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A MODEL OF PARTNERSHIP FORMATION WITH FRICTION AND MULTIPLE CRITERIA

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ABSTRACT. We present a model of partnership formation based on two discrete character traits. There are two classes of individual. Each individual observes a sequence of potential partners from the other class. The traits are referred to as attractiveness and character, respectively. All individuals prefer partners of high attractiveness and similar character. Attractiveness can be measured instantly. However, in order to observe the character of an individual, a costly interview (or date) is required. On observing the attractiveness of a prospective partner, an individual must decide whether he/she wishes to proceed to the interview stage. During the interview phase, the prospective pair observe each other's character and then decide whether they wish to form a pair. Mutual acceptance is required for both an interview to occur and a pair to form. An individual stops searching on finding a partner. A set of criteria based on the concept of a subgame perfect Nash equilibrium is used to define an equilibrium of this game. It is argued that under such a general formulation there may be multiple equilibria. For this reason, we define a specific formulation of the game, the so called symmetric version, which has a unique symmetric equilibrium. The form of this equilibrium has some similarities to the block separating equilibrium derived for classical models of two-sided mate choice and job search problems, but is essentially different.

1. INTRODUCTION

This paper presents a model of matching based on two traits. There are two classes of individual, and each individual wishes to form a partnership with an individual of the opposite class. Each individual observes a sequence of potential partners. Mutual acceptance is required for a partnership to form. On finding a partner, an individual ceases searching. One measure describes 'attractiveness'. Preferences are common according to this measure: i.e. each individual prefers highly attractive partners, and all individuals of a given class agree as to how attractive individuals of the opposite class are. Preferences are homotypic with respect to the second measure, referred to as 'character', that is, all individuals prefer partners of a similar character.

For convenience, it is assumed that individuals know their own attractiveness and character. Also, the distributions of character and attractiveness are assumed to be discrete with a finite support, and constant over time.

Key words and phrases. mutual mate choice, game theory, common preferences, homotypic preferences, subgame perfect equilibrium.

Together, the attractiveness, character and the class of an individual determine their 'type'. Individuals can observe the attractiveness and character of prospective partners perfectly. However, in order to measure character, a costly interview is required. In addition, individuals incur search costs at each stage of the search process.

At each moment an individual is paired with a prospective partner. First, both individuals must decide whether they wish to proceed to the interview stage on the basis of the attractiveness of the prospective partner. The final decision on pair formation is based on both the attractiveness and character of the prospective partners. Mutual acceptance is required for an interview to occur and a pair to form. At equilibrium, each individual uses a strategy appropriate to their type. The set of strategies corresponding to such an equilibrium is called an equilibrium profile.

Such a problem may be interpreted as a mate choice problem in which the classes are male and female, or a job search problem in which the classes are employer and employee. The assumption that attractiveness can be observed very quickly, but an interview (dating) is required to observe someone's character is obviously a simplification. However, in the case of human mate choice many traits that can be thought of as defining attractiveness (physical attractiveness, employment, economic status) are usually measured quickly, whilst observation of traits defining character (political and religious views, tastes and emotions) are generally more difficult to measure.

1.1. **Related Literature.** Our model has a resemblance to 'speed dating' models recently developed and tested by Fisman et al. (2006). The types of preferences we study have been analyzed (in continuous time) by Marimon and Zilibotti (1999). They found that these types of preferences, in this context, were quite tractable and somewhat equivalent to the formulation with ex post idiosyncratic uncertainty. Other recent directed search papers in the same vein are Albrecht et al. (2006) and Galenianos and Kircher (2009), looking at directed search in job applications.

The benchmark model in this literature is Smith (2006), which focuses on assortative matching and block segregation in 'marriage' models (Burdett and Coles, 1999; Burdett, 1978; Coles and Burdett, 1997). Relative to this literature, the novel aspect introduced in this paper is the 'character'-dimension of agents types and, in particular, the costly application/invitation-stage of the model, which follows Shimer (2005) and in the types of search frictions it assumes.

In the case of such a model of job search, it is assumed that a job's attractiveness (interpreted as pay, conditions, status) can be observed from an advert. The attractiveness of a job seeker (interpreted as qualifications and experience) are readily seen from his/her application. Similarity of character may be interpreted as the ability of the employer (or department) and employee to work together as a team. Several labour market studies have found empirical evidence that employers and employees are happiest with labour market choices they view as similar to themselves in some respects (eg. labour market type, class, educational level), Peterson et al. (2000); Beller (1982); Albelda (1981).

In certain cases, the character of an employee might be interpreted as his/her speciality. For example, suppose two different mathematics departments are looking for a professor. Both want someone with a large number of publications in prestigious journals (a measure of attractiveness). However, department A wants a fluid dynamicist and department B wants a game theorist, since such a professor would suit their profile/fulfil their requirements (i.e. suitability of characters). This interpretation is somewhat problematic within the framework presented here, since it is likely that an individual's speciality would be visible from the application.

In the literature to date, job search models typically adopt a matching function approach, where the employer and employee search for the perfect 'fit' using a set of costly criteria, see Coles and Burdett (1997). Equilibrium conditions are derived and tested for robustness once the model is built, and policy recommendations follow (Burdett, 1978; Pissarides, 1994; Drewlanka, 2006; Shimer and Smith, 2000). Jovanovic (1979), Hey (1982), and MacMinn (1980) are the classic studies. Devine and Piore (1991) and Shimer and Smith (2000) survey the more recent developments, while, as we have stated above, Smith (2006) and Chade and Smith (2006) provide the current benchmark model.

Another strand in the literature is the search-theoretic literature developed by McCall (1970) and extended by Diamond and Maskin (1979) and others, where the job search problem is conceived of as a dynamic program which has to be solved in finite time, so labour markets are best described by optimal control problems and solution methods. The literature here is vast and well studied.

Job search has also been modelled as a mating or network game, with representative contributions being (Albelda, 1981; Beller, 1982; Peterson et al., 2000; Coles and Francesconi, 2007; Fisman et al., 2008; Pissarides, 1994). In the biological literature, mate choice is modelled as sequential observation of prospective mates. The model presented here is a development of this strand. For models of mutual mate choice based on common preferences see Johnstone (1997); Alpern and Reyniers (2005), and Janetos (1980).

For a model of mate choice based on homotypic preferences see Alpern and Reyniers (1999). The models of Johnstone and of Alpern and Reyniers assume that the distribution of the value of available mates changes over time as partnerships form and individuals leave the mating pool. Ramsey (2008) considers a similar problem interpreted as a job search problem.

The general approach is presented in Section 2. Section 3 gives a comparison of the approach used here and classical models of two-sided job search and mate choice problems. It is intended that this section will give an intuitive feel for the approach to solving such problems and the added complexity involved when common and homotypic preferences are combined. Section 4 describes a model of partnership formation in which character forms a ring and neither the distributions of the traits nor the search costs depend on the class of a searcher. This model is called symmetric. Section 5 gives the set of criteria that we wish an equilibrium to satisfy. These conditions are based on the concept of a subgame perfect equilibrium (a refinement of the concept of Nash equilibrium). Section 6 describes a general method for calculating the expected utilities of each individual under a given strategy profile. Section 7 considers the dating subgame (when individuals decide whether to form a partnership) and the soliciting subgame (when individuals decide whether to go on to the dating subgame). Section 8 presents results on the existence and uniqueness of a symmetric equilibrium in the symmetric game. An algorithm for determining this equilibrium is also presented, together with an example. Section 9 briefly illustrates the problems involved in adopting such an approach to the more general formulation. Section 10 gives a brief conclusion and suggests directions for further research.

2. General Formulation

We present a model of a sequential decision process leading to the formation of a long term partnership. It is assumed that there are two classes of player and individuals in a partnership have to be of different classes (e.g. in job search problems employees form partnerships with employers, in mate choice problems males form partnerships with females). Individuals view a sequence of prospective partners. It is assumed that costs are incurred during the search process, so in general an individual should not continue searching until he finds his/her best possible partner. The following assumptions are made:

A: We consider the formation of long term relationships between two classes of player (e.g. marriage, employment). When a partnership is formed, the two individuals involved leave the population of searchers. Henceforth, we will refer to these two classes of players as males and females.

B: Interactions are bilateral and occur between a male and a female. The length of an interaction is assumed to be small (effectively of zero length) compared to the time between interactions. The pair must decide whether to form a partnership or continue searching. Mutual acceptance is required for a partnership to form. Individuals cannot return to a previously met prospective partner.

C: When an individual leaves the population, he/she is replaced by a clone, i.e. an individual of the same sex, attractiveness and character enters the population of searchers. Hence, the joint distribution of attractiveness and character is fixed. It might be more realistic to consider a steady-state model in which individuals enter the pool of searchers at a steady rate according to their sex, attractiveness and character. However, due to the novelty of the approach used and some of the issues involved in deriving equilibria, for the present we adopt the simpler clone replacement approach. It is intended that the steady state approach will be adopted in future work.

D: It is assumed that time is discrete and the search costs of males and females per unit time are c_1 and k_1, respectively. At each moment in time a player encounters a prospective partner. We assume random matching, i.e. the attractiveness and character of the female encountered by a male is chosen at random from the joint distribution of attractiveness and character among females. Using the assumption of clone replacement, it is also easy to adapt the model to assume that encounters occur as a Poisson process. It may be assumed that individuals find prospective partners at rate 1 and males and females pay search costs of c_1 and k_1 per unit time, respectively. Empirical evidence on search costs in real-life job search problems abounds in the literature. For instance, Peterson et al. (2000) considers job search costs to be the primary cause of 'sticky' wages and low labour market mobility, when contrasting the European and US labour markets. Devine and Piore (1991) also presents empirical evidence on search costs, and in the large Lucas/Prescott literature incorporating job search costs into macroeconomic models, Andolfatto (1996) is representative.

E: Encounters have two stages. In the first stage, both individuals must decide whether they wish to date based on the attractiveness of the prospective partner. For convenience, it is assumed that these decisions are made simultaneously. Dating only occurs by mutual consent. It is assumed that the costs of dating to males and females are c_2 and k_2, respectively. During a date, each observes the character of the other and then decides whether to accept the other as a partner. Again, it is assumed that these choices are made simultaneously. A partnership is formed only by mutual consent. In some scenarios—for example, when character is not important—it might pay a female to immediately accept a male without going through the dating process. However, to keep the strategy space as simple as possible, it is assumed that individuals must always date before forming a pair. The total utility of an individual is taken to be the utility gained from the partnership minus the sum of the search costs and dating costs incurred. It is assumed that utility is not transferable and individuals maximise their expected total utility from search.

Such an approach implicitly assumes that the number of males equals the number of females. However, the model can be easily adapted to allow the number of males and females to differ. Suppose that there are r times as many males as females. In this case, we may assume that at each moment a proportion $(r-1)/r$ of males meet a prospective partner who would give them an expected utility of $-\infty$. In reality, such males do not meet a prospective partner.

The supergame Γ is defined to be the game in which each player observes a sequence of prospective partners as described above. An encounter between two prospective partners will be referred to as the encounter game. The encounter game is split into two subgames, the soliciting subgame, when players decide whether to date or not and the dating subgame, when they decide whether to form a partnership or not.

3. Comparison with the Classical Partnership Formation Game

Two-sided problems are by nature game-theoretic and so we look for a Nash equilibrium solution at which no individual can improve their expected utility by changing their strategy. Note that there may be multiple Nash equilibria. For example, suppose that mate choice is based only on attractiveness and there are only two levels of attractiveness: high and low. Suppose individuals of high attractiveness only accept individuals of low attractiveness as partners. Similarly, individuals of low attractiveness only accept individuals of high attractiveness as partners. It can be seen that this is a Nash equilibrium, since e.g. a male of high attractiveness cannot gain by accepting a female of high attractiveness, since she would not accept him. Also, he could not gain by rejecting a female of low attractiveness, since he would not find a partner. However, one would expect that if a male accepts a female of attractiveness x, then he would accept any female of attractiveness $> x$. McNamara and Collins (1990) derive an equilibrium which satisfies such a condition, referred to as the **optimality criterion**. This criterion states that any individual accepts a prospective partner if and only if the utility from such a partnership is as least as great as the expected utility of the individual given that he/she continues searching. Such an equilibrium can be derived inductively. Consider a female of maximum attractiveness. She will be acceptable to any male. Hence, such females face a one-sided problem and their equilibrium strategy is of the form: accept the first male of attractiveness $\geq x_1$. Call such males first class. It follows that males of attractiveness $\geq x_1$ are acceptable to any female (i.e. face a one-sided problem) and their equilibrium strategy is of the form: accept the first female of attractiveness $\geq y_1$. Call such females first class. It follows that first class males will only pair with first class females. The problem faced by the rest of the population then reduces to a problem in which first class individuals are not present. Define $x_0 = y_0 = \infty$. Arguing iteratively, it can be shown that k classes of males and females can be defined, such that a male of attractiveness x is of class i if $x \in [x_i, x_{i-1})$ and a female of attractiveness y is of class j if $y \in [y_j, y_{j-1})$. Males of class i pair with females of class i. There may be a class of males or females who do not form partnerships.

In the problem considered here, individuals do not always agree on the desirability of a member of the opposite sex as a partner. It would be natural to try and reduce the game considered to a sequence of one-sided choice problems. However, for games within the general framework presented in Section 2 there are some technical problems associated with such an approach. For example, consider a problem in which attractiveness and character are independent, both have a uniform distribution over the integers $0, 1, 2, \dots, m$ regardless of sex. It is expected that individuals of maximum attractiveness, m, and close to median character will have a higher expected expected utility from search (i.e. be choosier) than individuals of attractiveness m and extreme character, either 0 or m (see Alpern and Reyniers (1999)). In the problem considered by McNamara and Collins (1990) it is relatively easy to order individuals according to how choosy they should be. This ordering is used to derive the unique equilibrium satisfying the optimality criterion. Such an ordering is not so easy in the problem considered here. For example, should a male of attractiveness m and character 0 be more or less choosy than a male of attractiveness $m-1$ and close to median character? Ramsey (2010) shows that multiple equilibria may exist in such a problem, i.e. in general there is no unique sequence of one-sided problems that can be solved to define an equilibrium.

4. The Symmetric Model with Character Forming a Ring

Due to the problems outlined in the previous section, we present a model which allows us to adopt a similar (but not identical) approach to the one used by McNamara and Collins (1990). Attractiveness and character are denoted X_a and X_c, respectively. The population is assumed to be large. It will be assumed that

- **a:** X_a and X_c are independent. The distribution of X_a does not depend on sex. The distribution of X_c in both sexes is uniform on the integers $0, 1, \ldots m-1$.
- **b:** The difference between characters is calculated according to modulo m, i.e. character can be thought of as a ring with 0 and $m-1$ being neighbouring characters.
- **c:** Search and dating costs are c_1 and c_2, respectively, independently of sex.
- **d:** The utility obtained by a type $\mathbf{x} = [x_a, x_c]$ individual from pairing with a prospective partner of type $\mathbf{y} = [y_a, y_c]$ is given by $g(y_a, |x_c - y_c|)$, i.e. the utility function is independent of sex.

Using such an approach, intuitively an individual's mating prospects do not depend on his/her character or sex. Such a game will be referred to as symmetric.

5. Equilibrium Conditions

For a game defined within the general framework, introduced in Section 2, we require an equilibrium profile to satisfy the following generalisation of the optimality criterion for the classical two-sided problem. Namely:

- **Condition 1:** In the dating subgame, an individual accepts a prospective partner if and only if the utility from such a pairing is at least as great as the individual's expected utility from future search (ignoring previous costs).
- **Condition 2:** An individual is only willing to date if their expected utility from the resulting dating subgame minus the costs of dating is as least as great as their expected utility from future search.
- **Condition 3:** The decisions made by an individual do not depend on the moment at which the decision is made.

It should be noted that the expected future utility of an individual from search, and thus the exact form of the dating and soliciting subgames, depends on the strategy profile used in the population as a whole. This dependency will be considered more fully in Section 7.

The most preferred prospective partners of a type $[x_a, x_c]$ individual are those of maximum attractiveness who have character x_c. Condition 1 states that in the dating subgame an individual will always accept his/her most preferred partner, since an individual's future expected utility from search must be less than the utility from obtaining his most preferred partner. Moreover, if in the dating subgame a male accepts a female who would give him a utility of k, then he must accept any female who would give him a utility of at least k. It follows that the acceptable difference in character is non-decreasing in the attractiveness of a prospective partner.

Condition 3 states that the Nash equilibrium strategy should be stationary. This reflects the following facts:

- **a:** An individual starting to search at moment i faces the same problem as one starting at moment 1.
- **b:** Since the search costs are linear, after searching for i moments and not finding a partner, an individual maximises his/her expected utility from search simply by maximising the expected utility from future search (i.e. by ignoring previously incurred costs).

We might also be interested in profiles that satisfy the following condition.

- **i:** In the soliciting subgame, an individual of attractiveness y_a is willing to date dates prospective partners of attractiveness above some threshold, denoted $t(y_a)$, such that if $y_1 > y_2$, then $t(y_1) \geq t(y_2)$, i.e. the more attractive an individual, then the choosier he/she is when choosing a dating partner.

However, it seems reasonable that individuals of low attractiveness may not solicit dates from highly attractive prospective partners, as by doing so they will incur dating costs while it is expected that such a date will not lead to pair formation. Hence, we do not require individuals to use threshold rules when deciding whether to solicit a date. For example, Härdling and Kokko (2005) argue that in certain circumstances small males should avoid courting attractive females to avoid the possibility of attacks from larger males.

Definition 5.1. *An equilibrium profile, denoted π^*, of Γ is a strategy profile under which the behaviour of all searchers satisfies Conditions 1 to 3 in each of the possible subgames. The value function of Γ corresponding to π^* is the set of expected utilities of each individual according to type under the strategy profile π^*.*

Note that π^* must define the appropriate behaviour in all possible dating subgames, even those that do not occur under π^*

In the particular case of the symmetric game, we wish to find an equilibrium which is symmetric with respect to character and sex. That is to say:

- **Condition 4:** If an individual of type $[x_a, x_c]$ is willing to date a prospective partner of attractiveness y_a, then any individual of attractiveness x_a is willing to date a prospective partner of attractiveness y_a.
- **Condition 5:** If an individual of type $[x_a, x_c]$ is willing to pair with a prospective partner of type $[y_a, y_c]$ in the dating subgame, then an individual of type $[x_a, x_c+i]$ is willing to pair with a prospective partner of type $[y_a, y_c + i]$ [addition is carried out $\mod (m)$].

An equilibrium which satisfies Conditions 4 and 5 will be referred to as a symmetric equilibrium. Note that under a symmetric strategy an individual's expected utility from search is independent of sex and character, i.e. only depends on attractiveness.

Note that at equilibrium, if an individual of type $[i, k]$ is willing to date an individual of attractiveness j, then he/she must be willing to pair with some prospective partners of attractiveness j (otherwise unnecessary dating costs are incurred). Hence, a type $[j,k]$ prospective partner (the most preferred partner of such attractiveness) must be acceptable in the corresponding dating subgame.

6. Deriving the Expected Utilities Under a Given Strategy Profile

Consider the symmetric game described above. We will look for a symmetric equilibrium profile, thus we may assume that the strategy profile used is symmetric (i.e. satisfies Conditions 4 and 5). Given the strategy profile used by a population, we can define which pairs of types of individuals proceed to the dating subgame and which pairs of types of individuals form pairs. From this, it is relatively simple to calculate the expected length of search and the expected number of dates of an individual of a given type.

Let $p(\mathbf{x})$ be the probability that an individual is of type $\mathbf{x}$. Define $M_1(\mathbf{y};\pi)$ to be the set of types of prospective partners that an individual of type $\mathbf{y}$ will date (under the assumption of mutual acceptance) under the strategy profile π. Define $M_2(\mathbf{y};\pi)$ to be the set of types of prospective partners that eventually pair with an individual of type $\mathbf{y}$. By definition $M_2(\mathbf{y};\pi) \subseteq M_1(\mathbf{y};\pi)$.

The expected length of search of an individual of type $\mathbf{y}$, $L(\mathbf{y};\pi)$, is the reciprocal of the probability of finding a mutually acceptable partner at any given stage. The expected number of dates of such an individual, $D(\mathbf{y};\pi)$, is the expected length of search times the probability of dating at any given stage. Hence,

$$L(\mathbf{y};\pi) = \frac{1}{\sum_{\mathbf{x}\in M_2(\mathbf{y};\pi)} p(\mathbf{x})}; \quad D(\mathbf{y};\pi) = \frac{\sum_{\mathbf{x}\in M_1(\mathbf{y};\pi)} p(\mathbf{x})}{\sum_{\mathbf{x}\in M_2(\mathbf{y};\pi)} p(\mathbf{x})}. \tag{1}$$

The expected utility of a type $\mathbf{y}$ individual from forming a pair under the strategy profile π is the expected utility from pairing given that the type of the prospective partner is in the set $M_2(\mathbf{y};\pi)$. Hence, the individual's expected total utility from search, denoted $R(y_a;\pi)$ since this expected utility depends only on an individual's attractiveness, is given by

$$R(y_a;\pi) = \frac{\sum_{\mathbf{x}\in M_2(\mathbf{y};\pi)} p(\mathbf{x}) g(x_a, |x_c - y_c|)}{\sum_{\mathbf{x}\in M_2(\mathbf{y};\pi)} p(\mathbf{x})} - c_1 L(\mathbf{y};\pi) - c_2 D(\mathbf{y};\pi). \tag{2}$$

Note that it is relatively simple to extend these calculations to non-symmetric games.

7. The Dating and Soliciting Subgames

Since this game is solved by recursion in the manner developed by Spear (1989, 1991), we first consider the dating subgame. In this section we consider the general formulation of the game.

7.1. **The Dating Subgame.** Assume that the population are following a symmetric strategy profile π. The male and female both have two possible actions: accept the prospective partner, denoted a, or reject, denoted r. Also, we ignore the costs already incurred by either individual, including the costs of the present date, as they are subtracted from all the payoffs in the matrix, and hence do not affect the equilibria in this subgame.

Suppose the male is of type $\mathbf{x}$ and the female is of type $\mathbf{y}$. The payoff matrix is given by

$$\begin{array}{rc} & \begin{array}{cc} \text{Female: } a & \text{Female: } r \end{array} \\ \begin{array}{r} \text{Male: } a \\ \text{Male: } r \end{array} & \left(\begin{array}{cc} [g(y_a,|x_c-y_c|),g(x_a,|x_c-y_c|)] & [R(x_a;\pi),R(y_a;\pi)] \\ [R(x_a;\pi),R(y_a;\pi)] & [R(x_a;\pi),R(y_a;\pi)] \end{array} \right) \end{array}.$$

Note that the payoff matrix depends on the strategy profile used via the expected utilities of the individuals involved in a subgame. These expected utilities were derived in Section 6.

From Condition 1, at equilibrium an individual accepts a prospective partner if and only if the utility gained from such a partnership is at least as great as the expected utility from future search. Hence, the appropriate Nash equilibrium of this subgame is for the male to accept the female if and only if $g(y_a,|x_c-y_c|) \geq R(x_a;\pi)$ and the female to accept the male if and only if $g(x_a,|x_c-y_c|) \geq R(y_a;\pi)$.

Note that for convenience, we assume that when $g(x_a,|x_c-y_c|) = R(y_a;\pi)$, a female always accepts the male (in this case she is indifferent between rejecting and accepting him). Similarly, if $g(y_a,|x_c-y_c|) = R(x_a;\pi)$, it is assumed that a male always accepts a female.

Note that if a male rejects a female, then the female is indifferent between accepting or rejecting the male. By a rule satisfying Condition 1, a female will make an optimal response whatever action the male takes (i.e. such an equilibrium is subgame perfect). Hence, there is a unique Nash equilibrium satisfying Condition 1. Let $\mathbf{v}(\mathbf{x},\mathbf{y};\pi) = [v_M(\mathbf{x},\mathbf{y};\pi), v_F(\mathbf{x},\mathbf{y};\pi)]$ denote the value of the dating subgame corresponding to this equilibrium, where $v_M(\mathbf{x},\mathbf{y};\pi)$ and $v_F(\mathbf{x},\mathbf{y};\pi)$ are the values of the subgame to the male and female, respectively.

We now consider the soliciting subgame.

7.2. **The Soliciting Subgame.** Once the dating subgame has been solved, we may solve the soliciting subgame and hence the game $G(\mathbf{x},\mathbf{y};\pi)$, played when a male of type $\mathbf{x}$ meets a female of type $\mathbf{y}$. As before, we assume that the population is following a symmetric strategy profile π.

Both players have two actions: a - accept (solicit a date) and r - do not solicit a date. Since the utility an individual expects from a date is independent of his/her character, the payoff matrix can be expressed as follows:

$$\begin{array}{ccc} & \text{Female: } a & \text{Female: } r \\ \begin{array}{c}\text{Male: } a \\ \text{Male: } r\end{array} & \left(\begin{array}{c} [\overline{v}_M(x_a, y_a; \pi) - c_2, \overline{v}_F(x_a, y_a; \pi) - c_2] \\ [R(x_a; \pi), R(y_a; \pi)] \end{array} \right. & \left. \begin{array}{c} [R(x_a; \pi), R(y_a; \pi)] \\ [R(x_a; \pi), R(y_a; \pi)] \end{array} \right) . \end{array}$$

Here $[\overline{v}_M(x_a, y_a; \pi), \overline{v}_F(x_a, y_a; \pi)]$ denotes the expected values of the dating subgame to the male and female, respectively, given the strategy profile used by the population, the measures of attractiveness of the prospective partners and the fact that a date followed.

From Condition 2, the female should solicit a date if and only if her expected utility from such a date is at least as great as the expected utility from future search, i.e.

$$\overline{v}_F(x_a, y_a; \pi) - c_2 \geq R(y_a; \pi).$$

Similarly, the male should solicit a date if

$$\overline{v}_M(x_a, y_a; \pi) - c_2 \geq R(x_a; \pi).$$

Note that Condition 2 requires the players to use the subgame perfect equilibrium in any soliciting subgame.

In the next section we present an algorithm to find a symmetric equilibrium of the symmetric game, as an algorithmic game in the tradition of Velupillai (1997); Nisam et al. (2007).

8. A Symmetric Equilibrium of the Symmetric Game

Theorems 8.1-8.3 describe the form of a symmetric equilibrium of the symmetric game. These results do not fully characterize such an equilibrium. However, they do justify the logic behind the algorithm presented in Subsection 8.1. The constructive form of this algorithm allows us to state the key result of this section, Theorem 8.5, on the existence and uniqueness of a symmetric equilibrium in the symmetric game.

Theorem 8.1. *At a symmetric equilibrium π^* of the symmetric game, the utility of an individual is non-decreasing in attractiveness.*

Proof. Assume that for some $i > j$, $R(i; \pi^*) < R(j; \pi^*)$. Consider the dating subgame. By assumption, type $[i, k]$ individuals are less choosy than type $[j, k]$ individuals, but are always preferred as partners. Hence, $F_2([j, k]; \pi^*) \subseteq F_2([i, k]; \pi^*)$. It follows that a searcher who is willing to date a prospective partner of attractiveness j will also be willing to a date one of attractiveness i (who is expected to be a better partner and at least as likely to be mutually acceptable). Hence, a searcher of type $[i, k]$ can obtain the same expected utility as a searcher of type $[j, k]$ as follows:

- **a:** In the soliciting subgame, solicit dates with any prospective partner who would date an individual of attractiveness j.
- **b:** In the dating subgame, accept any prospective partner who would pair with an individual of type $[j, k]$.

It follows that the expected utility of a searcher at such an equilibrium must be non-decreasing in his/her attractiveness.

■

Theorem 8.2. *At a symmetric equilibrium π^* of the symmetric game, searchers of maximum attractiveness, i_{max}, are willing to date prospective partners of attractiveness above a certain threshold.*

PROOF. From Theorem 8.1, if a searcher of attractiveness i_{max} accepts a prospective partner of type $[i, k]$ in the dating subgame, then acceptance is mutual. It follows that the expected utility of such a searcher from the dating subgame is non-decreasing in the attractiveness of the prospective partner (since the character is independent of attractiveness). A searcher should be willing to date a prospective partner if the expected utility from such a date is at least as great as the expected utility from future search. If this condition is satisfied for some level of attractiveness i, then it will be satisfied for all higher attractiveness levels. Note that a searcher of attractiveness i_{max} is willing to date a prospective partner of attractiveness i_{max}, since such dates give the highest possible expected utility. ■

Theorem 8.3. *At a symmetric equilibrium π^* of the symmetric game, a searcher of attractiveness i solicits dates with prospective partners of attractiveness in $[k_1(i), k_2(i)]$, where $k_2(i)$ is the maximum attractiveness of a prospective partner willing to date the searcher. In addition, $k_1(i)$ and $k_2(i)$ are non-decreasing in i and $k_1(i) \leq i \leq k_2(i)$.*

PROOF. The proof of this theorem is by recursion. Theorem 8.2 states that for $i = i_{max}$ the equilibrium strategies are of the appropriate form. Assume that Theorem 8.3 is valid for searchers of attractiveness $\geq i + 1$, where $i < i_{max}$.

First, suppose no prospective partner of attractiveness $> i$ will date a searcher of attractiveness i. By ignoring meetings with prospective partners of attractiveness $> i$, the game faced by searchers of attractiveness i can be reduced to a game in which they are the most attractive. From Theorem 8.2, it follows that searchers of attractiveness i are willing to date prospective partners of attractiveness in $[k_1(i), k_2(i)]$, where $k_2(i) = i < k_2(i + 1)$ and $k_1(i) \leq i < k_1(i + 1)$.

Now assume that $k_2(i) > i$. It follows that $k_1(i + 1) \leq i$. Firstly, we show that searchers are willing to date prospective partners of attractiveness j, where $i < j \leq k_2(i)$. If such a prospective partner finds a searcher of attractiveness i acceptable in the dating subgame, then acceptance is mutual. It follows that the expected utility of the searcher from such a date is greater than the expected utility of the prospective partner. Hence, from Theorem 8.1, a searcher of attractiveness i should be willing to date a prospective partner of attractiveness j.

Secondly, the proof that searchers of attractiveness i should be willing to date prospective partners of attractiveness j, where $j \leq i$, if and only if j is above some threshold is analogous to the proof of Theorem 8.2. Also, if a prospective partner of type $[k_1(i+1), l]$ is mutually acceptable to a searcher of type $[i+1, k]$ in the dating subgame, then from Theorem 8.1 a prospective partner of type $[k_1(i+1), l]$ accepts a searcher of type $[k_1(i+1), k]$ (and hence any searcher of the same character and greater attractiveness) in the dating subgame. Thus, by accepting exactly the same types of prospective partners of attractiveness $k_1(i+1)$ in the dating subgame as searchers of type $[i+1,k]$ do, a searcher of type $[i,k]$ will ensure the same expected utility from a date with a prospective partner of attractiveness $k_1(i+1)$ as a searcher of type $[i+1,k]$ obtains. This expected utility must be at least $R(i+1;\pi^*)$. It follows from Theorem 8.1 that $k_1(i) \leq k_1(i+1)$. ■

Hence, the general form of the equilibrium of the symmetric game is intuitive. Individuals date those who are of a similar level of attractiveness. It should also be noted that at such an equilibrium a type $[i,j]$ male will pair with a type $[i,j]$ female.

Due to the assumption that there are no costs associated with soliciting a date when dating does not follow, at such an equilibrium a searcher of attractiveness i is indifferent between soliciting and not soliciting a date with a prospective partner who is not willing to date. In this case we should check the condition based on the concept of subgame perfectness. This states that a searcher should solicit a date if the expected utility from dating after a 'mistaken' acceptance is greater than the expected utility from future search. Suppose a prospective partner of attractiveness j would not pair with any searcher of attractiveness i in the dating subgame. A searcher of attractiveness i should not solicit a date with a prospective partner of attractiveness j, in order to avoid the dating costs when there is no prospect of pairing.

It is possible that a prospective partner would wish to pair with a searcher of lower attractiveness in the dating subgame but, due to the costs of dating and the risks of obtaining a prospective partner of inappropriate character, would not solicit a date. This will be considered in the example given in Section 8.2.

Note: At the equilibrium of the classical problem considered by McNamara and Collins (1990), the population is partitioned into classes, such that class i males only form pairs with class i females. For the game considered here, such a partition only exists in very specific cases:

1: When the search and dating costs are low enough, type $[i,k]$ males only pair with type $[i,k]$ females.

2: When the costs of dating are high relative to the importance of character, mate choice is based entirely on attractiveness.

The difference between the two equilibria is illustrated by the example in Section 8.2.

After deriving some of the properties of an equilibrium, we now describe a procedure for deriving the equilibrium itself. Individuals of maximum attractiveness face a one-sided search problem. They should be willing to date a prospective partner if and only if the expected utility obtained from such a date minus the dating costs is at least as great as the expected utility from future search. Similarly, in the dating game a searcher should accept a prospective partner if and only if the utility gained from such a partnership is at least as great as the expected utility from future search. By following such a strategy, such individuals will maximise their expected utility from search, see Whittle (1982). Individuals of a lower level of attractiveness face a similar problem given the strategies followed by those of a higher level of attractiveness.

Since the solution of the corresponding set of inequalities is difficult to present in an explicit form, in Section 8.1 we describe an algorithm which derives a symmetric equilibrium. The constructive nature of this algorithm leads to the key theorem of the paper on the uniqueness and existence of a symmetric equilibrium in this game.

8.1. **The Algorithm.** Define $r = \lfloor \frac{m}{2} \rfloor$, i.e. the median character level if the number of character levels is odd and a character level neighbouring the median otherwise. Since the equilibrium is assumed to be symmetric with respect to character and sex, it suffices to consider males of character r. The game can be solved as follows

- **1:** Assume that males of maximum attractiveness are only willing to date females of maximum attractiveness. Consider strategy profiles π_t, $t = 0, 1, 2, \ldots, \lfloor m/2 \rfloor$, where under strategy π_t males of type $[i_{max}, r]$ pair with females whose characters do not differ by more than t (i.e. as t increases males accept successively less preferred females). We calculate $R(i_{max}; \pi_0), R(i_{max}; \pi_1), \ldots$ in turn until $R(i_{max}; \pi_t) > g(i_{max}, t+1)$ (i.e. the expected utility from search is greater than the utility from mating with any female of maximum attractiveness who is not acceptable) or $t = m/2$. This gives us the optimal rule of the form considered, see Whittle (1982), which is a lower bound on $R(i_{max}; \pi^*)$.
- **2:** If this lower bound is less than the utility obtained by a type $[i_{max}, r]$ male from pairing with a type $[i_{max} - 1, r]$ female minus the dating costs (i.e. the maximum possible reward from soliciting a date from a female of attractiveness $i_{max} - 1$), it may be optimal for males of maximum attractiveness to solicit dates with females of the second highest level of attractiveness. We can order females of the top two levels of attractiveness with regard to the preferences of a type $[i_{max}, r]$ male. By considering strategies under which type $[i_{max}, r]$ males are prepared to pair with successively less preferred females as in Point 1, we can derive the optimal strategy of males given they date females of the two highest levels of attractiveness.
- **3:** If required, in a similar way we can derive the optimal strategies of type $[i_{max}, r]$ males given that they date females of the u highest levels of attractiveness, where u is at least three and not more than the number of attractiveness levels. Hence, we can derive a strategy maximizing the expected utility of a type $[i_{max}, r]$ male. This strategy defines what attractiveness levels induce solicitation of a date from a male of maximum attractiveness and what types of females should be paired with after such dates (i.e. the pattern of dates and partnerships exhibited by individuals of maximum attractiveness at equilibrium).
- **4:** The strategy defined in Points 1-3 above should be extended to ensure subgame perfectness in all the possible derived dating subgames involving males of maximum attractiveness. The set of acceptable females in dating subgames can be easily found using Condition 1: i.e. a male of maximum attractiveness should accept a prospective partner in the dating subgame if and only if the utility he obtains from such a partnership is greater than his expected utility from search.

Note that the behaviour of males in dating subgames that do not occur under the equilibrium profile does not affect their expected utility from search at equilibrium.

Hence, the problem faced by a male of maximum attractiveness can be solved by solving a sequence of one-sided problems. The strategies used by other individuals of maximum attractiveness can be found using the symmetry of the profile with respect to character and sex. Note that it is assumed that if a male is indifferent between two strategies, then he uses the strategy which maximizes the number of attractiveness levels inducing willingness to date, together with the number of types of female that he will eventually pair with.

Suppose we have found the equilibrium strategies of individuals of attractiveness $> i$. The problem faced by a male of type $[i, r]$ reduces to a one-sided problem in which the set of females of higher attractiveness who are willing to date and pair with such males has been derived. The optimal response of such a male can be calculated in a way analogous to the one described in Points 1-4 above, with the following adaptations (which take into account the form of the equilibrium):

a: The initial strategy of a type $[i, r]$ male is as follows: A) solicit a date with i) any female of a greater attractiveness who would solicit a date from him, ii) any female of the same attractiveness and iii) females of lower attractiveness who would be solicited by males of attractiveness $i + 1$, B) in the dating game always pair with a female of the same type and any female who gives at least the same utility as the expected utility of an individual of attractiveness $i + 1$.

b: The set of prospective partners accepted in the dating subgame and solicited in the soliciting subgame is extended in an analogous way to the case of individuals of maximum attractiveness.

c: After determining the set of females that a male of type $[i, r]$ solicits a date with in the dating subgame and the females that he would pair with in the dating subgame, the behaviour of the male in the dating subgames that would not occur under the partial strategy profile derived as above is determined using the condition of subgame perfectness in the dating subgame.

The full strategy profile can be then defined using the assumption of the symmetry of the profile with respect to sex and character. It should be noted that although this algorithm has some similarities to the one presented by McNamara and Collins (1990), it is clearly different. Their algorithm is purely a one-dimensional search, which derives the sets of attractiveness levels that define the classes for each sex. The value of the symmetric game described here can be described by a one-dimensional function. However, in order to derive the equilibrium, a two-dimensional search over levels of both attractiveness and character is required.

The following theorem shows that the profile derived in this way is a symmetric equilibrium profile.

Theorem 8.4. *The strategy profile derived using the algorithm given above defines a symmetric equilibrium profile.*

Proof. First note that, by the definition of the algorithm, the form of strategy profile derived satisfies Theorems 8.2-8.3. Also, the behaviour of individuals in dating subgames that do not occur under the profile derived above explicitly satisfies the equilibrium conditions.

Now we show that under this strategy profile individuals of maximum attractiveness follow their optimal strategy. From the symmetry of the equilibrium and the strategy derived by the algorithm with respect to sex and character, we only need to consider males of character r.

First, consider males of type $[i_{max}, r]$. Suppose the maximum reward possible from a date with a female of attractiveness i minus the dating costs is exceeded by a lower bound on $R(i_{max}; \pi^*)$ (according to the algorithm males of maximum attractiveness do not solicit dates with such females). It follows from the conditions for subgame perfectness that a male of attractiveness i_{max} should not solicit a date with such a female (or any female of lower attractiveness). From Whittle (1982), it follows that given the set of females a male solicits dates from, a male should pair with a female if and only if the reward he obtains from pairing is greater than the expected reward from search. Hence, the algorithm considers all the possible optimal strategies of males of maximum attractiveness and chooses the best of these. From Theorem 8.1 males of maximum attractiveness essentially face a one-sided search problem, the algorithm thus derives the equilibrium strategy of males of maximum attractiveness.

Now suppose that this algorithm derives the equilibrium strategy of males of attractiveness $\geq i$. From Theorems 8.1 and 8.3 a male of type $[i-1, r]$ should solicit dates from any female of attractiveness $\geq i$ who solicits dates with him and pair with any female of attractiveness $\geq i$ who would pair with him in the dating game. The strategy of a male of type $[i-1, r]$ derived by the algorithm extends the sets of those females solicited and those paired with starting from a strategy which satisfies this condition. Given the strategies of the females of attractiveness $\geq i$, a male of attractiveness $i-1$ faces a one-sided search problem and the optimal strategy in this problem is derived in the same way as for individuals of maximum attractiveness (using the fact that a male of type $[i-1, r]$ should pair with any female that a male of type $[i, r]$ would pair with). Hence, the algorithm derives the equilibrium strategy of individuals of attractiveness $i-1$ given the strategies used by individuals of attractiveness $\geq i$. It follows by induction and the symmetry of the strategy profile derived that the algorithm gives a symmetric equilibrium profile. ■

Due to the form of the equilibrium and the finite number of types, the resulting strategy profile is well defined and unique. The theorem below follows directly from the construction of the symmetric equilibrium.

Theorem 8.5. *Assume that if any individual is indifferent between two strategies, then he/she uses the strategy which maximises the number of attractiveness levels inducing the solicitation of a date, together with the number of types of prospective partners that he/she will eventually pair with. There exists exactly one symmetric equilibrium of the symmetric game.*

One might ask whether an asymmetric equilibrium exists. Consider a finite-horizon game where each individual can observe up to n prospective partners. Suppose that in addition to Conditions 1-3, we require that an equilibrium profile in Γ is the limit of an equilibrium search profile in the finite-horizon game when $n \to \infty$. When $n = 1$, at the unique equilibrium profile each individual accepts any prospective mate (i.e. the equilibrium is symmetric). When n steps remain, an individual a) should solicit dates from prospective partners of attractiveness i if the expected utility from such a date is greater than the future expected utility from search (i.e. when $n-1$ steps remain) and b) pair with prospective partners in the dating subgame if the expected utility from pairing is greater than the future expected utility from search. Given the equilibrium profile in the $(n-1)$-step game is symmetric, all these expected utilities are independent of sex and character. Hence, the unique equilibrium profile in the n-step game is symmetric. It follows that there is a unique equilibrium profile of the symmetric game Γ, which satisfies Conditions 1-3 and is the limit of the equilibrium profile in the appropriately defined finite-horizon game.

8.2. **Example.** Suppose that the support of both X_a and X_c is $\{0, 1, 2, 3, 4, 5, 6\}$. It is assumed that the distributions of attractiveness and character are uniform. The search costs, c_1, and the interview costs, c_2 are equal to $\frac{1}{7}$. The utility obtained from a partnership is defined to be the attractiveness of the partner minus the distance (modulo 7) between the characters of the pair.

First we consider males of maximum attractiveness. Suppose they only solicit dates with females of attractiveness 6. The ordered preferences of a $[6, 3]$ male are as follows: first (group one) - $[6, 3]$, second equal (group two) - $[6, 2], [6, 4]$, fourth equal (group 3) $[6, 1], [6, 5]$ and sixth equal (group 4) - $[6, 0], [6, 6]$. Group 1, 2, 3 and 4 females give a utility from pairing of 6, 5, 4 and 3, respectively. We successively include females into the set of acceptable partners starting from the most preferred until no female of attractiveness 6 outside this set gives a greater utility than the current expected utility of a type $[6, 3]$ male. Let π_i denote any symmetric strategy profile in which type $[6, 3]$ males pair with females from the first i groups described above, $i = 1, 2, 3, 4$.

$$
\begin{aligned}
R(6; \pi_1) &= 6 - 49 \times \frac{1}{7} - 7 \times \frac{1}{7} = -2 \\
R(6; \pi_2) &= \frac{16}{3} - \frac{49}{3} \times \frac{1}{7} - \frac{7}{3} \times \frac{1}{7} = \frac{8}{3} \\
R(6; \pi_3) &= \frac{24}{5} - \frac{49}{5} \times \frac{1}{7} - \frac{7}{5} \times \frac{1}{7} = \frac{16}{5}.
\end{aligned}
$$

The expected utility of a type $[6, 3]$ male under π_3 is greater than 3. It follows that $[6, 0]$ and $[6, 6]$ females should not be accepted in the dating subgame.

Since our lower bound (3.2) on the expected utility of a type $[6, 3]$ male is less than the utility obtained from pairing with a female of the same character and the second highest attractiveness minus the costs of dating, $\frac{34}{7}$, we now consider strategy profiles in which type $[6,3]$ males solicit dates with females of attractiveness 5 and 6. We only have to consider:

1: Strategy profiles in which females of type $[5,3]$ are acceptable in the dating subgame. If this were not the case, then a type $[6,3]$ employer would be incurring unnecessary dating costs.

2: Prospective partners who give a utility higher than the current lower bound on the expected utility of a type $[6,3]$ male from search.

The ordered preferences of a type $[6,3]$ male among the set of females of attractiveness at least 5 who satisfy criterion 2 above is given by: group 1 is $\{[6,3]\}$, group 2 is $\{[6,2],[5,3],[6,4]\}$ and group 3 $\{[6,1],[6,5],[5,2],[5,4]\}$. We only need to consider strategy profiles of the following two types: a) type $[6,3]$ males pair with females from groups 1 and 2 above, denoted π_4, b) type $[6,3]$ males pair with females from all three groups, denoted π_5. We have

$$\begin{aligned} R(6;\pi_4) &= \frac{21}{4} - \frac{49}{4} \times \frac{1}{7} - \frac{14}{4} \times \frac{1}{7} = 3 \\ R(6;\pi_5) &= \frac{37}{8} - \frac{49}{8} \times \frac{1}{7} - \frac{14}{8} \times \frac{1}{7} = \frac{7}{2}. \end{aligned}$$

We now consider strategy profiles in which type $[6,3]$ males solicit dates with females of attractiveness at least 4. Since the present lower bound on $R(6;\pi^*)$ is 3.5, we only need to consider strategy profiles in which type $[6,3]$ males pair with the same types of females as in π_5 with the addition of type $[4,3]$ females. Denote such a strategy profile by π_6. We have

$$R(6;\pi_6) = \frac{41}{9} - \frac{49}{9} \times \frac{1}{7} - \frac{21}{9} \times \frac{1}{7} = \frac{31}{9} < \frac{9}{2}.$$

It follows that type $[6,3]$ males should not solicit dates with females of attractiveness 4. Hence, at a symmetric equilibrium, type $[6,3]$ males solicit dates with females of attractiveness 5 and 6 and pair with females of type in M_6, where

$$M_6 = \{[6,1],[6,2],[6,3],[6,4],[6,5],[5,2],[5,3],[5,4]\}.$$

It should be noted that a type $[6,3]$ male should pair with a type $[4,3]$ female given that they are dating. However, due to the costs of dating, the low probability of finding an acceptable partner of attractiveness 4 and the relatively small gain obtained from such a partnership compared to the expected utility from future search, a type $[6,3]$ male should not solicit a date with such a female. Define $M_i + [s,t]$ to be the set of $[k+s, j+t]$ where $[k,j] \in M_i$. From the symmetry of the equilibrium with respect to character and sex, an individual of type $[6, 3+t]$ is willing to date prospective partners of attractiveness 5 and 6 and pair with those in $M_6 + [0,t]$.

Note that searchers are not matched in a block-separated way as in McNamara and Collins (1990). For example, a type $[6,3]$ male will pair with a type $[6,1]$ female, who would mate with a type $[6,0]$ male. However, a type $[6,3]$ male will not pair with a type $[6,0]$ female.

Now we consider males of type $[5,3]$ and assume that individuals of maximum attractiveness follow the strategies derived above and males of attractiveness 5 solicit dates with females of attractiveness 5 and 6 (from the form of the equilibrium, a male should solicit dates with females of the same attractiveness). From the symmetry of the game with respect to sex and character, since males of type $[6,3]$ pair with females of type $[5,2],[5,3]$ and $[5,4]$, it follows that males of type $[5,3]$ will pair with females of type $[6,2]$, $[6,3]$ and $[6,4]$. They must also pair with females of type $[5,2],[5,3]$ and $[5,4]$ as such females give a type $[5,3]$ male a utility of $4 \geq R(6;\pi^*) \geq R(5;\pi^*)$. The expected utility of a type $[5,3]$ male under such a strategy profile, π_7, is

$$R(5;\pi_7) = \frac{29}{6} - \frac{49}{6} \times \frac{1}{7} - \frac{14}{6} \times \frac{1}{7} = \frac{10}{3}.$$

This is greater than the expected utility from accepting the next most preferred types ($[5,1]$ and $[5,5]$). Hence, we can now consider strategy profiles in which males of type $[5,3]$ solicit dates with females of attractiveness at least 4. The only case we need to consider is extending the set of acceptable females to include those of type $[4,3]$. Denote this new strategy profile by π_8. We have

$$R(5;\pi_8) = \frac{33}{7} - \frac{49}{7} \times \frac{1}{7} - \frac{21}{7} \times \frac{1}{7} = \frac{23}{7} < \frac{10}{3}.$$

It follows that males of type $[5,3]$ should solicit dates with females of attractiveness 5 and 6 and pair with females of a type in $\{[5,2],[5,3],[5,4],[6,2],[6,3],[6,4]\}$. In these cases acceptance is mutual. It should also be noted that males of type $[5,3]$ should accept females of type $[6,1]$ or $[6,5]$ in the dating subgame. However, in these cases acceptance is not mutual. Females of type $[4,3]$ would be accepted in the dating subgame by a type $[5,3]$ male, but such males would not solicit a date with such a female.

It can be seen that males and females of the top two levels of attractiveness do not date any other individuals. The problem faced by males of attractiveness 4 thus reduces to a problem analogous to the one faced by those of attractiveness 6 (they are the most attractive of the remaining males). It follows that $M_4 = M_6 - [2,0]$. Arguing iteratively, $M_i = M_{i+2} - [2,0]$ for $i = 1,2,3,4$. Males of attractiveness 2 or 4 solicit dates with females of the same attractiveness or of attractiveness one level lower. Males of attractiveness 1 or 3 solicit dates with females of the same attractiveness or of attractiveness one level higher. We have $R(4;\pi^*) = 1.5$ and $R(3;\pi^*) = \frac{4}{3}$, so in the dating subgame males of attractiveness 3 and 4 accept any females giving them a utility of at least 2 (in this game the utility from a pairing is by definition an integer). Also, $R(2;\pi^*) = -0.5$ and $R(1;\pi^*) = -\frac{2}{3}$, so in the dating subgame males of attractiveness 1 and 2 accept any female giving them a utility of at least 0.

Since females of attractiveness 5 and 6 do not solicit dates with males of attractiveness 4, such males are indifferent between soliciting and not soliciting dates with such females. We should check the relevant equilibrium condition based on the concept of subgame perfectness, i.e. if a female of attractiveness 6 did 'by mistake' accept a date with a male of attractiveness 4, should the male solicit a date? In the dating subgame, only a female of type $[6,3]$ would accept a male of type $[4,3]$. Hence, the expected utility of a type $[4,3]$ male from dating a female of attractiveness 6 is

$$\overline{v}_M(4,6;\pi^*) = \frac{1}{7}\times 6 + \frac{6}{7}\times 1.5 - \frac{1}{7} = 2 > R(4;\pi^*).$$

It follows that males of attractiveness 4 should solicit dates with females of attractiveness 6. Arguing similarly, such males should solicit dates with females of attractiveness 5 and males of attractiveness 2 should solicit dates with females of attractiveness 3 or 4.

No females of attractiveness 5 or 6 would pair with a male of attractiveness 3 in the dating subgame. It follows that males of attractiveness 3 should not solicit dates with females of attractiveness 5 or 6. Arguing similarly, a male of attractiveness 1 should not solicit dates with females of attractiveness above 2.

It remains to determine the strategy used by a type $[0,3]$ male. Since no female of greater attractiveness will date such a male, we only have to consider strategy profiles such that a male pairs with successively less preferred partners of attractiveness 0. Note that $R(1;\pi^*) = -2/3$ is an upper bound on $R(0;\pi^*)$, thus in the dating game individuals of attractiveness 0 must pair with any prospective partner who gives them a utility of 0 (i.e. with those of the same type). Denote the strategy profile under which individuals of attractiveness at least 1 use the strategies derived above and males of type $[0,3]$ pair with females of types in a) $\{[0,3]\}$, b) $\{[0,2],[0,3],[0,4]\}$ and c) $\{[0,1],[0,2],[0,3],[0,4],[0,5]\}$ by π_9, π_{10} and π_{11}, respectively. We have

$$\begin{aligned} R(0;\pi_9) &= -49\times\frac{1}{7} - 7\times\frac{1}{7} = -8 \\ R(0;\pi_{10}) &= -\frac{2}{3} - \frac{49}{3}\times\frac{1}{7} - \frac{7}{3}\times\frac{1}{7} = -\frac{10}{3} \\ R(0;\pi_{11}) &= -\frac{6}{5} - \frac{49}{5}\times\frac{1}{7} - \frac{7}{5}\times\frac{1}{7} = -\frac{14}{5}. \end{aligned}$$

The expected utility of a type $[0,3]$ male under π_{11} is greater than the utility obtained from pairing with females of type $[0,0]$ and $[0,6]$. Hence, at a symmetric equilibrium males of type $[0,3]$ should not accept such females.

It remains to consider the set of females that a male of attractiveness 0 should solicit a date with according to the conditions based on subgame perfection. At equilibrium, no female of attractiveness greater than 2 would ever pair with a male of attractiveness 0 in the dating subgame. Hence, males of attractiveness 0 should never solicit dates with females of attractiveness above 2. Females of type $[2,3]$ and $[1,3]$ would pair with a type $[0,3]$ male in the dating subgame. Arguing as in the case of males of attractiveness 4 soliciting dates with females of attractiveness 5 and 6, males of attractiveness 0 should solicit dates with females of attractiveness 1 and 2.

Table 1 gives a synopsis of the equilibrium strategy profile. Each individual should accept a prospective partner in the dating subgame if the utility from such a matching is at least as great as the utility from search. For ease of presentation, the set of such partners is not presented.

Attractiveness	Solicits dates with prospective partners of attractiveness	Expected Utility
6	{ 5,6 }	3.50
5	{ 5,6 }	3.33
4	{ 3,4,5,6 }	1.50
3	{ 3,4 }	1.33
2	{ 1,2,3,4 }	-0.50
1	{ 1,2 }	-0.67
0	{ 0,1,2 }	-2.80

Table 1. Brief description of the symmetric equilibrium for the example considered

9. Generalizing the Model

Suppose that character is placed along a line instead of around a ring, i.e. the difference between two characters is calculated according to the standard absolute difference. Considering the game presented in Section 8.2 (with unspecified search and dating costs), there is still a large degree of symmetry with respect to sex and character (e.g. the character levels j and $6-j$ can be interchanged without essentially changing the game).

We wish to derive an equilibrium which reflects this inherent symmetry. Suppose a type $[i,j]$ male solicits a date with a female of attractiveness k and pairs with a female of type $[k,l]$. Firstly, a type $[i,j]$ female should be willing to date a male of attractiveness k and pair with a male of type $[k,l]$. Secondly, a type $[i,6-j]$ male should solicit a date with a female of attractiveness k and pair with a female of type $[k,6-l]$.

It is expected that males of type $[6,3]$ have the highest expected utility from search and so we can treat the problem they face as a one-sided problem. However, it is unclear whether in a specific problem individuals of type $[6,2]$ or those of type $[5,3]$ should have the higher expected utility from search at such an equilibrium. Hence, it is unclear how the algorithm should proceed.

Hence, in order to solve more general problems, the algorithm presented in Section 8 must be further developed. However, it seems that the general approach of solving a sequence of appropriately defined one-sided problems may well be useful in deriving a strategy profile which is at least very similar to an equilibrium strategy profile (see Ramsey [2010] for a similar approach). Also, the form of the general problem and the usefulness of such an approach indicate that if there are multiple equilibria, then the behaviour observed at such equilibria should be qualitatively similar.

10. Conclusion

This paper has presented a model of partnership formation where both common and homotypic preferences are taken into account. The preferences of all searchers are common with respect to the attractiveness of prospective partners and homotypic with respect to character. Attractiveness can be assessed immediately, but in order to assess character a costly date (or interview) is required.

We have considered a particular type of such problems in which the distribution of attractiveness and character, as well as search and interview costs, were independent of the class (sex) of a player. Character was assumed to form a ring, such that the 'extreme' levels of character are neighbours. For convenience, the supports of attractiveness and character were assumed to be finite sets of integers. The distribution of character is uniform.

The form of a symmetric equilibrium profile which satisfies various criteria based on the concept of subgame perfectness was derived and an algorithm to find such a profile described. These criteria are a generalization of the optimality criterion used by McNamara and Collins (1990) to define the unique equilibrium in the classical two-sided job search problem. It is shown that such an equilibrium exists and is unique (assuming that if an individual is indifferent between accepting or rejecting a prospective partner at any stage, then he/she accepts). Although the equilibrium derived here does have some similarities to the equilibrium derived by McNamara and Collins (1990), it is essentially different, since it is not a block separating equilibrium.

The use of this combination of preferences would seem to be logical in relation to job search and mate choice. Although there is no perfect correlation in individuals' assessment of the attractiveness of members of the other class, there is normally a very high level of agreement, particularly among males in mate choice problems. These 'mixed' preferences seem to be both reasonably tractable within the framework of searching for a partner within a relatively large population and allow a general enough framework to model the preferences of individuals reasonably well (although it would seem that modelling character as a one-dimensional variable is rather simplistic). By using a larger number of types, we could approximate continuous distributions of attractiveness and character.

For simplicity it was assumed that individuals know their own attractiveness and character, whereas in practice they may have to learn about these measures over time (see Fawcett and Bleay (2009)).

Also, individuals are able to measure attractiveness and character perfectly, although at some cost. It would be interesting to consider different ways in which information is gained during the search process. For example, some information about the character of a prospective partner may be readily available. Hence, an improved model would allow some information to be gained on both the attractiveness and character of a prospective partner at each stage of an interaction.

In terms of the evolution of such procedures, it is assumed that the basic framework is given, i.e. the model assumes that the various search and dating (interview) costs are given. Hence, this model cannot explain why such a system has evolved, only the evolution of decisions within this framework.

Individuals may lower their search costs by joining some internet or social group. Such methods can also lead to biasing the conditional distribution of the character of a prospective partner in a searcher's favour. It is possible that dating (interview) costs are dependent on the types of the two individuals involved. For example, two individuals of highly different characters might incur low dating costs, as they realize very quickly that they are not well matched.

Also, the ability to incur dating costs may well transfer information regarding the attractiveness and/or character of an individual. In this case, it may be more costly to successfully date highly attractive prospective partners, since they would only accept partners who can pay high dating costs (i.e. are attractive).

In addition, it would be useful to investigate how the utility functions, together with the relative costs of searching and interviewing, affect the importance of attractiveness and character in the decision process. It should be noted that using attractiveness as an initial filter in the decision process will lead to attractiveness becoming relatively *more* important than character, especially if the costs of dating are relatively high.

It would also be useful to adapt the algorithm to problems in which the distribution of character is not uniform and/or the set of character levels do not form a ring. In this case, it is expected that individuals of extreme character will usually be less choosy than those of a central character for a given level of attractiveness. Two major problems result from this. Firstly, the form of an equilibrium will be more complex than the form of the symmetric equilibrium given here. Any algorithm to derive an equilibrium in this case will certainly be more complex than the algorithm outlined in this paper, which uses the fact that the problem can be reduced to a sequence of one-sided problems. The unique equilibrium derived here would be useful as a point of reference.

Finally, it would be interesting to consider games in which the distributions of traits and/or search costs depended on class. In this case, it would be natural to assume that the equilibrium is asymmetric with respect to class. In the spirit of the derivation of equilibrium points in the classical matching problem (see Gale and Shapley (1962)), it would be interesting to see whether equilibria analogous to male-choice and female-choice equilibria exist. For the types of model considered here, to find a male-choice equilibrium we would try to maximize the expected utility of males while adapting female choice to male choice.

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