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Prioritizing the "worse off" under attainability constraints: An indeterminacy problem for distributive fairness

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Abstract: Numerous theories of distributive fairness promote the idea that we ought to give extra weight to benefits to the worse off and can thereby be seen as promoting gap closures. This paper underlines the relevance of making a distinction between attainable and ideal target levels for individuals in populations affected by distributive fairness and show that in cases of scarce resources, theories that promote aggregate gap closures and prioritization of the worse off can in view of this distinction be interpreted in three mutually inconsistent ways.

Keywords: distributive fairness, equality, priority setting, capped objectives, social choice

JEL Classification: D63, D71, I31.

Introduction

A widespread idea in the literature on distributive fairness is that we should give extra weight, priority, to benefitting the worse off. Luck egalitarians argue that compensation for shortfalls that are due to bad luck, i.e. due to no fault or choices of the agent, contribute to the overall goodness of an outcome (cf. Arneson 1989; Cohen 1989; Temkin 1993, 2003). Sufficientarian approaches of different types suggest that we have special obligations to bring individuals up to a certain threshold level and thus ought to give extra weight to benefits to individuals that fall short of this level (cf. Anderson 2000; Crisp 2003; Frankfurt 1987; Shields 2012; Wiggins 1987). Prioritarian theories claim that the worse off an individual is in absolute terms, the more a benefit given to her is worth (cf. Broome 1991, 2015; Parfit 1997, 2012). According to proponents of the capability approach, finally, it is generally accepted that individuals that lack certain capabilities should be given priority (cf. Alkire 2005, Alkire et al. 2015, Nussbuam 2000, Sen 1992). In this paper, we argue that due to restrictions on what individuals can feasibly attain, e.g. incurable diseases that constrain attainable health states, learning disabilities that restrict an individual's educational attainment, wars that limit the effectiveness of anti-poverty programmes within a particular period of time etc., there are at least three different ways to understand the general notion that benefits to individuals that are characterized by some form of shortfall should be given extra weight, that all three interpretations are plausible, but that they are inconsistent.

Numerous theories of distributive fairness can be re-conceptualized in terms of proposals to close gaps. Egalitarianism can be understood as the view that promotes closing the *inequality gap*, i.e. the shortfall from perfect equality. Sufficientarian theories can be conceived of as the views that promote closing the *insufficiency gap*, i.e. the shortfall from the state of affairs where all individuals are at or above the sufficiency level. Prioritarianism can, at least for practical purposes, be seen as the view that promotes closing the *priority gap*, i.e. the shortfall from the state of affairs where individual benefits are worth the same, regardless of who enjoys them. The capabilities approach can be understood as the view that promotes that we should close the *capabilities gap*, i.e. the shortfall from fair distribution of capabilities. For each of these normative theories, the relevant gap can be mitigated by bringing all individuals in the population to the level at which she no longer contributes to the aggregate gap size, and the closer to this level individuals are, the better.

In this paper we address two aggregation problems that arise for any theory that promotes gap closures under conditions of scarcity**.** These problems arise once a well-defined ideal target for each individual in the population has been identified and the amount of available resources doesn't suffice to bring every individual to the ideal target. The first problem is well known and arises in relation to the two-dimensionality of gaps. Gaps relevant to distributive fairness are defined in terms of a *multitude* of individual shortfalls of different *magnitude*. Several individuals have shortfalls, and the shortfalls can be of different size. Theories that promote gap closures must provide an answer to how these dimensions relate to each other (cf. Shields 2012; Temkin 1993).

The second problem is generally overlooked and arises in relation to popular justifications of aggregations of the two dimensions that invoke the magnitude of shortfalls as justification of the aggregation and provide extra weight to larger shortfalls. We show that, even under the assumptions that we (i) have established a unique unit of measurement (e.g. resources, utility, capabilities) that enables scalar comparisons of the magnitude of different shortfalls, and (ii) have established a well-defined ideal target (e.g. X amount of resources, utility-level Y, capability set Z), justifications of a specific aggregation that rely on a notion of how bad off someone is are indeterminate. This indeterminacy is due to the fact that in many instances there are three distinct ways to establish how badly off someone is even when a unique unit of measurement and a definitive ideal target level have been established. This entails that theories that promote this type of solution to the first aggregation problem will be inconsistent in some applications.

Our argument relies on the identification of three distinct relevant *status levels* for each individual in terms of what matters (cf. Herlitz & Horan 2016a). An individual has a *current status*, i.e. her position with respect to what matters in the current state of affairs. There is for each individual an *ideal level*, i.e. the status that the individual ideally should be at. And there is for each individual an *attainable level*, i.e. the higher status that the individual ideally should be at and also feasibly *can* be at. Once these three status levels have been outlined, we show that there are at least three ways in which to understand how well an individual fares both in absolute and relative terms.

In order to present the argument we will use a generalization of a specific indicator of gap closures which is inspired by the so-called FGT-class indicators popular in poverty research (Alkire et al. 2015; Foster, Greer & Thorbecke 1985, 2010; Herlitz & Horan 2016, MS). This indicator has been designed to measure the extent to which heterogeneous individual shortfalls (defined as a technical concept that denotes the difference between a current status and a desired level) of a population are satisfied. We here extend this indicator to situations where attainability-constraints apply. The indicator provides us with a numerical representation of gap closures, which facilitates discussing gap closures and reveals the particular aggregation problem we are interested in.

Our argument is straightforward if at occasions somewhat technical. In section one we stress the importance of making a distinction between what we refer to as the *ideal level* and the *attainable level*. In combination with a *current level*, this distinction reveals that there are three distinct types of shortfalls: current level-ideal level, current level-attainable level, and attainable level-ideal level. In section two we show that once we have made this distinction there are three different ways to develop prioritization models, weighting schemes, in light of them. In section three we illustrate that the prioritization models that are compatible with the notion of compensating for shortfalls and closing gaps conflict. In section four we discuss some possible extensions of the paper.

I: Pure Allocation Problems:
The Distinction between Attainable and Ideal Target Levels

We are in this paper interested in choice situations in which some good (broadly conceived) is to be distributed across a heterogonous population, and normative criteria that apply to such situations. Choice situations that concern the distribution of goods can be partially characterized by involving a population with individuals at different *current levels* in terms of the good. By current level we here mean the amount of the good that an individual possesses at the moment of the choice, i.e. the amount of the good that the individual enjoys if there is no intervention. Thus, for example, when health improvements are distributed among a population every individual has a specific current level in terms of the good which is distributed, i.e. health. When monetary resources are distributed in a population, every individual has a specific current level in terms of the good which is distributed, i.e. money. When utility is distributed, every individual has a specific current level in terms of utility. And so on.

A familiar quarrel concerning theories that address and promote certain prioritization schemes when goods are distributed across a heterogonous population relates to the issue of which good, and in extension which unit of measurement, matters (cf. Cohen 1993; Dworkin 1981a, 1981b; Pogge 2002; Sen 1980). Some contend that what matters are resources (cf. Pogge 2002; Rawls 1971). Some hold that what matters is welfare (cf. Hare 1981, Kymlicka 1990). Some hold that what matters are so-called capabilities (Nussbaum 2000; Sen 1992). And some hold that it is opportunities for wellbeing that matter (Arneson 1989; Cohen 1989; Roemer 1998). Our argument assumes that a single well-defined good has been established, but the argument is compatible with a wide range of views about what this good is (e.g. resources, welfare, capabilities) and therefore also relevant to theories of distributive fairness regardless of what good they promote as relevant.

A common, if not always explicated, way of thinking about distributive fairness and about compensating for shortfalls is to compare current levels with some *uniform* target level and promote a *gap closure* so that individuals are brought toward the uniform target level. This is perhaps best exemplified in the more practical realm by recent research on poverty reduction (cf. Alkire et al. 2015; Foster, Greer & Thorbecke 1985, 2010), but also some theoretical approaches (e.g. sufficientarianism, the capabilities approach) make this feature very clear (cf. Nussbaum 2000; Shields 2012). Uniform target levels can be thought of in at least three different ways. Firstly, one can think of the uniform target level as a *mean value* of the current levels of all individuals in the population. Secondly, one can conceptualize the uniform target level in terms of a perfectionist ideal. Perfectionist ideals can be framed in terms of how well off the individual who is best off is (cf. Temkin 1993), in terms of some minimum level of what is needed for a decent life (cf. Alkire 2005a, Anderson 2000; Crisp 2003; Wiggins 1987), or in terms of an unattainably high level, which *in practice* amounts to a prioritarian position (cf. Parfit 1997, 2012; Broome 2015). Finally, one can infer a uniform target level based on every individual's relation to every other individual (Temkin 1993). Regardless of how one conceptualizes a target level, a straightforward way to compensate for shortfalls is to give additional priority to allocation alternatives that reduce the size of the aggregated gaps between current levels and the uniform target level.

A limitation of this general way of thinking about distributive principles is that it presupposes *uniformity* of the target levels. It is important to recognize that uniform target levels are in many situations unrealistic. All individuals cannot attain the same things. In many situations there will, therefore, for every individual in the population be *two* target levels: on the one hand an *attainable level*, and on the other hand an *ideal level*. The latter level will plausibly be uniform across a population. The former will plausibly not be uniform across a population. Consider, for example, distribution of health. When health resources are allocated across a population every individual will be on a certain, individuated current level in terms of health. For all individuals in the population there will also be a uniform ideal level, i.e. perfect health. The distance between these, we commonly think, denotes how severe the health condition of an individual is. However, every individual in the population will have individual, non-uniform attainable levels. A diabetic will be unable to reach the ideal level since there is no cure for diabetes. She can receive the best care there is for diabetics, but that will place her on her individual attainable level, not on the uniform ideal level. Similarly, of course, for individuals with celiac, the blind, the asthmatics, those with incurable cancer, Down syndrome and so on. Likewise, it is not obviously true that all individuals can feasibly reach an ideal level in the area of education. And, it could even be impossible for every individual to attain the same level of income, and it appears plausible that not every individual can attain the same level of welfare.

It is difficult to establish precise definitions of attainability constraints, but their relevance ought to be obvious enough. In order to proceed with our argument we suggest, following David Wiggins that the attainability constraints should be understood as a set of *time constraints*, *technology constraints*, and *ability constraints* (cf. Alkire 2005b; Wiggins 1987). Some gap closures are impossible due to time constraints. We might not have time to provide adequate amount of aid to individuals who due to no fault or choice of their own suffer the consequences of a natural disaster for example. Some shortfalls cannot be fully remedied because we lack the appropriate technology. We lack the technology to cure diabetes, but also to cure certain mental illnesses that significantly reduces wellbeing. Finally, some individuals lack the ability to reach the ideal level due to intrinsic qualities. Individuals are pathologically different: some are prone to depression, some have allergies, and so on. It is important to note that attainability constraints, being designed in this way, are not universal, and the three different classes of constraints are also somewhat intertwined. What appears to be a time constraint and what appears to be an ability constraint might turn out to be a technology constraint for example. Yet, this is hard to avoid since it is impossible to foresee what potential technology there is. The attainability constraints differ depending on where a policy is implemented as well as on when it is implemented.

More abstractly, we thus suggest that a general mistake that is often made when we think about gap closures and compensations for relevant shortfalls, and when we have embraced a unique unit of measurement, is that individual shortfalls are conceived of in terms of the distance between *merely two* well-defined points: a current level and a target level. With such a conception of what shortfalls are answers to the question of how unequal a society is can be generated by either looking at the current level of the worst off (Rawlsian maximin), by adding up the sum total of shortfalls, or by applying weighted addition of the shortfalls (Temkin, 1993). This, we contend, is an overly simplistic analysis. Instead, we suggest that one should think of the relevant shortfalls of an individual as defined by *three* points: a current status level, an ideal level, and an attainable level. What matters is how well someone *is* doing in terms of X, how well they *ideally* would be doing in terms of X, and their actual *attainable level*: how well they feasibly *can* be doing in terms of X. This suggestion has support in the literature on needs, recent work of capabilities approach theorists, development literature and also research on environmental challenges (cf. Alkire 2005, Alkire et al. 2015; Doyal & Gough 1991; Gough 2015; Wiggins 1987). Yet, to our knowledge, no one has yet incorporated the idea into a single framework that also accounts for other ways of thinking about shortfalls, and no one has to our knowledge pointed to the theoretical challenges that the conflicts that can be identified pose.

Three general types of shortfalls can ground both what we should remedy and priority weights that apply to individual benefits. First, there is the shortfall that is defined in terms of the difference between an individual's current level and an ideal level. We will call this *bad offness*. This is, it appears, what most theories of distributive fairness address. Second, there is the shortfall that is defined in terms of the difference between an individual's current level and her attainable level. We will call this *bad realization*. This shortfall is rarely discussed in the normative-theoretical literature even though it appears to be the most practically useful concept (cf. Alkire 2005). It is clearly a highly relevant type of shortfall when we attempt to remedy shortfalls. Indeed, it appears to increase the risk of resource waste to not use this concept of shortfall in such situations. Yet, it might also be relevant when we assign weights to individual benefits (cf. Herlitz & Horan MS).

Finally, there are the shortfalls that are defined in terms of the difference between attainable levels and ideal levels. We will call this *bad outlooks*. A seemingly reasonable, yet overlooked, idea that seems highly relevant to distributive fairness is that *this* is a valid ground for priority weights. A straightforward example of when this seems plausible is this: imagine two individuals, Matt and Lisa. Their current levels are identical. Let's say they each have 1 unit of what matters. The ideal levels for both are also identical. Let's say 100 units of what matters. They only differ in that the attainable level for Matt is 90 units, while it for Lisa is 10 units. Now, imagine that we face a choice situation in which there are only two alternatives: either we allocate 1 unit to Matt, or we allocate 1 unit to Lisa. It here seems plausible that we should allocate the unit to Lisa and compensate for the fact that she is worse off than Matt based on the fact that her outlooks are worse.

Some attention has recently been given to the potential relevance of individuals' *prospects* in discussions concerning distributive principles and in particular prioritiarianism (cf. Broome 2015; Parfit 2012; Voorhoeve & Fleurbaey 2012). It is worth underlining that whereas attainability constraints are related to prospects, prospects such as they have been discussed in the literature is a factor that is different from attainable levels. Prospects, such as they have been addressed in the literature, concern (at least partly) what an individual will achieve *without* an intervention. The idea of attainable levels is more specific. They concern the limitations on what an intervention can do. That being said, our argument can, we believe, be transferred to discussions about prospects more generally.

In light of the above discussion, we suggest that when conceptualizing choice situations in which distributive fairness is at stake, one needs to distinguish three different levels that might be relevant to the choice. First, there is the *current level* for each individual in the population. These often differ across a population. Second, there is the *attainable level* for each individual in the population. Also these at the very least occasionally differ. Not every individual can feasibly attain the same level in terms of that which matters to the choice, e.g. health, education, income, goods. Third, there is an *ideal level* for each individual in the population. This is often uniform for the population. Yet, also this level might be heterogonous, for example if one allows for dessert to be relevant for the choice.

II: Three Aggregate Gap Closure Indicators that Prioritize the "Worse Off"

In this section, we present a model and indicator to measure the extent of aggregate gap closure in a population of individuals with different current levels and heterogeneous closable gaps. We first introduce the three types of individual levels and derive our three notions of individual shortfall from differences between pairs of these levels. We then use the indicator to characterize resources according to their availability and develop weighting schemes based on different types of shortfall for situations where resources are scarce.

Suppose $N = \{1,2, \dots, n\}$ is a population of n individuals in which each individual $i \in N$ is characterized by three types of levels in line with the discussion in the previous section. Levels are assumed to be well-defined and measurable for all individuals with a common unit of measurement, e.g. utility, health, functionings. First, there is the ideal target level of an individual, where $t_i^{**} > 0$ measures the ideal level of individual i, e.g. the poverty line, good health, sufficiency threshold, population mean. Second, there is the attainable level of an individual: $a_i \geq 0$ measures this attainable level of i, e.g. the attainable health level. Finally, there is the current level of the individual: $c_i \geq 0$ denote the current level of individual i, e.g. the poverty of an individual, the (poor) health status, the insufficiency.

We consider the following type of intervention. Given an initial situation $s = ((c_i, a_i, t_i^{**}))_{i=1}^n$, suppose $R \geq 0$ is an amount of resources to be distributed across the population and x_i denotes the quantity of the good allocated to individual i. There are two types of feasibility constraints which restrict the effectiveness of this intervention. First, there is a *resource constraint*. An allocation $x_R = (x_i)_{i=1}^n$ is said to be achievable for resources R if the aggregate amount of the good allocated does not exceed the total amount of the good available, i.e. provided the following resource constraint holds $\sum_{i=1}^{n} x_i \leq R$.[1] Second, there is the *attainability constraint* discussed in the previous sections, namely that the achieved level or post-intervention status of an individual cannot exceed her attainable level, i.e. an individual's achieved level is given by $x_i + c_i$ provided this quantity does not exceed her attainable level, i.e. provided $x_i + c_i \leq a_i$. On the other hand, if this quantity exceeds her attainable level, i.e. $x_i + c_i > a_i$, then her achieved level is equal to her attainable level and we can say that an excess amount of resources has been allocated to individual i in light of her attainability constraint.

Two important features of attainable levels deserve to be mentioned at this point. Firstly, the ideal levels may not be attainable for all members of the population. In particular, if $a_i < t_i^{**}$ for some $i \in N$, then person i's ideal level is unattainable regardless of the quantity of resources allocated to person i. Secondly, it is possible that for at least some members of the population, their attainable level exceeds their ideal target level even if their current level does not, i.e. $c_i < t_i^{**} < a_i$ for some $i \in N$. For example such cases can arise frequently in poverty analysis and according to sufficientarian approaches to distributive fairness, where the attainable incomes of some of the poor may exceed, perhaps greatly, the poverty line/sufficiency threshold. In view of the objective of aggregate gap closure, e.g. poverty eradication or securing a basic minimum of welfare for each individual (cf. Dorsey 2012), what matters for these individuals is whether a resource allocation helps the individual achieve her ideal target level or not, e.g. the poverty line or the basic minimum, and not whether she exceeds her ideal level since achieved levels beyond this ideal level add no extra value to the objective. With this in mind, the simplest way of dealing with the second feature of attainable levels in light of the capped objective is to define a capped attainable level, denoted a_i^*, such that $a_i^* = a_i$, if $a_i < t_i^{**}$ and $a_i^* = t_i^{**}$, if $a_i \geq t_i^{**}$. For ease of exposition, we shall hereafter refer to the capped attainable

[1] If an individual gains resources then $x_i > 0$. Whereas if she loses resources $x_i < 0$. Otherwise $x_i = 0$.

level simply as the attainable level, although this capped restriction will be implicitly present throughout the analysis.

In this paper, we focus on situations in which shortfalls arise because current levels do not exceed attainable and ideal levels, ideal levels are capped and may not be attainable for all members of the population, i.e.

$$c_i \leq a_i^* \leq t_i^{**} < \infty, \text{ for all } i \in N$$

Each individual is thus characterized by a triple (c_i, a_i^*, t_i^{**}) and our three notions of individual shortfall are defined in terms of differences or gaps in the pairs of each of these levels. Firstly, an individual's *realization shortfall* is measured by the difference between the individual's current level and her attainable level, i.e. $c_i - a_i^* \leq 0$. Negative values indicate the presence of a gap, and its magnitude measures the extent of the individual's realization shortfall (cf. Herlitz & Horan MS). In particular, individuals with larger realization shortfalls are considered worse off from a bad realization perspective. Importantly, this type of gap is a closable gap, i.e. if sufficient resources exist, realization shortfalls can be satisfied for all members of the population.

Secondly, an individual's *bad-offness shortfall* is measured by the difference between her current level and her ideal target level, i.e. $c_i - t_i^{**} \leq 0$. This difference measures an individual's idealized gap, i.e. the individual's current distance to the ideal. Idealized gaps are not necessarily closable. They are closable only in the case where an individual's attainable level is equal to her ideal level, i.e. $a_i^* = t_i^{**}$, otherwise they are not. Individuals with larger bad-offness shortfalls are considered worse-off from this perspective.

Finally, an individual's *bad-outlooks shortfall* is measured by the difference between her feasibility-constrained target level and her ideal level, i.e. $a_i^* - t_i^{**} \leq 0$. This type of shortfall is non-closable, i.e. regardless of the quantity of available resources, bad-outlooks shortfalls cannot be satisfied, due to, for example, time or technological constraints. Bad-outlooks shortfalls thus limit an individual's prospects with respect to the ideal target, irrespective of resource availability. Individuals with larger bad-outlooks shortfalls are considered worse-off from a bad-outlooks perspective.

The following are two relevant special cases of our model, examples of which we will consider in the following section. First, there is the situation discussed in the Lisa and Matt example of the previous section. In this class of situations, all members of the population have individuated attainable levels, the same ideal level $t > 0$, and a common current level $c > 0$, i.e. $t_i^{**} = t$ for all i and $c_i = c$ for all i. Thus, $s = ((c, a_i^*, t))_{i=1}^n$ refers to a situation characterized by a uniform ideal target, a common starting point and heterogeneous attainable levels. Individuals with relatively higher attainable levels, i.e. better prospects, experience larger realization shortfalls, but smaller bad-outlooks shortfalls. All individuals have the same level of bad-offness shortfall.

Second, there are situations characterized by a common ideal level $t > 0$, i.e. $t_i^{**} = t$ for all i, and individuated current and attainable levels. Thus $s = ((c_i, a_i^*, t))_{i=1}^n$ refers to a situation characterized by a uniform ideal target, and heterogeneous current and attainable levels. Individuals with relatively lower current levels, i.e. lower starting points, experience higher bad-offness shortfalls, and individuals with relatively higher attainable levels, i.e. better prospects, have lower bad-outlooks shortfalls

Following Herlitz and Horan, the indicator we present measures aggregate gap closures as a weighted sum of the individual closable gaps of the population (cf. Herlitz & Horan MS). In our model, the relevant closable gap is the individual's realization shortfall, i.e. $c_i - a_i^*$.[2] Suppose $\omega_i \geq 0$ is the weight given to individual i's realization shortfall, where the sum of the weights is standardized to equal 1, i.e. $\sum_i \omega_i = 1$. Given an initial situation $s = ((c_i, a_i^*, t_i^{**}))_{i=1}^n$, resources $R \geq 0$, allocation $x_R = (x_i)_{i=1}^n$ and weighting scheme $\omega = (\omega_i)_{i=1}^n$, the value of the indicator of aggregate gap closures, denoted v, is given by

[2] Focusing on the realization shortfall ensures that the indicator takes into account the attainability constraint and the capped nature of attainable levels in light of the objective.

$$v_\omega(s, x_R) = 1 + \sum_{i=1}^{n} \omega_i \min\{0, x_i + c_i - a_i^*\}$$

where min{ } is the minimum function and the allocation satisfies the following resource constraint $\sum_{i=1}^{n} x_i \leq R$. The minimum function picks the lowest number in the set $\{0, x_i + c_i - a_i^*\}$ which ensures that an excess amount of resources allocated to an individual adds no extra value to the indicator and thus the indicator takes into account both the attainability constraint and the capped nature of the objective, i.e. if $x_i + c_i > a_i^*$, all else equal, then $\min\{0, x_i + c_i - a_i^*\} = 0$.

The maximum value of the indicator is equal to 1 for all weighting schemes, i.e. $v_\omega(s, x_R) = 1$ for all ω satisfying $\sum_i \omega_i = 1$. To see this, note that the objective of aggregate gap closures is completely fulfilled for the population if the realization shortfalls of all individuals in the population who receive positive weight are satisfied, i.e. given an achievable allocation x_R and weighting scheme $\omega = (\omega_i)_{i=1}^{n}$, if $x_i + c_i - a_i^* \geq 0$ for all i with $\omega_i > 0$, then $\min\{0, x_i + c_i - a_i^*\} = 0$ for all i with $\omega_i > 0$, and thus $v_\omega(s, x_R) = 1$.

The indicator satisfies the following monotonicity axiom which ensures that larger values of v indicate a greater degree of aggregate gap closure within the population (cf. Herlitz & Horan MS).

Monotonicity Axiom: Given other things, a reduction in the closable gap of an individual strictly increases the indicator of aggregate gap closure provided the individual receives positive weight.[3]

The satiable property of the gap closure objective allows us to characterize available resources in terms of sufficiency, abundance and scarcity (SAS)

[3] More formally, suppose $x_R = (x_i)_{i=1}^{n}$ and $x'_R = (x'_i)_{i=1}^{n}$ are two achievable allocations for resources R such that x' is Pareto superior to x, i.e. $x_i \leq x'_i$ for all i, and $x_j < x'_j$ for at least one individual j. It can be easily shown that if ω is any arbitrary weighting scheme, then $v_\omega(s, x_R) \leq v_\omega(s, x'_R)$, and if $\omega_j > 0$ and $x_j < a_j^*$, then $v_\omega(s, x_R) < v_\omega(s, x'_R)$.

according to the achievability of the following three types of intervention.[4] Given an initial situation s, resources R are said to be *sufficient* if there exists an achievable allocation such that the attainable levels of *all* individuals in the population who receive positive weight are achieved and none of these individuals receives an excess amount of the good, i.e. given $R > 0$, there exists an allocation x_R satisfying $\sum_i x_i \leq R$ such that $v_\omega(s, x_R) = 1$ and $x_i = a_i^* - c_i$ for all i with $\omega_i > 0$. Secondly, resources R are said to be *abundant* if there exists an achievable allocation such that the attainable levels of *all* individuals in the population that receive positive weight are achieved and at least one of these individuals receives an excess amount of resources, i.e. given $R > 0$, there exists x_R satisfying $\sum_i x_i \leq R$ such that $v_\omega(s, x_R) = 1, x_i \geq a_i^* - c_i$ for all i with $\omega_i > 0$ and $x_j > a_j^* - c_j$ for at least one individual j with $\omega_j > 0$. Finally, resources R are said to be *scarce* if for all achievable allocations, there is at least one individual in the population that receives positive weight for whom their attainable level is not achieved, i.e. given $R > 0$, we have that $v_\omega(s, x_R) < 1$ for all allocations x_R satisfying $\sum_i x_i \leq R$.

In case of scarcity, it becomes necessary to address which weighting scheme is an appropriate one so that different gap closures can be aggregated and alternative allocation alternatives compared. The first problem of aggregation occurs here**.** Several approaches to developing weighting schemes can be pursued. For example, weighting schemes could be individually specified, construed based on population characteristics, characteristics of the initial situation or outcome dependent considerations.

In this paper we focus on weighting schemes derived from the view that individuals should be compensated for shortfalls that are not due to any decision of their own. We thus develop three different weighting schemes on the basis of the three types of shortfall we identified in the initial situation. An advantage of developing weights that depend on an initial situation is that the weights only

[4] For a comprehensive review of the literature on SAS, see Daoud (Daoud 2011).

have to be elicited once and can then be applied to all social evaluations.[5] Each type of shortfall will thus be reflected in a specific weighting scheme.

First, consider a weighting scheme that gives extra weight to gap closures for individuals with greater realization shortfalls, i.e. the shortfall arising from the difference between an individual's attainable and her current level, i.e. $a_i^* - c_{i,}$. Many weighting schemes could be specified to capture such a prioritization. In this paper, we focus on weighting schemes in which the magnitude of an individual's realization shortfall directly determines the priority weight. Suppose $\omega_i = \frac{\max\{a_i^* - c_{i,}0\}}{\sum_j \max\{a_j^* - c_{j,}0\}}$ is the weight applicable to the benefits given to individual i.[6] Incorporating this weighting scheme, the bad-realization indicator of aggregate gap closure, denoted v_1, is given by

$$v_1(s, x_R) = 1 + \sum_{i=1}^{n} \left(\frac{\max\left\{a_i^* - c_{i,}0\right\}}{\sum_j \max\left\{a_j^* - c_{j,}0\right\}}\right) \min\{0, x_i + c_i - a_i^*\}$$

Under this weighting scheme, benefits to individuals with larger realization shortfall receive relatively higher priority weight. The indicator thus favors allocations which distribute more resources to individuals with relatively larger realization shortfalls because benefits to such individuals receive higher weight under the prioritization and thus gap closure for these individuals contributes relatively greater value to the indicator. On the other hand, allocating resources to individuals with relatively smaller realization shortfalls gives a lower score to the indicator and thus contributes less to the overall objective of aggregate gap closure. This indicator thus captures the prioritization view that seeks to compensate individuals who are worse-off in terms of realization shortfalls.

[5] A disadvantage of weighting schemes that depend on an initial situation is that weights do not depend on the outcome and thus the weights may in some circumstances become less relevant for social evaluations if the outcome differs substantially from the initial situation. For an illuminating discussion on the relative merits of outcome-based weighting schemes and weights that depend on an initial situation, see Bleichrodt et al. (Bleichrodt, Diecidue & Quiggin 2004)

[6] We assume that from here on in that there is at least one individual i in the population for whom $c_i < a_i^* < t_i^{**}$. This assumption ensures that all three of our weighting schemes are well defined, i.e. the denominator is non-zero and each of the weighting schemes sums to equal 1.

Second, consider now the prioritization view that seeks to compensate individuals who are worse off in terms of bad-offness shortfalls, i.e. the shortfall arising from differences between their current and ideal levels. Under this view, benefits to individuals with larger idealized gaps ought to be given higher priority weight. Suppose $\omega_i = \frac{\max\{t_i^{**}-c_{i,}0\}}{\sum_j \max\{t_j^{**}-c_{j,}0\}}$ is the weight applicable to benefits given to individual i. Incorporating this weighting scheme, the value of bad-offness indicator of aggregate gap closures, denoted v_2, is given by

$$v_2(s, x_R) = 1 + \sum\nolimits_{i=1}^{n} \left(\frac{\max\{t_i^{**} - c_{i,} 0\}}{\sum_j \max\{t_j^{**} - c_{j,} 0\}}\right) \min\{0, x_i + c_i - a_i^*\}$$

This indicator typically favors interventions that distribute relatively more resources to individuals with larger idealized gaps, i.e. large bad-offness shortfall, since benefits to such individuals receive higher priority weight. Thus, instead of giving relatively higher weight to individuals with larger closable gaps, the indicator gives extra weight to benefits to individuals who face the greatest distance between their current situation and the ideal target level. Such individuals will only have a large closable gap if their attainable target level is reasonably close to their ideal target level, otherwise, their closable gap is smaller, but the weighting scheme still gives extra weight to their gap closure.

Finally, consider the prioritization view that seeks to compensate individuals who are worse off in terms of bad-outlooks shortfalls, i.e. the difference between their attainable and ideal target levels, i.e. $t_i^{**} - a_i^*$. Under this view, the closable gaps of individuals with greater bad-outlooks shortfalls ought to receive relatively higher priority weight. Suppose $\omega_i = \frac{\max\{t_i^{**}-a_i^*,0\}}{\sum_j \max\{t_j^{**}-a_j^* 0\}}$ is the weight applicable to benefits given to individual i. Note for this weighting scheme, individuals with larger bad-outlooks shortfalls are considered worse off since they receive higher priority weight. Incorporating this weighting scheme, the value of bad-outlooks indicator of aggregate gap closure, denoted v_3, is given by

$$v_3(s, x_R) = 1 + \sum_{i=1}^{n} (\frac{\max\{t_i^{**} - a_i^*, 0\}}{\sum_j \max\{t_j^{**} - a_j^*, 0\}}) \min\{0, x_i + c_i - a_i^*\}$$

Under this prioritization, the indicator tends to give higher value to interventions that favor gap closure for individuals who experience relatively greater bad-outlooks shortfall since benefits to such individuals receive higher priority weight. The closable gaps of such individuals may be large or small. On the one hand, the attainable level tends to be relatively further from the ideal target level, but on the other hand, their current level could also be quite low. However, the idea of compensating for bad-outlooks shortfalls requires that their closable gaps, whether small or large, receive extra weight in the prioritization.

In situations characterized by a uniform ideal level, the following standardization technique can be applied to ensure that the values of the gap closure indicator are non-negative, regardless of the specific weighting scheme chosen.

Suppose $s = (c_i, a_i^*, t)_{i=1}^n$ is a situation with a uniform target level $t > 0$ for all individuals. Given an achievable allocation x_R and weighting scheme $\omega = (\omega_i)_{i=1}^n$, the value of the standardized indicator of aggregate gap closures, denoted $\hat{v}$, is given by

$$\hat{v}_\omega(s, x_R) = 1 + \sum_{i=1}^{n} \omega_i \min\{0, \frac{x_i + c_i - a_i^*}{t}\}$$

where the sum of the weights is equal to 1, i.e. $\sum_i \omega_i = 1$. The values of the standardized indicator are non-negative with a maximum value of 1. Its minimum value depends on the weighting scheme and is given by $1 - \sum_i \omega_i \frac{(a_i^* - c_i)}{t} \geq 0$. The standardized indicator thus satisfies the following $0 \leq 1 - \sum_i \omega_i \frac{(a_i^* - c_i)}{t} \leq \hat{v}_\omega(s, x_R) \leq 1$, for all weighting schemes ω with $\sum_i \omega_i = 1$, and achievable allocations x_R. We will make extensive use of this standardization technique in the next section when we consider three examples, each of which has a uniform ideal level.

III: Illustrative Examples of Prioritizations that Conflict

We will now proceed and illustrate how the three weighting schemes can imply different prioritization decisions for one and the same choice situation, and thereby be inconsistent. For reasons of space, we don't use real data for these illustrations.[7] The examples are abstract, but we hope that it is easy enough for the reader to see the resemblance between them and actual social choice situations.

Example 1: Consider a population of $n = 3$ individuals who share a common current level and uniform ideal target, but have heterogeneous attainable levels. Table 1 illustrates one such situation. In this example, whereas individual i has the lowest attainable level, and thus worst prospects, individual k has the highest attainable level, and thus best prospects. Suppose 25 units of resources are available to be distributed within the population, that every individual has the same *capability* of transforming resources into goods (cf. Moreno-Ternero & Roemer 2006), and consider the following two types of interventions. Intervention x allocates all of the extra resources to the person with the best prospects, i.e. to person k. By contrast, intervention y prioritizes persons with lower prospects, allocating resources to person i until their attainable level is achieved, and then distributing all of the remaining resources to the individual with the second lowest attainable level, i.e. person j.

Table 1 reports values of the standardized indicator of aggregate gap closure for weighting schemes that seek to compensate individuals for realization shortfalls and bad-outlooks shortfalls. Notice that the bad-realization indicator ranks

[7] The spreadsheets underlying the computations presented in these examples are available from the authors upon request.

intervention x as the best intervention, i.e. $\hat{v}_1(x) > \hat{v}_1(y)$, whereas the bad-outlooks indicator gives a higher rank to intervention y, i.e. $\hat{v}_3(y) > \hat{v}_3(x)$.

Table 1: Bad-realization and bad-outlooks prioritizations

Population N	Ideal target t^{**}	Attainable target a^*	Current level c	Intervention x	Intervention y	Bad-realization weights ω_1	Bad-outlooks weights ω_3
i	100	10	5	0	5	0.0370	0.6
j	100	50	5	0	20	0.3333	0.3333
k	100	90	5	25	0	0.6296	0.0667
Total	300	150	15	25	25	1	1
Bad-realization indicator: $\hat{v}_1$				0.4704	0.3815		
Bad-outlooks indicator: $\hat{v}_3$				0.78	0.86		

In this situation, the bad-realization prioritization views the person with the best prospects as the worst off since her closable gap is the largest. On the other hand, the person with the worst prospects is viewed as the best off since her closable gap is the smallest. Hence, the bad-realization prioritization gives the highest weight to benefits to person k, and the lowest weight to benefits to i. Consequently, the bad-realization indicator favors interventions that prioritize distributing resources to k rather than i or j and the indicator thus gives the higher value to intervention x.

In contrast, the bad-outlooks prioritization views the person with the worst prospects as the worst off, i.e. person i, since her attainable level is lowest and furthest from the ideal target level. On the other hand, the person with the best prospects is viewed as the best off since her attainable level is highest and closest to the ideal level. Consequently, the bad-outlooks prioritization gives the highest weight to i and the lowest weight to k. In this situation, the bad-outlooks prioritization favors interventions that distribute more resources to i than k and consequently it gives the higher value to intervention y.

Example 2: Consider now a population of $n = 3$ individuals characterized by a uniform ideal target, but heterogeneous attainable levels and heterogeneous current levels. Table 2 illustrates a situation in which individual k has the best starting point but the worst prospects and i has the worst starting point but the best prospects. Suppose 20 extra units of resources are available, every individual has the same capability of transforming resources to goods and we consider two different types of interventions. Intervention x distributes all of the extra resources to the person with the worst prospects, i.e. person k, whereas intervention y distributes all of the extra resources to the person with the worst starting point, i.e. person i.

Table 2 reports, for each of these interventions, values of the standardized indicator of gap closure for weighting schemes based on bad-offness and bad-outlooks shortfalls. Notice that whereas the bad-offness indicator gives higher rank to intervention y, i.e. $\hat{v}_2(y) > \hat{v}_2(x)$, the bad-outlooks indicator gives higher ranking to intervention x, i.e. $\hat{v}_3(x) > \hat{v}_3(y)$.

Table 2: Bad-offness and bad-outlooks prioritizations

Population N	Ideal target t^{**}	Attainable target a^*	Current level c	Intervention x	Intervention y	Bad-offness weights ω_2	Bad-outlooks weights ω_3
i	100	70	0	0	20	0.3704	0.2857
j	100	65	10	0	0	0.3333	0.3333
k	100	60	20	20	0	0.2963	0.3810
Total	300	195	30	20	20	1	1
Bad-offness indicator: $\hat{v}_2$				0.4981	0.5130		
Bad-outlooks indicator: $\hat{v}_3$				0.5405	0.5214		

From a bad-offness perspective, the person with the worst starting point is the worst off since her current level is furthest from the uniform ideal level. Hence, benefits to person i receive the highest weight, whereas benefits to the person with the best starting point, i.e. person k, receive the lowest weight under this prioritization. The bad-offness prioritization favors intervention y since this intervention distributes all of the extra resources to the person with the largest idealized gap in the initial situation.

On the other hand, the bad-outlooks perspective views the person with the worst prospects as the worst off since her attainable level is furthest from the uniform ideal. Consequently in this situation, the bad-outlooks prioritization gives the highest weight to benefits to the person with the best starting point and worst prospects and the lowest weight to benefits to the person with the best prospects. The bad-outlooks prioritization thus favors intervention y since this intervention gives all of the extra resource to the person with the largest bad-outlooks shortfall, i.e. person k.

Example 3: Consider finally a population of $n = 3$ individuals with a uniform ideal target, but heterogeneous attainable levels and heterogeneous current levels. Table 3 illustrates a situation in which individual k has the best starting point and the best prospects, and individual i has the worst starting point and worst prospects. Suppose 20 units of additional resources are available, uniform capability to transform resources to goods and consider the following two types of interventions. Intervention x distributes all of the extra resources to the person with the best starting point and best prospects, whereas intervention y allocates all of the additional resources to the person with the worst starting point and worst prospects.

Table 3 reports values of the standardized indicator for weighting schemes that aim to compensate individuals for realization shortfalls and bad-offness shortfalls. In particular, although the bad-realization indicator gives a higher rank to intervention x, i.e. $\hat{v}_1(x) > \hat{v}_1(y)$, the bad-offness indicator gives the higher rank to intervention y, i.e. $\hat{v}_2(y) > \hat{v}_2(x)$.

Table 3: Bad-realization and bad-offness prioritizations

Population N	Ideal target t^{**}	Attainable target a^{*}	Current level c	Intervention x	Intervention y	Bad-realization weights ω_1	Bad-offness weights ω_2
i	100	40	20	0	20	0.1818	0.3810
j	100	70	30	0	0	0.3636	0.3333
k	100	90	40	20	0	0.4545	0.2857
Total	300	200	90	20	20	1	1
Bad-realization indicator: $\hat{v}_1$				0.6818	0.6273		
Bad-offness indicator: $\hat{v}_2$				0.7048	0.7238		

From the bad-realization perspective, the person with the worst starting point and worst prospects has the smallest closable gap and consequently benefits to her receive the lowest weight under this prioritization. On the other hand, the person with the best starting point and best prospects has the largest closable gap and benefits to her thus receive the highest weight under the bad-realization perspective. Consequently in this example, the bad-realization indicator favors the intervention that distribute relatively more resources to the person with the best starting point and best prospects, i.e. intervention y.

From the bad-offness perspective, the person with the worst starting point and worst prospects has the largest idealized gap and consequently benefits to her receive the highest weight under this prioritization. On the other hand, the individual with the best starting point and best prospects has the lowest idealized gap and benefits to her thus receive the lowest weight. As a result, in this example, the bad-offness indicator favors intervention y since this intervention distributes more resources to the individual whose starting point is furthest from the uniform ideal level.

IV: Conclusion

In this paper, we have underlined the importance of the distinction between attainable levels and ideal levels for assessments of distributive fairness. Once this distinction is recognized, it is revealed that the notion of prioritizing the worse off has three distinct interpretations, all of which are to some extent reasonable. We provided examples of these interpretations in terms of weighting schemes for gap closures, and illustrated that they under certain circumstances support different and incompatible prioritization decisions.

Nothing prevents us, of course, from recognizing the validity of a plurality of grounds for priority weights. However, since the different weighting schemes occasionally conflict, a proponent of this sort of pluralism needs to explain how we should deal with such conflicts. This can be done in different ways. Either, one can develop a way of aggregating the weighting schemes, or one can introduce lexical orders of weighting schemes so that, for example, priority weights that are based on the difference between attainable levels and ideal levels only are relevant when other priority weights fail to establish a unique alternative that is better than all other alternatives. Whichever approach one takes to this issue, it requires an explanation and a defense, and such are not possible to discern in the current literature on substantial principles of distributive fairness.

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