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Thomas Conlon
Smurfit School of Business
University College Dublin

John Cotter
Smurfit School of Business
University College Dublin

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Downside Risk and the Energy Hedger's Horizon

Thomas Conlon^a, John Cotter^a

*^aSmurfit School of Business, University College Dublin, Carysfort Avenue, Blackrock,
Co. Dublin, Ireland.*

Abstract

In this paper, we explore the impact of investor time-horizon on an optimal downside hedged energy portfolio. Previous studies have shown that minimum-variance hedging effectiveness improves for longer horizons using variance as the performance metric. This paper investigates whether this result holds for different hedging objectives and effectiveness measures. A wavelet transform is applied to calculate the optimal heating oil hedge ratio using a variety of downside objective functions at different time-horizons. We demonstrate decreased hedging effectiveness for increased levels of uncertainty at higher confidence intervals. Moreover, for each of the different hedging objectives and effectiveness measures studied, we also demonstrate increasing hedging effectiveness at longer horizons. While small differences in effectiveness are found across the different hedging objectives, time-horizon effects are found to dominate confirming the importance of considering the hedgers horizon. The findings suggest that while downside risk measures are useful in the computation of an optimal hedge ratio that accounts for unwanted negative returns, hedging horizon and confidence intervals should also be given careful consideration by the energy hedger.

Keywords: Energy Hedging, Futures Hedging, Wavelet Transform, Hedging Horizon, Downside Risk.

1. Introduction

The practise of risk management often involves the use of futures contracts to manage the price risk associated with a given spot market position. The optimal futures hedge ratio necessary to reduce the risk associated with a given spot position is commonly determined using variants of the minimum-variance approach (Chen et al., 2003; Brooks et al., 2002). However, the minimum-variance approach treats positive and negative fluctuations equally, while hedgers may prioritize the reduction of downside risk only (Hung & Lee, 2007). Another important hedging consideration is the horizon over which a hedger wishes to reduce risk, with improved effectiveness found for longer horizons (Ederington, 1979). However, these and other studies have only considered the impact of the hedging horizon on the minimum-variance hedge ratio (For example, In & Kim (2006)). In this study, we build on the previous literature by considering the effect of the hedging horizon on both the optimal futures hedge ratio and associated effectiveness for a variety of downside risk hedging objectives.

The primary aim of corporate risk management is to provide protection against the possibility of dangerous tail-risk events (Stulz, 1996). In the context of futures hedging, a range of alternative downside risk measures have been proposed to estimate and subsequently limit the impact of low probability tail-risk events.¹ Value-at-risk (VaR) and conditional value-at-risk (CVaR or expected shortfall) are two approaches to measure potential loss of a portfolio over a given period² and have been applied as risk objectives in a portfolio allocation setting (Adam et al., 2008). In recent research VaR and CVaR have also been adopted for use as the hedging risk objective function. Using VaR and CVaR as the measure of hedge portfolio risk, Chang (2011) and Harris & Shen (2006) determined the optimal futures hedge ratio required to minimize the tail risk of the hedge portfolio. An alternative downside risk measure often applied in the literature is the semivariance, which measures the expected value of deviations below a target value. The semivariance may also be applied to calculate the optimal hedge ratio necessary to reduce the semivariance risk of a spot position hedged by futures (Turvey & Nayak, 2003). In this paper, we move beyond the minimum-variance framework to consider the impact of the energy hedging horizon and the

¹A thorough review of the different objective functions applied to minimize risk in futures hedging can be found in Chen et al. (2003). Recent studies have also examined the impact of investor preferences such as risk aversion on the optimal hedging strategy, finding significant variation from risk minimization strategies (Cotter & Hanly, 2012, 2010).

²Additional details and applications of both VaR and CVaR can be found in Bertsimas et al. (2004); Basak & Shapiro (2001)

investor confidence investor on differing risk objective functions including value-at-risk, conditional value-at-risk and semivariance.

Given the range of alternative risk objective functions available to energy futures hedgers, it is important to understand their performance for different hedging horizons. The effectiveness of a standard minimum-variance energy hedge has been explored using a variety of performance metrics, including variance, semi-variance, VaR and CVaR. Further, performance of downside objective functions has been assessed using a spectrum of performance measures (Harris & Shen, 2006). However, previous studies have only evaluated downside effectiveness from the perspective of an investor considering a single confidence level. In this study, we expand futures hedging performance measurement to incorporate a range of confidence intervals at different hedging horizons.

Earlier studies considered data sampled at weekly, fortnightly and monthly horizons to demonstrate an increase in variance reduction effectiveness for hedgers at longer horizons (Benet, 1992; Ederington, 1979). However, Benet (1992) found a lack of stability in the effectiveness at longer horizons out-of-sample, due to the sample reduction problem. To overcome the problem of reduced data at longer horizons, wavelet multiscaling techniques have been adopted.³ Applying a minimum-variance framework, In & Kim (2006) demonstrated that S&P index hedgers achieve greater effectiveness at longer horizons. Examining a wide range of futures contracts, Lien & Shrestha (2007) showed both increased hedge ratios and out-of-sample effectiveness in the case of a minimum-variance hedger for increasing horizon. Finally, Conlon & Cotter (2012) applied a minimum-variance framework to introduce a dynamic time and horizon dependent futures hedge ratio.

This paper makes a number of contributions to the literature. First, in the case of energy hedging, we demonstrate a decrease in hedging effectiveness for increased levels of risk uncertainty at all hedging horizons. Next, we explore the impact of different risk objective functions with an increase in the optimal heating oil hedge ratio and improved effectiveness at longer horizons found, regardless of the objective function used to minimize

³The wavelet transform has been applied to a range of problems in economics and finance, including the characteristics of international diversification at differing horizons (Rua & Nunes, 2009), the relationship between stock returns and inflation for various horizons Kim & In (2005) and the determination of the beta of a stock and the expected returns at varying horizons (Gençay et al., 2005). Moreover, wavelets have been applied to separate short-term noise and long term trends in crude oil prices (Naccache, 2011; de Souza e Silva et al., 2010). Further details on wavelet multiscaling can be found in Gençay et al. (2001).

hedge portfolio risk. While small differences in effectiveness are found across the different hedging objectives, time-horizon effects are found to dominate confirming the importance of considering the hedger's horizon. The findings suggest that while downside risk measures are useful in the computation of an optimal hedge ratio that accounts for unwanted negative returns, the hedging horizon and confidence intervals should also be given careful consideration by the hedger.

This remainder of this paper is structured as follows: Section 2 introduces the wavelet multiscaling technique and outlines the different objective functions examined. In section 3, the empirical energy data and their properties are outlined. Section 4 discusses the empirical findings, while conclusions are given in section 5.

2. Methodology

In determining the optimal hedge ratio necessary to reduce the risk associated with a given spot position, market participants need to consider preferences related to hedging horizon and risk objective. In this section, we introduce wavelet multiscaling techniques to understand the behaviour of financial time-series at different horizons and then consider a variety of alternative hedging objective functions including the downside metrics value-at-risk, conditional value-at-risk and semivariance.

2.1. Wavelet Analysis

Here, we provide a brief introduction into the application of wavelet multiscaling techniques to the time-horizon decomposition of financial time-series.⁴ Wavelets can be interpreted as 'small waves' with limited duration and are capable of decomposing a time-series in both time and frequency. By incorporating information from all available data at different horizons, wavelets overcome the problem of reduced data at longer horizons. Wavelet analysis can be considered a generalization of the Fourier transform, which employs harmonic functions as a basis, characterized by frequency. While the wavelet transform also uses oscillatory functions, in contrast to the Fourier transform these decay rapidly to zero. The family of functions generated using the wavelet transform are dilations and transformations of a single function, the mother wavelet, providing self-similarity. This can be

⁴Readers are referred to In & Kim (2006); Gençay et al. (2001); Percival & Walden (2000) for further technical detail.

distinguished from the windowed Fourier transform, where the frequency, width and position of the window are all independent. The wavelet transform is particularly suitable for financial data, due to the ability to accurately capture high frequency, high amplitude events such as spikes in returns, to decompose non-stationary data and also, due to self-similarity properties.

The discrete wavelet transform (DWT) provides an efficient means of studying the multiresolution properties of a signal, allowing decomposition into different time-horizons or frequency components (Gençay et al., 2001; Percival & Walden, 2000). The DWT consists of two basic wavelet functions, the father ϕ and mother ψ wavelets, given by:

$$\phi_{j,k}(t) = 2^{-\frac{j}{2}} \phi(2^j t - k) \quad (1)$$

$$\psi_{j,k}(t) = 2^{-\frac{j}{2}} \psi(2^j t - k) \quad (2)$$

where $j = 1, \dots, J$ in a J -level decomposition and k is a translation parameter. The father wavelet integrates to one and reconstructs the longest time-scale component of the series, (the trend), while the mother wavelet integrates to zero and is used to describe deviations from the trend. It is possible to show that a discrete signal $f(t)$ can be decomposed as a sequence of projections onto the father and mother wavelets. In particular, the orthogonal wavelet series approximates a continuous signal $f(t)$ as

$$f(t) \approx \sum_k s_{J,k} \phi_{J,k}(t) + \sum_k d_{J,k} \psi_{J,k}(t) + \dots + \sum_k d_{1,k} \psi_{1,k}(t) \quad (3)$$

where J is the number of multiresolution levels (or *scales*) and k ranges from 1 to the number of coefficients in the specified level. The coefficients $s_{J,k}$ and $d_{j,k}$ are the wavelet transform coefficients, while $\phi_{J,k}$ and $\psi_{j,k}$ are the approximating wavelet functions, where coefficients from level $j = 1 \dots J$ are associated with scale $[2^{j-1}, 2^j]$.

In this paper, we apply an extended version of the DWT, the maximum overlap discrete wavelet transform (MODWT), a variation of the orthogonal discrete wavelet transform (Percival & Walden, 2000). The MODWT is considered preferable to the DWT as it is asymptotically more efficient. This extension helps to overcome two

major drawbacks of the DWT, dyadic length requirements for the data and the fact that the DWT is non-shift invariant. Like the DWT it produces a set of time-dependent wavelet and scaling coefficients with each basis vector associated with a location t and a unitless scale $\tau_j = 2^{j-1}$ for decomposition level $j = 1, \dots, J$. It retains all coefficients at each scale, incorporating additional location information discarded by the DWT. This reduces the tendency for larger errors at lower frequencies, important in our application of calculating optimal futures hedge ratios using downside risk measures. Further, the MODWT produces a variance estimate that is asymptotically more efficient than that of the DWT, providing us with insight into the relationship between time-horizon and hedging strategy of the investor.

Decomposing a signal using the MODWT to J levels involves the application of a rescaled father wavelet $\tilde{\phi}_{j,l}$ to yield a set of smooth coefficients

$$\tilde{s}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{\phi}_{j,l} f_{t-l \bmod N} \quad (4)$$

and a rescaled mother wavelet $\tilde{\psi}_{j,l}$ to yield a set of detail coefficients

$$\tilde{d}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{\psi}_{j,l} f_{t-l \bmod N} \quad (5)$$

for all times $t = \dots, -1, 0, 1, \dots$, where f is the original signal to be decomposed (Percival & Walden, 2000). The MODWT applies a set of scale-dependent localized differencing and averaging operators, $\tilde{\phi}_{j,t} = \frac{\phi_{j,l}}{2^j}$ and $\tilde{\psi}_{j,l} = \frac{\psi_{j,l}}{2^j}$ which can be considered as rescaled versions of the originals. The coefficients $\tilde{d}_{j,t}$ are associated with changes at horizon j .

Once a signal have been decomposed into constituent horizons, the coefficients can then be used to calculate the contribution of each horizon to the overall variability of the time-series (Percival & Walden, 2000). Specifically, the wavelet coefficients, $d_{j,t}^f$ and $d_{j,t}^g$, associated with a particular time-horizon j and time t for functions f and g can be used to calculate a horizon dependent wavelet covariance (Percival & Walden, 2000). An unbiased

estimator of the wavelet covariance⁵ at scale $\tau_j = 2^{j-1}$ is given by

$$cov_{fg}(\tau_j) = \frac{1}{M_j} \sum_{t=L_j-1}^{N-1} d_{j,t}^f d_{j,t}^g \quad (6)$$

where $M_j = N - L_j + 1$ is the number of coefficients remaining after discarding the boundary coefficients. The wavelet variance for function f at a particular scale is similarly defined,

$$variance_f(\tau_j) = \frac{1}{M_j} \sum_{t=L_j-1}^{N-1} \left[d_{j,t}^f \right]^2. \quad (7)$$

The wavelet variance and covariance decompose the statistics of the financial time-series at increasingly higher resolutions, (on a scale-by-scale basis), and allows the exploration of the signal at different time-horizons. In the case of the daily data examined in this study, the wavelet variance-covariance structure can be found at 1 – 2, 2 – 4, 4 – 8, 8 – 16 and 16 – 32 day horizons, incorporating both short- and long-run horizons.⁶

2.2. Hedge Ratios and Downside Risk

In this paper, we compare the hedging effectiveness of a number of different downside risk objectives to a minimum-variance hedge at different hedging horizons. Assuming the hedger has a long spot position in the underlying commodity, the return, r_h , on the hedge portfolio is given by

$$r_h = r_s - hr_f \quad (8)$$

where r_s and r_f are the log returns of the spot and futures prices respectively. The hedge ratio h corresponds to the quantity of futures contracts that must be short sold to remove the risk associated with the hedgers objectives. Different approaches to the risk objective function include value-at-risk, conditional value-at-risk, semivariance

⁵At the boundary of a time-series, the wavelet transform uses 'mirrored' coefficients, potentially introducing a bias to the data. To alleviate any bias, the coefficients affected by the boundary are removed from the calculation of the statistics

⁶For ease of exposition, throughout this study we take an average of each of the above horizons, referring instead to 1.5, 3, 6, 12 and 24 day horizons.

and variance, and are described in turn. We also consider the same vector of measures to assess the performance of the respective hedge ratios.

2.2.1. Value-at-Risk

Value-at-risk (VaR) has been applied to measure tail-risk across a range of asset classes, (Cabedo & Moya, 2003; Basak & Shapiro, 2001). Recently, VaR has been introduced both to estimate the optimal futures hedge ratio, (Chang, 2011; Harris & Shen, 2006), and to determine the effectiveness of the hedge portfolio relative to the spot (Conlon & Cotter, 2012). For a given confidence level, γ , VaR is defined as the maximum expected loss on a portfolio over a given time-period and given by

$$VaR(\Phi_r, \gamma) = -\Phi_r^{-1}(1 - \gamma), \quad (9)$$

where Φ , the cumulative distribution function, is the probability that the portfolio returns will be less than a given value. In order to find the optimal VaR hedging strategy, we apply the historical simulation methodology (Dowd, 2008; Harris & Shen, 2006; Cabedo & Moya, 2003). Using a grid search approach, a range of alternative hedge ratios are tested in order to find the hedge ratio that results in a global minimum for the hedge portfolio. Although the historical simulation VaR may not be a global convex function of the hedge ratio, a global minimum can be found using a grid search of the parameter space. At a given confidence level, this approach selects the minimum historical simulation VaR of the hedge portfolio from a range of possible hedge ratios.

Given a particular hedge ratio, the hedging effectiveness can also be calculated using the VaR framework. For a hedge portfolio with hedge ratio h , the VaR hedging effectiveness is given by

$$HE_{VaR_\gamma} = 1 - \frac{VaR_\gamma(r_h)}{VaR_\gamma(r_s)} \quad (10)$$

where $VaR_\gamma(r_h)$ and $VaR_\gamma(r_s)$ are the VaR of the hedge portfolio and spot at confidence level γ . The effectiveness measure, HE_{VaR_γ} , determines the proportion of the original spot VaR that is removed by futures hedging for a given hedge ratio.

2.2.2. Conditional Value-at-Risk

Conditional Value-at-Risk (CVaR) or expected shortfall is a downside risk measure applied for both risk management and asset allocation purposes in finance (Bertsimas et al., 2004). CVaR is a coherent, spectral measure of financial risk and determines the expected loss of a given portfolio conditional on the event that the VaR is exceeded. The CVaR of a portfolio with returns r is given by

$$CVaR(\Phi_r, \gamma) = -\frac{1}{1-\gamma} \int_0^{1-\gamma} \Phi_r^{-1}(p) dp \quad (11)$$

and is a weighted average of returns less than the VaR for a given confidence interval γ . Similar to the approach applied for VaR minimization, the optimal CVaR hedge ratio is found using the historical simulation method. A range of possible hedge ratios are tested incrementally using a grid search and the hedge ratio with the lowest portfolio CVaR is selected (see also Harris & Shen (2006)).

The CVaR hedging effectiveness for a hedge portfolio with hedge ratio h can then be found using

$$HE_{CVaR_\gamma} = 1 - \frac{CVaR_\gamma(r_h)}{CVaR_\gamma(r_s)} \quad (12)$$

where $CVaR_\gamma(r_h)$ and $CVaR_\gamma(r_s)$ are the VaR of the hedge portfolio and spot at confidence level γ .

2.2.3. Semivariance

While the variance is a two-sided measure that applies equal weight to both negative and positive returns, the semivariance focuses on downside risk by measuring the variability of returns below a certain target return (Turvey & Nayak, 2003). The semivariance is defined as follows

$$SV(\Phi_r, T) = - \int_{-\inf}^T (T-r)^2 d\Phi(r) \quad (13)$$

with the target return given by T , r the portfolio return and Φ the distribution of returns.⁷ The minimum semivariance futures hedge ratio is found by iteratively calculating the semivariance for a range of possible hedge ratios and selecting the ratio corresponding to the smallest semivariance (Eftekhari, 1998).

The semivariance can also be applied as a measure of hedging effectiveness in the following way,

$$HE_{SV} = 1 - \frac{SV(r_h)}{SV(r_s)} \quad (14)$$

where $SV(r_h)$ and $SV(r_s)$ are the semivariance of the hedge and spot portfolios respectively.

2.2.4. Variance

The final hedge ratio and performance evaluation method uses the most commonly applied variance measure. The minimum-variance hedge ratio can be derived as the slope coefficient of spot price changes on futures price changes, (Ederington, 1979), and is given by

$$h_{mv} = \frac{Cov(r_s, r_f)}{Variance(r_f)} \quad (15)$$

where $Cov(r_s, r_f)$ and $Variance(r_f)$ are the covariance between spot and futures returns, and the variance of futures returns respectively.

The variance is a standard two-sided measure of risk in finance and can be used to calculate the effectiveness of a given hedge ratio in the following way,

$$HE_{Variance} = 1 - \frac{Variance(r_h)}{Variance(r_s)} \quad (16)$$

where $Variance(r_h)$ and $Variance(r_s)$ are the variance of the hedge and spot portfolios respectively.

⁷Throughout this paper we assume a target return of zero, to capture the preferences of investors who wish to avoid negative returns.

2.2.5. Horizon dependent hedge ratios

In section 2.1 we described how the wavelet transform allows the decomposition of a time-series into a set of coefficients associated with different time-horizons. Treating each of the time-horizons separately, we can then use the wavelet coefficients to calculate optimal hedge ratios associated with a given horizon using equations (9, 11, 13, 15). For example, in the case of VaR the objective function is minimized for the hedge portfolio at each time-horizon τ ,

$$VaR(\Phi_\tau, \gamma, \tau) = -\Phi_{r(\tau)}^{-1}(1 - \gamma), \quad (17)$$

resulting in a set of VaR optimized hedge ratios, $HR_{VaR}(\gamma, \tau)$. In a similar fashion, we obtain a set of hedge ratios associated with different time-horizons for the alternative downside hedging objectives CVaR and semivariance.

In the case of the horizon dependent minimum-variance hedge ratio, using the horizon dependent variance (equation 7) and covariance (equation 6), we can determine the hedge ratio $h_{mv}(\tau)$ at horizon τ using equation 15. The treatment of the horizon dependent minimum-variance hedge ratio is in keeping with the methodology applied in previous studies (for example, Lien & Shrestha (2007); In & Kim (2006)).

3. Data and Preliminary Analysis

In order to understand empirically the impact of time-horizon and the risk objective function for an energy hedger, we consider the case of a long spot position in heating oil hedged by futures contracts.⁸ The associated futures contract is the New York Harbor No. 2 Heating Oil Future Contract traded on the New York Mercantile Exchange (NYMEX) and daily log returns are found using data from January 1, 1997 to December 31, 2010, a total of 3,506 days. Heating oil is examined as it is a large, active market with historical losses of large magnitude and so constitutes a useful dataset to test downside hedging measures (Ederington & Lee, 2002; Sadorsky, 2000).

Summary statistics for both the spot and futures contracts are given in table 1. Over the time-period studied both contracts displayed a positive return, with the spot position found to be more volatile. Also, there is evidence

⁸Additionally, other energy commodities crude oil, natural gas and gasoline were also tested, with similar results obtained. For brevity, only heating oil is considered in this study, with additional results available on request from the authors.

for non-normality, with each contract displaying negative skewness and excess kurtosis. Large one-day losses are evident in both contracts, with maximum losses of 47% and 21% found. This suggests that downside risk measures are appropriate in the case of heating oil. Also, the VaR and CVaR for each asset is detailed at a range of confidence intervals illustrating the potential losses. Over the time-period studied, the correlation between the spot and futures is found to be 0.72 using original unfiltered data.

[Table 1 about here.]

In order to study the importance of time-horizon on downside risk hedging, we first decompose both spot and futures returns applying the MODWT (section 2.1). The MODWT is chosen as it enables us to consider a time-series of any size and to align the wavelet coefficients with the original data. The least asymmetric wavelet, LA8, with filter width 8 was chosen for this study as it allows us to meaningfully relate events at different horizons to those in the original time-series, important when considering low-probability events such as those captured by downside risk. Only wavelet coefficients unaffected by the boundary are applied in the calculation of the hedge ratio and associated performance, removing any potential boundary condition problems. Wavelet coefficients are calculated up to a 24-day horizon, as beyond this the number of coefficients impacted by the boundary reduced the data available.

In this paper, we apply a horizon-dependent rolling window analysis (Conlon & Cotter, 2012). Here, the optimal hedge ratios are calculated at each time-horizon in-sample using a rolling window of 1,000 days. In order to capture the performance, the following 1,000 days are then used to measure the out-of-sample hedging effectiveness. Then, the rolling window is moved forward by one day, where hedge ratios and associated effectiveness are recalculated.

4. Empirical Results

In this section, we present our empirical findings for optimal heating oil hedge ratios determined using the various risk objective functions described (section 2.2). Table 2 shows the optimal futures hedge ratio for each objective function at different time-horizons in the case of heating oil. Considering first the minimum-variance

hedging objective, the average hedge ratio is found to increase for longer hedging horizon, in keeping with previous findings (Lien & Shrestha, 2007; In & Kim, 2006). In fact, for a hedger with a horizon greater than 6 days, the optimal hedge ratio is greater than the naive ratio (one), suggesting that the hedging strategy requires that the hedger sells more than one futures contract for each short position held. Next, we consider the case of a hedger minimising the semivariance of the hedge portfolio (assuming a target return of zero, the hedger wishes to reduce negative outcomes). Just as in the previous case, the optimal semivariance hedge ratio is found to increase considerably from short to long horizons. However, hedge ratios at shorter horizons are found to differ from those prescribed by the minimum-variance strategy, suggesting a different hedging requirement to reduce downside risk. Further, the optimal semivariance hedge ratio calculated using the original data is greater than that found for the shortest wavelet decomposed horizons. This may suggest greater levels of noise in high frequency data, creating difficulty in measuring the hedge ratio accurately (Benet, 1992).

Hedge ratios calculated using historical simulation VaR are also shown for different confidence level preferences in table 2. Considering the impact of horizon first, the optimal hedge ratio is found to have larger magnitude at longer horizons for all confidence levels. However, the range between short- and long-horizon hedge ratios is found to be smallest at the lowest confidence level studied (75th percentile). Moreover, the range between short- and long-horizons hedge ratios is smaller for all confidence levels than that found for variance and semivariance minimizing objectives. Examining the confidence intervals, the trend in VaR hedge ratios differ according to horizon. At short horizons, the optimal hedge ratio is smallest for high uncertainty (99% confidence level) while at long horizons hedge ratios are found to be greatest for high uncertainty. Finally, CVaR optimized hedge ratios are also detailed in table 2. While the optimal CVaR hedge ratios differ from those found for VaR optimization, the impact of both horizon and confidence level on hedge ratios is found to be similar. For all confidence levels, the optimal CVaR hedge ratio increases monotonically with the horizon. For the shortest horizons, the largest hedge ratio is found for low confidence intervals (75%), while at the longest horizons the largest optimal hedge ratio is at high confidence interval (99%). The implication of these finding for futures hedging is the importance of considering preferences on both the hedgers horizon and confidence interval in computing both the VaR and

CVaR minimizing energy hedge ratios, two findings not previously addressed in the literature.

[Table 2 about here.]

To understand the importance of hedging horizon on the optimal heating oil hedge ratio for different hedging risk objectives, we now consider the out-of-sample hedging performance detailed in tables 3 and 4. For each of the hedge ratios previously calculated, we measure the hedging effectiveness by determining the proportion of spot variance, semivariance, VaR and CVaR that is removed by hedging at each time-horizon. Regardless of the hedging objective function applied or the performance metric used, hedging effectiveness is found to increase for longer horizons. While small differences in effectiveness are found across the different hedging objectives, these are found to be dominated by the large performance gains found at longer horizons for all objectives.

In table 3a, we examine the variance hedging effectiveness for the different heating oil hedge ratios shown in table 2 using the various objective functions described. Regardless of the underlying hedging objective, variance hedging effectiveness is found to increase monotonically from short to long horizons. In particular, at the longest horizon studied, 96% – 97% of the variance risk of the spot position is removed for all hedging objectives. Comparing the hedging objectives, at lower confidence intervals the VaR and CVaR optimized hedge ratios are found to reduce the level of variance risk by a greater amount than the minimum-variance hedge. These findings of improved hedging effectiveness using the VaR and CVaR reduction objectives are in keeping with Chang (2011); Harris & Shen (2006). However, the small performance improvements achieved for the VaR and CVaR hedging objectives are dominated by the effectiveness gains at longer horizons. Comparing the variance and semivariance objectives, the minimum-variance hedge ratios are found to reduce the spot variance by more than the minimum-semivariance ratios across all time-horizons.

[Table 3 about here.]

Next, we measure the effectiveness of the different heating oil hedge ratios proposed in table 2 from the point of view of semivariance reduction, given in table 3b. As demonstrated in the case of variance reduction, the semivariance effectiveness is also found to increase for longer time-horizons regardless of the hedging objective.

This means that hedgers with long hedging horizon remove a larger proportion of negative spot fluctuations than those with short horizons. It is worth noting that the minimum-variance hedge ratios are found to reduce the spot semivariance by at least the same amount as the semivariance hedge ratios at all time-horizons. Comparing the performance across the various hedging objectives and confidence intervals, the horizon dependent minimum-variance hedge ratios are also found to be at least as effective as the VaR and CVaR methods in reducing portfolio semivariance.

In order to calculate the hedging effectiveness using VaR and CVaR, we apply a range of confidence intervals. In the case of VaR and CVaR hedge ratios, we measure the hedging effectiveness using the same confidence interval applied in calculating the hedge ratio. For comparative purposes, we also measure the VaR and CVaR performance of the minimum-variance and minimum-semivariance hedge ratios across the range of confidence intervals studied. Performance, measured first using VaR reduction, is given in table 4a. As previously shown for variance and semivariance, the VaR effectiveness is found to increase monotonically for longer horizons, regardless of the hedging objective or confidence interval examined. Across the hedging objectives, a heating oil futures hedger with the longest horizon studied (24 days) removes on average 82% of tail risk, while only 56% is removed on average at the shortest horizon. An agent wishing to reduce their downside risk, measured using VaR, will achieve better performance if they hedge for a longer horizon.

[Table 4 about here.]

Next, we compare the effectiveness across the various VaR confidence intervals studied. The first result to note is that hedging effectiveness improves as the confidence interval decreases for all hedging objectives. In other words, as the hedger becomes more concerned about tail-risk, the hedging performance they achieve deteriorates. This result is also robust to using different values of the hedge ratio, as can be seen in figure 1(A). Across the wide range of hedge ratios examined, our findings are unchanged with better effectiveness achieved at low confidence intervals. This suggests that residual tail risk remains after hedging, regardless of the hedging objective followed. However, as proposed, this residual tail risk can be reduced for hedgers with long hedging horizons.

[Figure 1 about here.]

Considering the VaR performance of the different hedging objectives at the 99% confidence interval, we find that the variance hedge ratio performs best across the majority of horizons. At high levels of uncertainty, the small quantity of available data to calculate the hedge portfolio VaR may result in underperformance in comparison to the variance objective out-of-sample. However, for the other confidence intervals studied the VaR hedging objective is found to outperform other objectives across the majority of horizons. This is in keeping with previous findings, where the VaR minimization objective was shown to reduce portfolio VaR by slightly more than other methods (Chang, 2011; Harris & Shen, 2006).

Finally, we investigate the out-of-sample heating oil hedging performance for each of the hedging objectives using the CVaR measure, table 4b. Considering first the horizon effects, we find considerable improvement in CVaR hedging effectiveness between long and short-horizons for all hedging objectives and confidence intervals. For example, at a 99% confidence interval, 27% of spot CVaR is removed on average at the shortest horizon studied, while at a 24 day horizon 83% is removed. Given the relatively small differences between effectiveness for different hedging objectives, the horizon effects are again found to dominate. As found in the case of VaR effectiveness, CVaR performance is also found to improve monotonically for lower confidence intervals. In other words, hedgers find it more difficult to reduce tail risk when there is more uncertainty involved. The robustness of this result is illustrated in figure 1(B), where CVaR effectiveness is measured for a variety of hedge ratios. We show that independent of the hedge ratio applied, better CVaR effectiveness is achieved for lower confidence intervals.

Contrasting the CVaR effectiveness achieved by the various objective functions at different confidence intervals, the minimum-variance hedge is found to have the best performance at a 99% level. However, for the lower confidence levels considered the VaR hedge ratio is found to have the best CVaR effectiveness for the majority of time-horizons. The outperformance of the VaR hedging objective is a result of the small quantities of data available for calculation of the CVaR hedge ratio, resulting in decreased out-of-sample performance.

Comparing the various performance metrics detailed, we find for all hedging objectives that the optimal heating oil hedge ratio at each horizon achieves the highest levels of effectiveness when measured using variance. On

the other hand, the lowest effectiveness levels are measured using CVaR, suggesting that all hedging objectives perform best in terms of reducing two sided variance risk as opposed to tail risk. Contrasting the different objective functions, greater levels of uncertainty are associated with the value-at-risk based methods as opposed to the variance technique due to decreased quantities of available data for risk calculation. This effect is highlighted by the further decrease in effectiveness found at high confidence intervals for the VaR and CVaR risk objectives (figure 1).

We now turn our attention to the impact from hedging on the distributional properties of returns at different horizons. We consider the impact of each hedging objective on the skewness and kurtosis of the hedge portfolio at different horizons. Table 5a presents the change in skewness, measured as the skewness of the hedge portfolio minus the skewness of the spot. Considering the case of a hedger unconcerned about horizon, a hedged portfolio results in a positive change in skewness for all objectives. However, at the shortest horizons studied, 1.5 to 6 days, we find that the skewness of the hedge portfolio is lower than for the spot. In contrast, at long horizons, hedging is found to increase the portfolio skewness relative to the spot.

[Table 5 about here.]

The change in kurtosis between the spot and hedged heating oil portfolios is detailed in table 5b. As found in previous studies, hedging increases the kurtosis of the portfolio for all hedging objectives (Harris & Shen, 2006). However, the magnitude of the increase in hedge portfolio kurtosis is found to be *time-horizon dependent*, with substantially smaller changes in kurtosis found at longer time-horizons. Contrasting the various hedging objectives, the 99% CVaR hedging objective is found to have the smallest change in kurtosis across the majority of wavelet decomposed time-horizons. Also by moving to lower confidence intervals, the change in kurtosis is found to be greater for both the CVaR and VaR objectives. Given the earlier findings of decreased tail-risk at lower confidence intervals, this suggests that the increase in kurtosis is due to a higher concentration of returns around the mean (or peakedness).

In the empirical analysis, we explored and contrasted a number of alternative futures hedging objectives, considering optimal heating oil hedge ratios for a variety of hedging preferences. For all objective functions,

the optimal hedge ratio was found to increase at longer horizons. While small differences in effectiveness were found between various hedging objectives, large performance gains were demonstrated moving from short to long time-horizon regardless of the objective. The results suggest that while downside risk measures are useful in the computation of the optimal hedge ratio, the hedging horizon and confidence intervals should also be given careful consideration by the energy hedger. If the hedger's preference is in reducing the downside risk of the portfolio, the confidence interval chosen will have an impact on the performance achieved. Higher confidence intervals were shown to be associated with lower performance, a result not detailed in previous studies. Finally, we demonstrated that the highest levels of effectiveness are measured using the variance measure, with the lowest levels of effectiveness found using the CVaR performance measure for all objectives and horizons studied.

5. Conclusions

In this paper, we explored the impact of time-horizon on the optimal downside hedged portfolio for heating oil investors. The optimal downside risk hedge ratio was calculated using a variety of risk metrics, including value-at-risk, conditional value-at-risk and semivariance. In each case, the returns time-series were first decomposed into the underlying time-horizons using the wavelet transform. Then, the appropriate downside risk hedge ratio was calculated at each horizon. For the VaR and CVaR hedging objectives, we demonstrated that the optimal hedge ratio depends on the confidence interval chosen by the hedger, with the lowest optimal hedge ratios found at the highest uncertainty levels. Further, our findings for the various objective functions support those previously found for the minimum-variance hedge, with increasing hedge ratios found for longer time-horizons for all objective functions. In order to minimize downside risk using a futures hedge, an energy hedger with a long time-horizon is required to sell a greater number of futures contracts than the corresponding hedger with a short horizon.

We then proceeded to measure and compare the out-of-sample effectiveness of each energy hedging objective applying a range of performance metrics. Investigating the impact of uncertainty on the hedging performance, we found the highest levels of effectiveness for low VaR and CVaR confidence intervals, at all hedging horizons. This was also shown to be independent of the heating oil hedge ratio applied, suggesting that energy hedgers find it more difficult to reduce tail risk when there are higher levels of uncertainty involved.

Next, we demonstrated that across the range of different hedging objectives and effectiveness measures studied, hedging effectiveness increases for longer horizons. Moreover, while VaR and CVaR hedge ratios were found to have better VaR and CVaR effectiveness than the minimum-variance objective, these performance gains were shown to be small relative to those experienced by investors with long horizons. Finally, examining the change in kurtosis between the hedge and spot portfolios, we found a smaller increase in excess kurtosis for investors with longer hedging horizons.

Our results build considerably on previous studies investigating different futures hedging risk objectives. The findings suggest that while downside risk measures are useful in the computation of the optimal hedge ratio, the hedging horizon should also be given careful consideration by the hedger. Moreover, if the hedger's preference is in reducing the downside risk of the portfolio, the confidence interval chosen will have an impact on the performance achieved.

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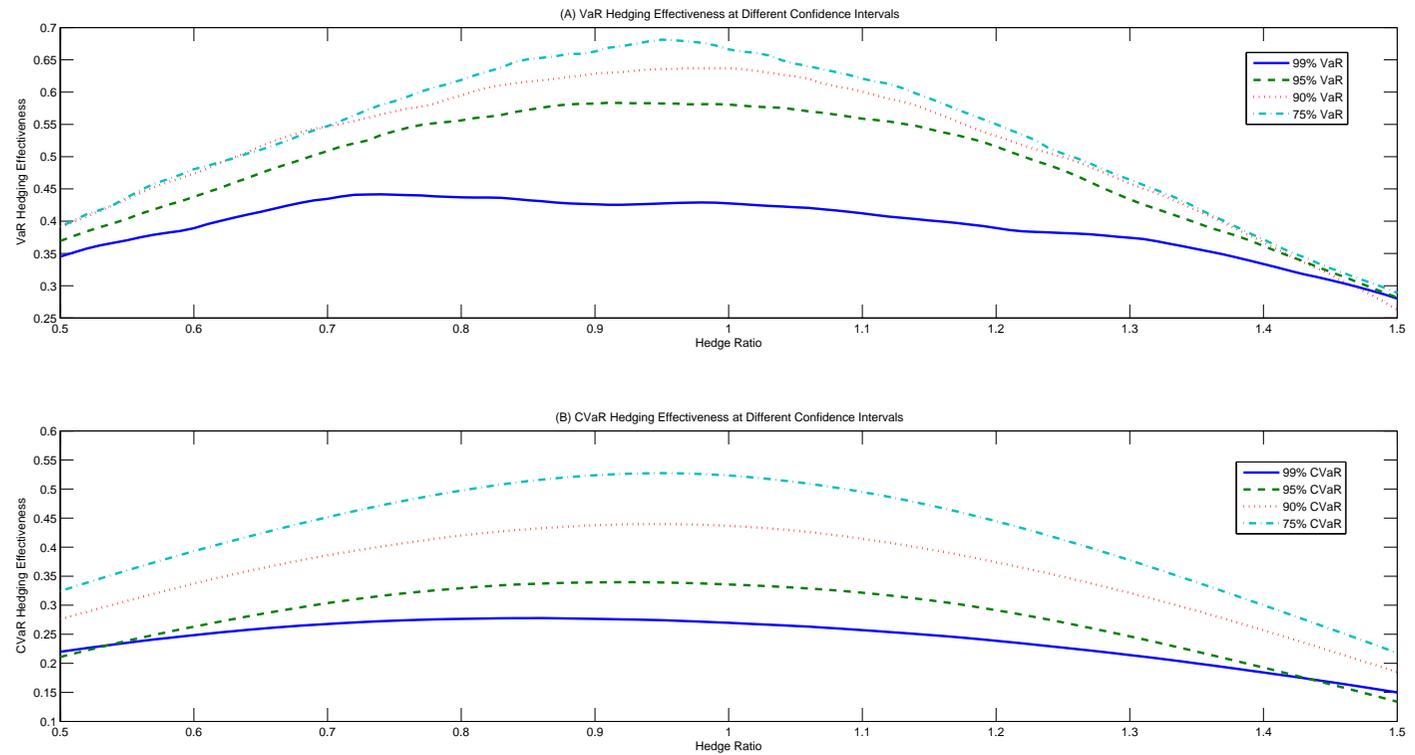


Figure 1: **VaR and CVaR hedging effectiveness for different hedge ratios.**

In-sample VaR and CVaR hedging effectiveness (proportion of risk reduced) at different confidence intervals for a range of hedge ratios. A 1000 day moving window approach is used, with the hedging effectiveness for each hedge ratio corresponding to the average across all windows.

	Heating Oil	
	<i>Spot</i>	<i>Futures</i>
Mean (%)	0.04	0.04
Standard Deviation (%)	2.78	2.46
Skewness	-1.53	-0.46
Kurtosis	39.88	7.29
Maximum loss (%)	47.01	20.96
99% VaR (%)	6.35	6.45
95% VaR (%)	3.98	3.75
90% VaR (%)	2.90	2.74
75% VaR (%)	1.44	1.36
99% CVaR (%)	10.18	9.04
95% CVaR (%)	5.88	5.51
90% CVaR (%)	4.64	4.37
75% CVaR (%)	3.10	2.93

(a) Summary Stats

	<i>Spot</i>	<i>Futures</i>
<i>Spot</i>	1.00	0.72
<i>Futures</i>	0.72	1.00

(b) Spot & Futures Correlation

Table 1: Descriptive statistics and correlation for log returns of crude futures and spot times-series.

Notes: Sample period is January 1997 to December 2010. Mean return and standard deviation statistics and one-day value-at-risk and conditional value-at-risk are given in percentage terms, while a skewness of zero indicates no skewness and a kurtosis of 3 indicates no excess kurtosis is present.

Horizon	Variance	Semi-Variance	Value-at-Risk				Conditional Value-at-Risk			
			99%	95%	90%	75%	99%	95%	90%	75%
Original	0.82	0.98	0.78	0.92	0.98	0.96	0.85	0.91	0.95	0.94
1.5 Days	0.73	0.65	0.82	0.96	0.92	0.93	0.49	0.80	0.88	0.91
3 Days	0.85	0.82	0.73	0.94	0.93	0.98	0.68	0.83	0.88	0.93
6 Days	0.90	0.79	0.77	0.98	0.98	0.98	0.80	0.91	0.96	0.97
12 Days	1.04	1.04	1.16	1.02	0.99	0.98	1.15	1.08	1.03	1.00
24 Days	1.19	1.19	1.05	1.11	1.05	1.03	1.20	1.18	1.13	1.07

Table 2: Optimal heating oil hedge ratios for different objective functions at various time-horizons, 1997-2010.

Notes: Hedge ratios are calculated at various horizons in-sample using different objective functions. These include variance, semi-variance, value-at-risk and conditional value-at-risk minimisation (99%, 95%, 90% and 75% confidence intervals). A 1000-day moving window approach is used, with average hedge ratios over all windows given in each case. The data is transformed into different time horizons using the wavelet filter (LA8) up to a 24 day horizon.

Horizon	Variance	Semi-Variance	Value-at-Risk				Conditional Value-at-Risk			
			99%	95%	90%	75%	99%	95%	90%	75%
Original	0.81	0.80	0.78	0.82	0.81	0.82	0.81	0.82	0.82	0.82
1.5 Days	0.73	0.70	0.74	0.74	0.75	0.75	0.59	0.75	0.75	0.75
3 Days	0.84	0.83	0.79	0.85	0.85	0.84	0.77	0.84	0.84	0.85
6 Days	0.92	0.87	0.86	0.92	0.92	0.92	0.88	0.92	0.92	0.92
12 Days	0.95	0.94	0.93	0.95	0.95	0.95	0.93	0.94	0.95	0.95
24 Days	0.96	0.96	0.95	0.97	0.97	0.97	0.96	0.96	0.97	0.97

(a) Variance Effectiveness

Horizon	Variance	Semi-Variance	Value-at-Risk				Conditional Value-at-Risk			
			99%	95%	90%	75%	99%	95%	90%	75%
Original	0.68	0.64	0.66	0.67	0.66	0.67	0.67	0.67	0.67	0.67
1.5 Days	0.58	0.58	0.58	0.55	0.56	0.57	0.50	0.59	0.57	0.57
3 Days	0.74	0.74	0.71	0.73	0.74	0.73	0.70	0.74	0.74	0.74
6 Days	0.88	0.84	0.83	0.88	0.88	0.88	0.85	0.88	0.88	0.88
12 Days	0.93	0.92	0.92	0.93	0.93	0.93	0.92	0.93	0.93	0.93
24 Days	0.96	0.96	0.94	0.97	0.97	0.96	0.95	0.96	0.96	0.97

(b) Semi-Variance Effectiveness

Table 3: Out-of-Sample heating oil hedging effectiveness at different horizons 1997-2010.

Notes: Hedge ratios are first calculated at various horizons in-sample using different objective functions. These include variance, semi-variance, value-at-risk and conditional value-at-risk minimisation (99%, 95%, 90% and 75% confidence intervals). For each objective function, the performance is then measured out-of-sample using variance and semi-variance effectiveness (proportion of spot risk removed). A 1000-day moving window approach is used, with the first 1000 days used in the calculation of the hedge ratio and the next 1000 days used to measure effectiveness with averages over all windows given in each case. The data is transformed into different time horizons using the wavelet filter (LA8) up to a 24 day horizon.

Horizon	Hedging Objective															
	Variance				Semi-Variance				Value-at-Risk				Conditional Value-at-Risk			
	99%	95%	90%	75%	99%	95%	90%	75%	99%	95%	90%	75%	99%	95%	90%	75%
Original	0.55	0.63	0.66	0.70	0.53	0.65	0.68	0.72	0.51	0.67	0.71	0.76	0.54	0.67	0.71	0.76
1.5 Days	0.47	0.56	0.58	0.62	0.43	0.51	0.53	0.56	0.49	0.62	0.66	0.70	0.34	0.59	0.65	0.70
3 Days	0.54	0.66	0.70	0.74	0.54	0.65	0.68	0.72	0.50	0.68	0.73	0.77	0.49	0.65	0.71	0.76
6 Days	0.66	0.74	0.77	0.80	0.64	0.67	0.69	0.73	0.63	0.75	0.78	0.81	0.64	0.74	0.79	0.82
12 Days	0.73	0.81	0.83	0.84	0.71	0.79	0.80	0.83	0.71	0.80	0.82	0.85	0.71	0.79	0.83	0.86
24 Days	0.81	0.79	0.81	0.81	0.79	0.80	0.82	0.81	0.74	0.84	0.88	0.90	0.78	0.80	0.85	0.89

(a) Value-at-Risk Hedging Effectiveness

Horizon	Hedging Objective															
	Variance				Semi-Variance				Value-at-Risk				Conditional Value-at-Risk			
	99%	95%	90%	75%	99%	95%	90%	75%	99%	95%	90%	75%	99%	95%	90%	75%
Original	0.35	0.53	0.58	0.62	0.31	0.53	0.58	0.63	0.34	0.54	0.60	0.66	0.35	0.54	0.60	0.66
1.5 Days	0.28	0.45	0.50	0.54	0.28	0.43	0.46	0.50	0.27	0.48	0.54	0.60	0.24	0.47	0.54	0.60
3 Days	0.39	0.57	0.61	0.66	0.39	0.56	0.60	0.64	0.38	0.57	0.63	0.68	0.37	0.56	0.62	0.68
6 Days	0.60	0.68	0.71	0.74	0.58	0.63	0.65	0.68	0.57	0.68	0.72	0.75	0.58	0.68	0.72	0.76
12 Days	0.72	0.75	0.78	0.80	0.70	0.74	0.76	0.79	0.69	0.75	0.78	0.80	0.69	0.75	0.78	0.81
24 Days	0.79	0.79	0.80	0.80	0.78	0.79	0.80	0.80	0.75	0.82	0.85	0.87	0.78	0.79	0.83	0.86

(b) Conditional Value-at-Risk Hedging Effectiveness

Table 4: Out-of-Sample heating oil hedging effectiveness at different horizons 1997-2010.

Notes: Hedge ratios were first calculated at various horizons in-sample using different objective functions. These include variance, semi-variance, value-at-risk and conditional value-at-risk minimisation (99%, 95%, 90% and 75% confidence intervals). For each objective function, the performance is then measured out-of-sample using value-at-risk and conditional value-at-risk at a range of confidence intervals. For the VaR and CVaR objective functions, the effectiveness (proportion of spot risk removed) is measured using the corresponding hedge ratio for that confidence interval. A 1000-day moving window approach is used, with the first 1000 days used in the calculation of the hedge ratio and the next 1000 days used to measure effectiveness with averages over all windows given in each case. The data is transformed into different time horizons using the wavelet filter (LA8) up to a 24 day horizon.

Horizon	Variance	Semi-Variance	Hedging Objective							
			Value-at-Risk				Conditional Value-at-Risk			
			99%	95%	90%	75%	99%	95%	90%	75%
Original	0.43	0.76	0.27	0.79	1.01	1.04	0.71	0.61	0.90	0.93
1.5 Days	-0.29	-0.21	-0.21	0.06	-0.05	-0.16	-0.24	-0.20	-0.08	-0.08
3 Days	-0.39	-0.31	-0.39	-0.42	-0.43	-0.42	-0.34	-0.39	-0.41	-0.42
6 Days	-0.16	-0.16	-0.20	-0.12	-0.11	-0.13	-0.22	-0.11	-0.13	-0.12
12 Days	0.09	0.05	0.04	0.09	0.07	0.07	0.04	0.09	0.10	0.08
24 Days	0.09	0.10	0.24	0.19	0.32	0.37	0.09	0.10	0.17	0.30

(a) Change in Skewness

Horizon	Variance	Semi-Variance	Hedging Objective							
			Value-at-Risk				Conditional Value-at-Risk			
			99%	95%	90%	75%	99%	95%	90%	75%
Original	33.75	52.23	26.79	51.59	61.06	60.95	41.34	48.67	56.76	57.32
1.5 Days	18.18	12.66	26.95	44.82	40.67	38.55	4.86	26.19	37.42	39.51
3 Days	18.23	16.66	7.40	24.30	24.58	26.89	6.37	15.85	20.10	24.55
6 Days	6.85	4.05	3.85	8.89	8.98	8.95	4.35	7.44	8.59	8.98
12 Days	4.36	3.92	2.18	4.06	4.51	4.42	2.25	3.59	4.43	4.74
24 Days	0.02	0.62	3.84	1.98	4.37	5.32	0.57	0.63	2.20	4.52

(b) Change in Kurtosis

Table 5: Heating oil hedge portfolio change in skewness and kurtosis at different horizons, 1997-2010.

Notes: Hedge ratios were first calculated at various horizons in-sample using different objective functions. These include variance, semi-variance, value-at-risk and conditional value-at-risk minimisation (99%, 95%, 90% and 75% confidence intervals). The change in skewness and kurtosis is measured as the difference between the skewness and kurtosis of the hedged portfolio compared to the unhedged (spot) portfolio. A 1000-day moving window approach is used, with the first 1000 days used in the calculation of the hedge ratio and the next 1000 days used to assess the distributional properties of the hedge portfolio with averages over all windows given in each case. The data is transformed into different time horizons using the wavelet filter (LA8) up to a 24 day horizon.