed signs	$x^2 – 17x + 30$	2.25

Overall, the demonstration lasted 14.25 minutes, amounting to approximately one third of total class time. The vignette clearly illustrates the didactic approach to teaching mathematics that was evident in all of the lessons in our study. The pedagogical approach is clearly that of drill and practice in a controlled and organised learning environment. In the course of the demonstration the teacher led the students through five examples alternating between problems with mixed and all negative signs.

For the first four examples, the teacher led the demonstration; here student involvement amounted to answering questions on the particular steps of the procedure. For the fifth example, the teacher requested a volunteer to do the problem by asking the class: 'Who would be able to do this one?' and then selected one student from all those who raised their hands. While she called out the steps of the procedure the teacher prompted her, praising her when she answered correctly: 'Good girl'; 'yes, exactly'; 'yeah, now isn't she right?') and gently reprimanding her when she made a mistake: 'Now think carefully', 'shh – now take your time and think – don't take what you think would be the obvious pair'. On completion, she praised her, saying: 'I think you are very good altogether now'. In all, this last example took approximately half a minute to complete.

In terms of mathematical content, the teacher covered more material than was the case in any of the other lessons. In terms of pace, the lesson moved quickly through the demonstration and onto the student practice aspect of the lesson. The teacher had the complete attention of the class and there were no instances of disruption throughout the lesson.

The fast pace of the demonstration conveys the teacher's confidence in the ability of all the students to understand the material; teaching is directed towards the whole class with no attention being directed to individual students who appeared to have difficulty following the procedure.

A bottom stream class in Mourne (VCC Fr D)

The second extract (see Appendix 4, Illustrative Vignette) involves a designated disadvantaged, non-fee paying (so-called free scheme), coeducational community college (Mourne VCC Fr D, first lesson). It involves a demonstration by a very experienced female teacher to a bottom track group. Two thirds of the students are from a working-class background and achieved a low mean score (13.3 out of 40) on the TIMSS-based mathematics test.

In this extract, the teacher is demonstrating how to factorise quadratic expressions with plus signs. The teacher has been working on grouping expressions with minus signs for a number of lessons. This lesson began with the correction of homework (see Illustrative Vignette 2 in the next section on Student Practice). The teacher indicated that she 'wants to show them today how to get the next lot of factors'. She then re-stated what factors are: 'factors are two things that multiply for your answer'. Following this, the teacher described the next step as follows:

> So you remember when we were learning factors we first of all showed you how to get it by multiplying out – so I want … everybody to multiply **x** plus **7** by **x** plus **8** – and then we are going to try to factorise something by going back.

The teacher then walked around checking whether the students remembered the method. When they did not look like they knew what to do she gave a hint by writing on the board:

$$\mathbf{x(x + 8) + 7(x + 8)}$$

Giving the students another couple of minutes to complete the multiplication, she demonstrated the correct approach by writing on the board:

$$\mathbf{x^2 + 8x + 7x + 56}$$
$$\mathbf{= x^2 + 15x + 56}$$

The teacher instructed the students that she wanted them to practice multiplying out another expression:

> We want to see how you will end up with something like this [*pointing to* $\mathbf{x^2 + 15x + 56}$] – in other words, how do you go backwards – if you start with that – how you end up back there – so I want you to multiply **(x+5) (x + 4)**.

In this lesson the teacher demonstrated how to factorise trinomial expressions with all plus signs. Following the introduction described above, the teacher presented a particular method for factorising trinomials with all plus signs as follows:

Factorise: $x^2 + 15x + 56$
Explanation: We want to end up with (x + 7) (x + 8). The middle term is the one we split up (15x).
Rule: The factors we come up with must multiply together to give 56 (third term) and add together to give 15x (middle term).
Step 1: Factorise the x^2 ***(x + ?) (x + ?)***

Step 2: Factorise the 56 in terms of the rule. Because the middle term is 15x we require the factors that ADD UP to +15, i.e., 7 and 8.

(x + 7) (x + 8)
Step 3: Check if this agrees with rule – 7 times 8 = 56 and (+7x) + (+8x) = +15x

As the excerpt indicates, the teacher did not attempt to make a connection with other mathematical topics. Her introduction involved writing the trinomial expression on the chalkboard, saying: 'Now we want to find the factors for this.' The key to understanding the method was understanding and memorising a particular rule.

The demonstration involved three identical examples and the teacher used a combination of whole class and individual questioning to work through the explanation. Overall the demonstration lasted 5.75 minutes or seventeen per cent of lesson time. Similar to the previous example, this lesson comprised drill and practice of a particular method, rather than the explanation of the concepts involved. Use was made of mathematical terminology to explain the difference between quadratic expressions with four terms, and quadratic trinomials with three terms. The teacher's tone was positive, she provided prompts when students were unsure of what to do next and she praised students when they get the 'correct' answer: 'Good girl Majella'; good lad'; 'very good'.

Student Practice

Practice of the demonstrated procedures by the students was the second prevailing pedagogical practice in the lessons observed. It included the doing and correcting of problems assigned to the class by the teacher – either during class time (student work*)* or at home (homework*)*. The two extracts that follow are from the same lessons referred to in the previous section on demonstration.

The first extract is from Barrow (SSG F). The students are learning how to factorise quadratic expressions using the trial and error method. Following the demonstration, the teacher moved the lesson on to the student practice phase of the lesson, saying: 'Now would you like to do one for me and see who can get it done first'. Students were assigned two problems, both with all negative signs. Following the correction of these problems the teacher moved on to solving quadratic equations where the coefficient of x was greater than 1.

Student Practice (Student Work: Doing): Illustrative Vignette 1

Time code (minutes:seconds)	**Activity**	**Lesson text**
22:12	Student work: <u>Example 6:</u> All negative signs	*Teacher:* Will you do number 26 for me now – I'll put it on the board so that I can do it – and you tell me what the answer is, *and writes:* **$x^2 - 11x - 42$** *Students are given approximately half a minute to do the question in their exercise books and then the teacher indicates by nodding to a particular student that she wants her to call out the answer.*
22:44	Student work: correction whole class. Teacher asks Sheila question.	*Teacher:* Yes – Sheila?
22:46	Sheila responds.	*Sheila:* Will I do it now? *Teacher:* Yes – tell me what it is and then we can decide. *Sheila:* Ummh, x + 3 and x – 14. *Teacher* *writes:* **(x – 14) (x + 3)** *Teacher:* Yes Sheila, she's a very quick little girl isn't she – I've noticed you before Sheila – you are a very fast little girl when I give you work like this in class. Now everybody else is she right – x minus 14 and x plus 3 – well? – Minus 14 by the plus 3 gives us the minus 42 and certainly x by x is x^2 – Now how do we check the middle term to make sure we are right – x by plus 3 is 3x and x by minus 14 is minus 14x? And yes when we add them we get, *and writes*: **–14x** **<u>+ 3x</u>** **–11x** *Teacher:* So we know we are right – there is no way you should not know that you have done your sum correctly. OK – I will give you one more of these and then I think we could go on to something harder – one more of these – number 25 – because you are too good for this – it's too easy for you.

24:00	Student work: Example 7: All negative signs	*Teacher cleans the blackboard and writes problem 2 on the board*: $\mathbf{x^2 - 33x - 70}$
		This time the teacher waits about 15 seconds before asking a student to call out the answer.
24:15	Teacher asks Ellen question.	*Teacher:* Okay – Ellen?
24:16	Student work correction: whole class Ellen answers question.	*Ellen:* Ummh – 2 and 35. *Teacher:* No, no say it properly for me – oh dear, oh dear – factors of a quadratic expression? *Ellen:* minus 35 and plus 2. *Teacher:* I'm looking for factors – you are giving me 2 numbers! *At this point Ellen pulls a face as if she feels like she has really slipped up!*

By assigning practice examples, the teacher is able to ascertain whether students have listened to and understood what was said during the explanation/demonstration phase. In the above excerpt, the emphasis was on speed rather than understanding; in this case the 'quickest' girl was rewarded with a lot of praise. It would appear that the teacher takes it for granted that all of the students were able to understand and keep up with the pace of the lesson. In lessons in other schools, the time given to student practice in class was usually much longer and teachers took the opportunity to walk around and monitor the progress of individual students and assist those having difficulty.

We now turn to the second illustrative vignette from Mourne (VCC Fr D). The lesson begins with the correction of homework from the previous lesson; the homework involved grouping quadratic expressions with minus signs. The previous lesson was devoted to student practice of identical quadratic expressions. Before correcting the homework presented in the excerpt, the teacher checks each student's homework and gives a star to those who attempted all of the assigned problems. While checking the homework in this way, the teacher ascertained that one of the questions caused a problem and she proceeds to correct this question.

Student Practice (Homework: whole class correction): Illustrative Vignette 2

Time code (minutes:seconds)	**Activity**	**Lesson text**
	Homework assigned in previous lesson (Lesson 8)	**Q. 14: $2a^2 + 2b - 4ab - a$** **Q. 15: $3x^2 + 4y - 6xy - 2x$** **Q. 16: $2xa - 4ay + 2by - bx$** **Q. 17: $6mp - 2mq + 3mp - mq$** **Q. 18: $mp + 2mp - 6mq - 3mq$**
2:29	Homework correction: whole class Q14 Teacher: asks whole class question.	*Teacher:* OK, so we are going to correct these now – 14 seems to have been the problem …Okay question 14 was … *Teacher calls out and writes the question on the board. Joe is whistling but teacher ignores it.* **$2a^2 + 2b - 4ab - a$** *Teacher:* So, again did you have to rearrange this?
2:45	Class answers question. Teacher asks another whole class question. Class answer whole class question.	*Class, some say:* No *Teacher:* Some people didn't re-arrange it – did it work if you didn't re-arrange it? *Class, some say*: Yes
3:00	Teacher asks Joe question.	*Teacher:* Who got it to work without re-arranging it – did you re-arrange it *(looking at Joe)* *Joe answers:* No *Teacher:* Can you tell me how to do it so?
3:02	Joe answers question.	*Joe:* No *Teacher repeats 'no' after him with a tone of resignation.*
3:05	Teacher asks Stephen question.	*Teacher:* Did anyone else get it to work without re-arranging it? Stephen did you say you did?
3:08	Stephen answers question.	*Stephen:* No
3:09	Teacher asks Margaret a question.	*Teacher:* Margaret did you re-arrange it? *There is quite a lot of noise outside the classroom which makes it fairly difficult to hear what is being said.*

3:15	Margaret answers Teacher question.	*Margaret: (inaudible – due to noise)* *Teacher:* Tell us what you did and we will have a look at it. *Margaret:* 2a into a plus b and minus a into plus 2b plus one … *Teacher writes this on the board.* **2a(a+b) – a(2b+1)**
3:28	Teacher asks whole class question.	*Teacher walks away from the board and looks at what she has written –* *Teacher:* Will this work?
4:05	Class answer whole class question.	*Class:* No *Teacher:* Why won't it work? *Some answer from class:* two different numbers … *Teacher:* Two different factors – so we will have to go back again and try to get the same factors in both of them – okay so we have to re-arrange it
4:15	Teacher asks Anne question. Anne answers teacher question.	*Teacher:* Anne could you suggest how we can re-arrange it? *Anne:* 2a squared minus a plus 2b minus 4ab *Teacher writes response on board* $\mathbf{2a^2 - a + 2b - 4ab}$ *Teacher:* Now let us take a look at that
4:35	Teacher asks Stephen a question.	*Teacher:* Now Stephen Murphy – what is common in the first 2 of those?
4:40	Stephen answers teacher's question.	*Stephen:* a … *Teacher:* So a times what? *Stephen:* a times 2a plus one *Teacher:* Perfect! *She writes:* **a(2a + 1)** *Teacher:* What's common in the second one? *Stephen:* b … *Teacher:* So 2 times? *Stephen:* 2 minus 4a **a(2a – 1) + b(2 – 4a)**
5:06	Teacher asks whole class question.	*Teacher:* Are we in trouble again?

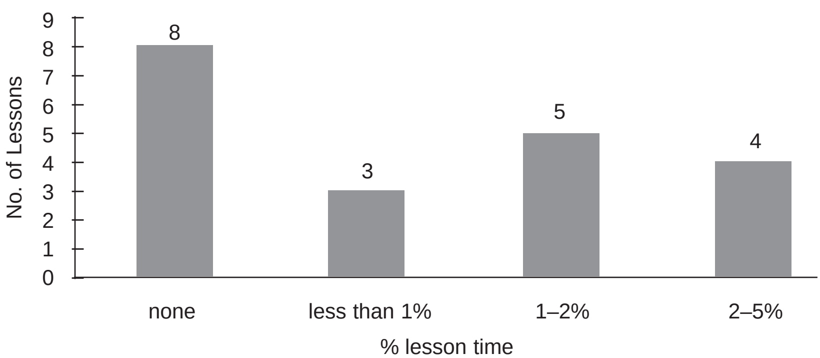
5:08	Class answer question	*Class:* Yeah *Teacher:* Why? *Class:* Mumbled response *Teacher:* We should try a 2 here and see what happens *– pointing to the place beside the b as follows: 2b ( )* – so if we have a 2a here what do we have inside – 1 minus 2a – **a(2a – 1) + 2b(1 – 2a).**
5:22	Teacher asks whole class question.	*Teacher:* Are we right now?
5:23	Class answer teacher whole class question.	*Class, some answer*: Yeah *and others* No.
5:26	Teacher asks whole class question.	*Teacher:* Why are we not right?
5:27	Class answer whole class question.	*Class:* The 2 are not the same *(meaning the 2 factors)* *Teacher:* Because one of them is 2a minus 1 and the other is 1 minus 2a – what's wrong with that? – how can we make that right?
5:45	Teacher asks 3 whole class questions – class respond to all.	*Class:* Change them around *Teacher:* So if that becomes minus then this becomes? *Class:* Plus *Teacher writes on the board:* a (2a – 1) + 2b (1 – 2a) becomes: **a (2a – 1) – 2b (2a – 1)** *Teacher:* Are we right now? *Class:* Yeah *Teacher:* Because why – because we have a 2a in both and a minus 1 in both – so now it is right – so the factors are: **(a – 2b) (2a – 1)** *Teacher:* So you had to play around a bit to get it right.
	Teacher asks whole class question to check who got it right.	*Teacher:* So who got it right?

In total, 25.75 minutes or seventy-six per cent of lesson time was spent on homework correction. Following the section of the lesson contained in the above extract, the teacher corrects all the remaining questions. Analysis of the discourse, which will be discussed later in this chapter, illustrates some of the core practices. The language used suggests an approach to mathematics as problematic: ‘Q14 seems to have been the problem’; ‘Why won’t it work?; ‘Are we in trouble again’, with a definite ‘right’ approach: ‘Are we right now?’; ‘Why are we not right?’; ‘So you had to play around a bit to get it right’; ‘So who got it right?’. In addition, appraisal was conducted in the public domain; here ‘right’ answers were praised with ‘perfect’.

Lesson Aims

By stating what material will be covered in the lesson and what relationship this lesson has with topics already covered, the teacher is providing an important learning context for the students. The findings in Figure 4.1 show, however, that the time spent by the teacher on this particular pedagogical practice was marginal. In eight of the twenty mathematics lessons, there was no lead in to the lessons at all, while in the other twelve lessons this pedagogical practice represented a minor part of teaching.

Figure 4.1: Percentage of lesson time spent on stating lesson aims (Base: 20 lessons)



The following vignette is taken from the lesson in which the highest proportion of time was spent on the statement of lesson aims (4.4 per cent) (in Blackstairs (VCC Fr D), first lesson). The excerpt occurs at the beginning of the lesson. The new topic being covered is co-ordinate geometry, specifically finding the midpoint of a line. In this extract we see the teacher providing a context for the lesson. He sets the scene for students, using a mixture of clarification, recall, demonstration and questioning. He also locates the work of the day in the context of the text book.

This extract involves a very experienced male teacher in a designated disadvantaged vocational community college, with a mixed ability group, of whom just over half are female. The school is located in a rural area. One third of the students in the class observed are from a middle class background and overall the students achieved a high mean score of 27.2 out of a possible 40 on the TIMSS-based mathematics test.

Illustrative Vignette: Lesson Aims – Lead In

Time code (minutes:seconds)	**Activity**	**Lesson text**
3:10	Lesson aims: Revision mapping co-ordinates	*Teacher:* … just before we start, before we start our class we'll just run back very briefly for about 2 minutes on the co-ordinates.
3:37	Revision: Questioning whole class Reference to U shaped graphs – Chapter 24: Graphs of quadratic functions	*Teacher:* Remember the **U** shape graphs we were drawing. We did that didn't we?
3:38	Class answer whole class question.	*Class:* Yeah.
3:40	Revision: Teacher – centred Reference to straight line graphs – Chapter 19: Graphing Lines – Simultaneous Equations	*Teacher:* We did the straight line graphs. *Class:* Yeah.
3.47	Teacher demonstrates plotting co-ordinates.	*Teacher:* And before I use the ruler there I'll do it out roughly. Before I begin, before I start the class just to recap on the points – where you had your X and Y axis – and you had your 1, 2, 3 and –1, –2, –3, back here. We'll say. And you had 1, 2, 3 up to 4 and the same distance apart down along here just so that you'll be able to know where the points are.

4:12	Revision: Questioning whole class	*Teacher:* What point for example is one there, one there – that point *(teacher marks in the co-ordinate (1, 3) on graph)?* *Class:* (1, 3) *Teacher:* (1,3), right. Just so that we'll know. And if you were to move down to this one down here – that point would be? *(Teacher marks in the co-ordinate (2, –3) on graph).* *Class:* (2 –3) *Teacher:* (2 –3), correct! And if you were to move over to this quadrant over here, move up to here – hands up for that?
4:48	Revision: Questioning individuals Teacher asks Vinnie question.	*Vinnie:* (–1, 4)
4:52	Teacher asks Amy question	*Teacher:* (–1, 4) correct! Right, and we'll take one more and then we'll be ready to start. Move down to there! Hands up for that? Yeah? (*pointing to Amy)* *Amy:* (–4, –1). *Teacher:* (–4, –1). Correct!
5:11	Lesson aims	*Teacher:* So that is the basis of what we are going to be starting. The basis of this chapter – near the end of the book – we are going to be finding the distance between two of those points (*teacher points to two of the co-ordinates on the diagram)*. We're going to be finding the slope of say that one (*teacher pointing to the line joining two of the co-ordinates)*. We're going to find the midpoint of something from there to there (*teacher again points to same line)*. And we're going to be finding, now that I've two sides drawn we're drawing a third side *(teacher joining up another two of the co-ordinates, now forming a triangle on the diagram)*. And maybe finding the area of that triangle *(teacher points to same triangle)*. What else will we be doing? We have distance, we have midpoint, we have slope, we have the area of a triangle, we might even find, if you're able for it in second year, the equation, find the equation of that line. Find the equation of this line (*teacher points to one of the lines on the diagram)*. And that's about it!

Conclusion

The findings show that all of the twenty lessons observed involved, what is referred to in the literature as, a traditional approach. The traditional approach reflects a view of mathematics as ‘a static, structured system of facts, procedures and concepts’ (Henningsen and Stein, 1997, p. 524). It is assumed that mathematics is a fixed, clearly defined subject matter. A procedural rather than a conceptual and/or problem solving approach to the subject prevails.

> Specific objectives, which students are to master, have been stated; the teacher’s role has been to demonstrate how a manipulation is to be carried out or to explain how a concept is defined; and students have been expected to memorise facts and to practice procedures until they have been mastered. (Romberg and Kaput, 1999: 4)

As is the case in traditional mathematics classrooms, the teachers in our study used a combination of lecture and drill and practice format. Typically, the teacher lectured to the class as a whole and asked questions. Students watched, listened, took notes or copied examples into their exercise books when instructed and answered questions when called upon. Rarely did they ask questions about the material or make comments. This contrasts with a problem-centred approach to school mathematics. This approach is described as ‘dynamic and exploratory’ (Romberg, 1994) and ‘requires one to focus on the active, generative processes engaged in by doers and users of mathematics’ (Henningsen and Stein, 1997; Schoenfeld, 1992). In this context, students are encouraged to draw on their previous mathematical knowledge and experience to construct meanings. They do this by actively engaging with other students as well as the teacher.

The use of traditional approaches to teaching is not confined to mathematics however. Our observations in English classes indicate that while there are important epistemological differences between mathematics and English that impact on how these subjects are taught in schools, there are also important similarities in the pedagogical approaches applied in both. In the English classes we observed in this study, demonstration and student practice constituted the core work just as it did in mathematics. English classes were also very similar to mathematics classes in that questioning was strongly teacher-led (see Chapter 8 below for a more detailed discussion). Moreover, a study of classrooms across a wide range of subjects suggests that while a traditional approach to teaching is frequently more pronounced in mathematics than in other subjects, it is by no means confined to this subject. Irish second level classrooms generally operate along strong didactical lines within which subject matter is presented as a relatively fixed and unproblematic body of knowledge (Lynch and Lodge, 2002).

The traditional approach to teaching mathematics that we observed therefore is part of a wider set of cultural and pedagogical practices in teaching. To represent it as a matter of individual teacher choice and responsibility would be to simplify a complex problem in teaching and learning. Mass public education has a long history in western European societies; it is a history that demonstrates a strong allegiance to drill and practice for all subject teaching, not just for the teaching of mathematics (Bowles and Gintis, 1976; Coolahan, 1981; Foucault, 1977).

While teachers are relatively autonomous in their own classrooms, they are also subject to a range of internal as well as external controls. School principals and senior management regulate their access to resources and structure their time and location through timetabling (Meyer and Rowan, 1988). In general, they work in isolation from their colleagues (van Veen et al., 2001) with an expectation that they will maintain a high level of vigilance over their students (Travers and Cooper, 1996). A growing culture of bench marking, quality assurance and the pressures of state examinations, not only create a stressful climate, but also curtail teachers' willingness to use exploratory or innovative teaching methods (Broadfoot, 1979; Murphy and Torrance, 1988; Hargreaves et al., 1996). Furthermore, the multiple roles that teachers and schools are now expected to fulfil have become increasingly complex (van Veen et al, 2001). Teachers experience their own role as both powerful and powerless and this, in turn, influences their capacity and motivation to be innovative or experimental (Davies, 1996).

To understand the prevalence of traditional approaches to the teaching of mathematics, therefore, we need to be mindful of the wider educational and socio-political contexts within which teaching takes place. Even the design and architecture of school buildings and classrooms need to be examined if one is to fully understand the dynamics of teaching and learning. The architecture of the classrooms we observed strongly facilitated a traditional, hierarchical approach to teaching. In all classes students were seated in individual desks that were arranged in rows facing the teacher at the front. While the teacher could circulate around the class when the students were engaged in individual work, and some did, in other classes there was little space to move between desks. The nature of classroom furniture and the size of the classrooms often militated against flexibility in pedagogical approach even when the teachers may have desired it.

Appendix to Chapter 4

Illustrative Vignette – Demonstration (From Mourne (VCC Fr D)

Time Code (minutes:seconds)	**Activity**	**Textbook:** Condon and Regan *Junior Certificate Mathematics I* , Folens **Chapter:** 17 page 198 **Topic:** Factorising Quadratic Trinomials – page 202
22:12	Demonstration Example 1: $X^2 + 15x + 56$ Teacher asks whole class question	*Teacher: goes back to the board and writes:* $\mathbf{x^2 + 15x + 56}$ *Teacher:* Now we want to see how to get the factors for this – what it is, is that we start here (*pointing to $x^2 + 15x + 56$)* and then we want to end up with this as your answer – (*pointing to (x + 7) (x + 8)* – and we are going to take the steps exactly backwards – so what happens if you go from this line (*pointing to $x^2 + 15x + 56$*) this time – now that's exactly the same ((*pointing to x^2)* and that's exactly the same ((*pointing to 56)* and then the 15x gets split up into what? – into 8x plus 7x – Now can anybody think of a reason why it is 8 and 7? – why? – I mean you could split 15x into 14x and 1x and 13x and 2x or 12x and 3x – why is it 8 and 7?
23:24	Stephen answers Teacher asks whole class question	*Stephen:* Because it is more even. *Teacher:* What's more even? *Stephen:* 8 and 7 *Teacher considers this for a moment*
23:32	Teacher asks same whole class question	*Teacher:* Is there any other reason why 8 and 7?
23:37	Rachel volunteers answer	*Rachel:* Because when you multiply 8 times 7 you get 56. *Teacher:* Good girl Rachel – (*and repeats Rachel's answer)* when you multiply 8 times 7 you get 56.
23:48	Teacher asks whole class question	*Teacher:* so what's the number that guides you then?
23:51	Class answer whole class question	*Class and Teacher together:* its 56.
23:53	Teacher explanation	*Teacher:* So you split up the 15 – and the way you split it up is guided by the 56 – and how do you work out what way to do it? – You find 2 numbers to multiply for 56 – so two numbers that multiply for 56 and add for 15? – and we call that they 'multiply for' <u>the product</u> – so we say their product is 56 – and when

		you 'add the two of them together' you call that their sum – so we say their sum is 15 – so you look for 2 numbers that will multiply to give you 56 and add together to give you 15 – and we know the answer to that already – so it is 8x plus 7x plus 56 – so we have changed this line and we have rewritten it by splitting up that middle one – now we are going to do it exactly the same way as you have done all the ones up to now –
24:45	Teacher asks whole class question	*Teacher:* So what would you think you would do next?
24:50	Class answer whole class question	*Class:* Take the x out. *Teacher:* you look at the first two and you take the x out – right! – so that's x times x plus 8, *and writes:* **x(x + 8)**
24:54	Teacher asks whole class question	*Teacher:* And what do you do then?
24:56	Class answer whole class question	*Class: … (inaudible) something about* 'the next two terms'. *Teacher:* Get the second two …
25:04	Teacher asks Paul question	*Teacher:* … and take out what Paul?
25:05	Paul answers question	*Paul:* the 7 *Teacher:* Take the 7 out – so it is 7 times x plus 8 – and how do you finish it? *Paul:* x plus 7 times x plus 8, *and teacher writes*: **(x + 7) (x + 8)** *Teacher:* so the factors for x^2 plus 15x plus 56 are x plus 7 times x plus 8 – right let's try one more.
25:22	Teacher asks Conor question Example 2: $X^2 + 8x + 12$	*Teacher:* suppose we want to get factors for x^2 plus 8x plus 12, *and writes*: $\mathbf{x^2 + 8x + 12}$ *Teacher:* Now we are going to have to do it like that, so we are going to have to leave the x^2 alone and we are going to leave the 12 alone and we are going to split up the 8x – how are we going to split it Conor?
25:45	Conor answers Teacher question	*Conor:* 2x plus 6x *Teacher:* Good lad and why is that? *Conor:* Because two sixes are twelve. *Teacher:* Because we want two numbers that are going to multiply together for 12 and are going to add for 8 and those two numbers are 6 and 2 – so its x^2 plus 6x plus 2x plus 12, *and writes:* $\mathbf{x^2 + 6x + 2x + 12}$

26:04	Teacher asks Paula question	*Teacher:* Now Paula, what will we do with these first two?
26:10	Paula answers Teacher questions	*Paula: (with teacher prompting her along)*: x times x plus 6, *and teacher writes:* **x(x + 6)**
26:18	Teacher asks Margaret question	*Teacher:* and Margaret what will we take out of the next two?
26:18	Margaret answers Teacher question	*Margaret:* two *Teacher:* and we get 2 times: *Margaret:* x plus 6, *and teacher writes:* **2(x + 6)**
26:28	Teacher asks Paul question	*Teacher:* so what are the factors then Paul?
26:30	Paul answers Teacher question	*Paul:* x plus 2 *Teacher:* times? *Martin:* x plus 6, *and teacher writes:* **(x + 2) (x + 6)**
26:35	Teacher asks Joe question <u>Example 3:</u> $X^2 + 7x + 12$	*Teacher:* very good – so we'll try one more – suppose we want to get factors for x^2 plus 7x plus 12, *and writes* $\mathbf{x^2 + 7x + 12}$ *Teacher:* we are going to leave the x^2 alone, we are going to leave the 12 alone and we are going to split up the 7x – Joe how will we do it?
26:53	Joe answers Teacher question	*Joe:* 3 and 4 *Teacher:* pardon? *Joe: louder*, 3 and 4 *Teacher:* yeah, we are going to split it into 3x plus 4x plus 12, *and writes* $\mathbf{x^2 + 3x + 4x + 12}$ *Teacher:* we wanted two numbers whose product is 12 and whose sum is 7 – so we wanted 2 numbers that will multiply for 12 and add for 7
27:09	Teacher asks Lisa question	*Teacher:* so okay Lisa what will we do now?
27:10	Lisa answers Teacher question	*Note: at this point there is quite a bit of noise outside the classroom.* *Lisa:* x *Teacher:* x times? *Lisa:* x plus 3 *Teacher:* Good, plus? *Lisa:* 3, no 4

		Teacher: 4 times? *Lisa:* x plus 3
27:28	Teacher asks Teresa question	*Teacher:* Teresa how do we finish it off?
27:30	Teresa answers Teacher question	*Teresa:* x plus 4 and x plus 3 *Teacher:* Very good and writes: **(x + 4) (x + 3)**
27:30	Teacher asks whole class question	*Teacher:* Does everyone know what do?
27:32	Class answer whole class question	*Class:* yeah!
27:35	Teacher asks whole class question	*Teacher:* Are you ready to try one yourselves?
27:36	Class answer whole class question	Class: yeah!

Table A4.1: Case-study schools – content of mathematics lessons

		Mathematics subject		Level
School and Lesson		Topic	Specific	
Barrow (SSG F)	1	Algebra	Quadratic Expression with mixed signs	Continuation
	2	Algebra	Factors by grouping and difference between two squares	Revision L1; Continuation
Nore (SSG Fr)	1	Statistics	Bar charts	New
	2	Statistics	Bar charts and Pie charts	Continuation L1
Suir (SSG Fr D)	1	Statistics	Mean and mode	Revision
	2	Statistics	Mean and mode	Revision
Liffey (SSB Fr D)	1	Simple Interest	Finding principal, interest, time and rate	New
	2	Simple Interest	Finding principal, interest, time and rate	Continuation L1
Lee (SSB F)	1	Algebra	Quadratic equations: problems	Continuation
	2	Algebra	Quadratic equations: problems	Continuation L1
Lagan (SC Fr)	1	Geometry	Length of a circle	New
	2	Geometry	Length of a circle and running tracks	Continuation L1
Errigal (CS Fr)	1	Algebra	Equations with fractions	Continuation
	2	Algebra	Equations with fractions	Continuation L1
Mourne (VCC Fr D)	1	Algebra	Factors: grouping expressions with minus signs	Continuation
	2	Algebra	Factors: grouping expressions with minus signs	Continuation L1
Blackstairs (VCC Fr D)	1	Co-ordinate geometry	Plotting co-ordinates and midpoint of a line	New
	2	Co-ordinate geometry	Length of a line; distance between two co-ordinates	Continuation L1
Nephin (VCC Fr)	1	Money	Electricity bills	Revision
	2	Algebra	Introduction and removing brackets	Continuation

Notes

[1] The classes in which student practice was clearly the prevailing pedagogical style were Barrow 2, Suir 2, Lee 1 and 2, Errigal 2, Mourne 1 and 2, Nephin 2. Teacher demonstration was the prevailing style in Barrow 1, Nore 1, Lagan 1 and Errigal 1.The remaining classes have a more even balance between the two pedagogical practices.