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Abstract

In this paper, the impact of investment horizon on asset co-skewness is examined both empirically and theoretically. We detail a strong horizon-based estimation bias for co-skewness. An asset that has positive co-skewness in one horizon may have negative co-skewness in another. This phenomenon is particularly evident for small-capitalization stocks. We propose a theoretical model to estimate long-horizon co-skewness using the shortest horizon data, which emphasizes the role of adjustment delays in pricing market-wide information among securities. Moreover, in the absence of intertemporal correlation, we show that co-skewness remains horizon-dependent. Our findings are robust to alternative specifications and have strong implications for asset pricing or portfolio allocation with co-skewness.

Keywords: Co-skewness; The Horizon Effect; Intertemporal Correlation; Asset Pricing

JEL Classification: G10, G12, G14

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1. Introduction

The capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) has been supplemented in various ways to better explain expected asset returns. In particular, an extensive literature has provided evidence that systematic skewness, denoted as co-skewness or gamma, further characterizes the risk of an individual security relative to the market (Kraus and Litzenberger, 1976, 1983; Harvey and Siddique, 2000). The importance of accounting for co-skewness both in asset pricing (Conrad et al., 2013; Kostakis et al., 2012) and optimal portfolio allocation (Jondeau and Rockinger, 2012; Martellini and Ziemann, 2010) has been documented. The existing literature has, however, evaluated co-skewness over a range of arbitrarily chosen horizons, with little consideration given to the impact of alternative estimation horizons. Monthly or daily returns are widely used without particular justification.[1] In this paper, we find that return horizon plays a central role in the estimation of co-skewness and the strength of this horizon effect is significantly related to the delay in security price adjustment to market wide information.

While a long line of research has considered the effect of the investment horizon on the estimation of financial parameters, little, if any, consideration has been given to the horizon effect on the calculation of co-skewness.[2] The single-period model underpinning most studies considering co-skewness is silent on the applicable length of an investment period. In a framework where investors are assumed to be heterogeneous with respect to their investment horizon, previous research has documented that the systematic risk (beta or β) of an asset or a portfolio is found to change as the horizon is extended under the single-period Sharpe-Lintner CAPM (Perron et al., 2013; Gençay et al., 2005; Handa et al., 1989; Hawawini, 1980*b*; Cohen et al., 1980). A considerable body of work has also yielded similar horizon effects, for example, in

[1] We examined 45 papers investigating asset co-skewness, published in top peer-reviewed journals including the Journal of Finance, Journal of Financial Econometrics, Review of Financial Studies, Journal of Financial and Quantitative Analysis, Journal of Banking and Finance, Journal of Business and Management Science between 1990 to 2015. 37 of these studies use monthly and/or daily returns. 13 consider returns measured using two or more horizons. For example, Conrad et al. (2013) test the impact of ex-ante skewness on expected stock returns using both daily and monthly returns. Chung and Schill (2006) investigate the pricing roles of the Fama and French (1993) three factors against higher-order comoments using daily, weekly, monthly, quarterly and semi-annual returns.

[2] The 'horizon effect' has, inter alia, been referred to in the literature as: the intervaling effect, the frequency problem, the investment horizon problem, and the holding period problem, among others.

examining common risk factors when the Fama-French three-factor model is considered (Kamara et al., 2016; Brennan and Zhang, 2019), in utilizing the GARCH-M framework through the use of the conditional CAPM (Brailsford and Faff, 1997), and in inspecting the multi-index return generating process underlying the arbitrage pricing theory (Parhizgari et al., 1993).

This paper extends the analysis to co-skewness, also in a single-period framework as underpins the work of Harvey and Siddique (2000). Holding mean and variance constant, prudent investors should prefer assets for which returns are right-skewed, relative to those that are left-skewed (Harvey and Siddique, 2000). Hence, assets with negative co-skewness to the market portfolio, with a resultant decrease to a portfolio's skewness, require higher expected returns; and vice versa. This paper highlights the possibility that portfolios with positive co-skewness in one horizon may have negative co-skewness when measured using another horizon. We find persuasive empirical evidence that co-skewness is highly sensitive to the length of the investment horizon. The signs of estimated co-skewness parameters are highlighted as prone to reversal across differing horizons for size-sorted portfolios, a finding which has not been reported in the literature previously.[3] Exploring the term structure of estimated co-skewness, we find that, for securities where the firm size is greater (less) than the market average, the strength of the horizon effect on co-skewness is positively (inversely) related to their market capitalization.

An additional contribution of our study is the provision of theoretical and economic underpinnings for the horizon effect on co-skewness. The literature documents two alternative explanations for the horizon effect. First, studies focus upon the estimation bias which occurs when using different lengths of investment horizon in the calculation of financial estimates. For example, research suggests that there may be delays in price adjustment for certain stocks to market-wide news (Lo and MacKinlay, 1990; Brennan et al., 1993; Badrinath et al., 1995; Zhang, 2006). The heterogeneous speeds among firms in releasing information and the adjustment of stock prices to market-wide information induce cross-serial correlation in security returns, which may also lead to autocorrelation in market index returns. Moreover, Levhari

[3]While the impact of horizon on co-skewness has not previously been examined, skewness of asset returns has been shown to be horizon dependent (Lau and Wingender, 1989; Hawawini, 1980*b*).

and Levy (1977) find that, even if returns are independent and identically distributed over time, the mathematical calculations (e.g. multiplications) between n-period returns and 1-period returns constitutes "a complex relation" between 1-period betas, $\beta_i(1)$, and n-period betas, $\beta_i(n)$. Brennan and Zhang (2019) confirm the significant role of the multiplicative relation. Estimation using short horizon data may also be biased due to thin trading as identified by Dimson (1979) and Scholes and Williams (1977), which leads to a lack of synchronization between the observed prices of the security and the market portfolio.

Accordingly, following this literature we develop a model of the horizon effect on co-skewness, theoretically indicating that co-skewness measured at any given T- day horizon can be expressed as a function of the daily co-skewness, the length T of the return horizon in days, and the security's intertemporal cross-correlation to the market as well as market autocorrelations in unit returns.[4] We show that our model produces accurate long-horizon estimates of co-skewness using only data from the shortest horizon. Furthermore, our model affirms that the estimation bias resulting from intertemporal correlations among security returns is an important source of the horizon effect on co-skewness. We provide evidence that the horizon effect on co-skewness is more conspicuous when higher order serial correlations are accounted for. Moreover, for a given order of serial correlation, increasing or decreasing the magnitudes of the intertemporal correlation coefficients is also shown to induce estimation bias in co-skewness. These results help to explain our empirical findings for size-sorted portfolios.[5]

A second possible explanation for the horizon effect is linked with a line of literature which documents evidence on the variations in investment horizons across investors and over time, from a behavioural perspective. Investment strategy is found to be horizon dependent. For example, Dierkes et al. (2010) investigate interactions between investment preferences and the investment horizon. Dierkes et al. (2010) find that investing in stocks is a better strategy for in-

[4]In the context of this paper, intertemporal correlations refer to the range of non-contemporaneous lagging and leading correlations between assets and the market, including those involving quadratic terms.

[5]Theobald and Yallup (2004) suggest that the speed of pricing adjustment for larger firms is greater than that of smaller counterparts. The authors establish a lead/lag relationship and demonstrate that large firms have faster speeds of adjustment than small firms. Their results are consistent with various other papers, such as Jegadeesh and Titman (1995) and Lo and MacKinlay (1990), which have established and investigated the lead/lag effects across size sorted portfolios and show that returns of large capitalization stocks lead those of small capitalization stocks. This supports why we find that larger firms have a relatively small horizon effect on co-skewness.

vestors with long investment horizons, while a pure bond strategy is a more optimal choice for those with short horizons. Aït-Sahalia and Brandt (2001) also find that the selection of optimal portfolios and the corresponding weights vary across investment horizons. Furthermore, clientele with different trading frequencies may have different perceptions of risk (Hur and Singh, 2016; Andries et al., 2015) and heterogeneous preference for skewness (Mitton and Vorkink, 2007). If investors are risk-averse, prudent and temperate, literature suggests that firms whose returns exhibit negative co-skewness should require higher premia relative to those with positive co-skewness (Kostakis et al., 2012; Harvey and Siddique, 2000). Thus, investors who trade frequently may have higher risk-tolerance and prefer to invest in securities with larger negative co-skewness to seek a higher risk premium, while longer-horizon investors may be willing to accept a low premium and only pursue securities with low co-skewness.

Our theoretical model and related findings also link to this behavioural reasoning by showing that in the case of co-skewness, intertemporal cross-correlation and autocorrelations are not solely responsible for the horizon effect. We find that, while intertemporal cross-correlation and auto-correlation help to drive the horizon dependence of co-skewness, the horizon effect is present even without such characteristics. While a similar finding has been detailed for skewness (Hawawini, 1980*a*), previous theoretical work linked the horizon effect in beta to frictions, manifesting as intertemporal cross-correlation and auto-correlation. Hawawini (1980*b*) proposed an early model to explain the relation between the estimated betas and intertemporal cross-correlations. This model suggests that there is no horizon effect on beta under the independence assumption in an efficient market, where all securities' prices are assumed to immediately adjust to new market-wide information. Our model, however, shows that the length of the investment horizon also plays a significant role, indicating a "scaling law" of co-skewness in the absence of intertemporal correlation. Therefore, any research or investment strategy related to the estimation of co-skewness should be cautious in choosing the investment horizon, in contrast to the case where only beta is considered.

Finally, given the evidence of the horizon effect on co-skewness, we examine the implications of this effect for higher-order asset pricing. In particular, we examine whether the cross-sectional variation in asset returns can be explained by exposure to co-skewness using the Fama and MacBeth (1973) two-step method. First, when using returns collected from the CRSP daily tape, we examine the pricing role of co-skewness in seven horizons (from one-day to one-quarter). Co-skewness is found to be consistently priced for horizons of up to 40 days. Second, we have also tested the horizon effect on pricing roles of co-skewness based on returns collected from the CRSP monthly tape. When longer-horizon returns are considered, it is more evident that co-skewness is only significantly priced for horizons shorter than quarterly. This meets theoretical expectations as, for increasing horizon, asset returns tend to be more normally distributed. Our results contribute to the literature by considering the impact of investment horizons and the changing pricing roles of higher-order moment factors. Using an arbitrary length of return horizon may lead to inconsistent conclusions.

Our findings for the impact of the sampling horizon on co-skewness is important for both portfolio selection and asset pricing. As proposed earlier, investors' preference for positive skewness typically leads to a desire for positive and higher co-skewness, which represent higher probabilities of extreme positive outcomes for a security relative to market returns. Our results suggest that an asset which is selected for a portfolio based on its positive co-skewness using one investment horizon may have negative co-skewness when using another horizon, with resultant implications for asset and portfolio selection (higher order moments have been considered for portfolio optimization by Jondeau and Rockinger (2012), Martellini and Ziemann (2010) and Guidolin and Timmermann (2008), for example.) In particular, one implication of the horizon dependence of systematic skewness is that related risk premia may also be sensitive to the investment horizon. Our empirical and theoretical findings convey a word of caution for empirical researchers who estimate asset co-skewness, especially for securities with extreme firm size (largest or smallest) .

The paper is structured as follows. Section 2 introduces the asset pricing implications of co-skewness from first principles and develops our model of horizon-dependent co-skewness. Section 3 describes the data employed and presents empirical results related to the estimation of co-skewness coefficients. Section 4 presents further tests on the pricing of co-skewness factor and its relation to the horizon effect. Concluding remarks are given in Section 5.

2. Co-skewness and Modelling Process

2.1. Asset Pricing Implication of Co-skewness and Investment Horizon

As noted by Brennan and Zhang (2019), the basic capital asset pricing model (CAPM) is a single-period model, where the horizon or time period is not specified. Similarly, in our set-up for a single-period model, an investor is holding a risky asset i for one period, with the length of the period (investment horizon) unspecified. The first-order condition is:

$$E[(1+R_{i,t+1})M_{t+1}|\Omega_t] = 1, \tag{1}$$

where $1+R_{i,t+1}$ is the total return on asset i at $t+1$, M_{t+1} is the marginal rate of substitution of the investor between a single period from t and $t+1$, and Ω_t is the information set available to the investor at time t. The length of the investment horizon (from t to $t+1$) may affect the content in this information set and thus the marginal rate of substitution, highlighting a potential horizon effect on pricing asset i. The estimation of beta in the traditional framework of CAPM may result in a horizon effect, often attributed to thin trading (Scholes and Williams, 1977), differences in the speed of adjustment to common information (Lo and MacKinlay, 1990) or a complex multiplicative relationship between short- and long-interval returns (Levhari and Levy, 1977).

Different pricing kernels vary primarily in the set-ups they use for the M_{t+1}. Kraus and Litzenberger (1976) and Harvey and Siddique (2000) extend the static CAPM to nonlinear forms of the risk-return trade-off by considering systematic skewness, developing the notion that moments of returns other than variance are relevant to maximizing investors' expected utility. Similarly, the horizon effect remains an issue when the single-period third-order asset pricing model is considered. In particular, in the theoretical framework developed by Harvey and Siddique (2000), a valid quadratic functional stochastic discount factor is proxied for the marginal rate of substitution:

$$M_{t+1} = a_t + b_t R_{m,t+1} + c_t R^2_{m,t+1}. \tag{2}$$

Taking a second-order Taylor series expansion of $U'(W_{t+1})$ around W_t, the quadratic SDF

(Equation 2) can be expressed as:

$$M_{t+1} = 1 + \frac{U''(W_t)W_t}{U'(W_t)} R_{m,t+1} + \frac{U^{(3)}(W_t)W_t^2}{2U'(W_t)} R_{m,t+1}^2 + o(W_t), \tag{3}$$

where the $b_t = \frac{U''(W_t)W_t}{U'(W_t)}$ matches the definition of the Arrow-Pratt measure of Relative Risk Aversion and $\frac{U^{(3)}(W_t)W_t^2}{2U'(W_t)}$ is one-half of the product of risk aversion and $c_t = \frac{U^3(W_t)W_t}{U''(W_t)}$ (a measure of Relative Prudence defined by Kimball (1990)). Thus, we have $b_t < 0$ and $c_t > 0$, since $U^{(3)}(W_t) > 0$ based on the assumption of non-increasing absolute risk aversion for a risk-averse utility-maximizing agent (Arrow, 1964; Scott and Horvath, 1980). Kimball (1990)) also suggests prudence with non-increasing absolute risk aversion for a risk-averse utility-maximizing agent is associated with the desire to avoid disappointment.

Therefore, for a risk averse and prudent investor, increases in total skewness are preferred. As for co-skewness, an asset with negative co-skewness to the portfolio may not be selected, as adding an asset with negative co-skewness to a portfolio results in a more negative skewness for the portfolio. In this paper, we build upon the extensive literature examining the effect of investment horizon on beta (for example, Perron et al. (2013), Handa et al. (1989), Cohen et al. (1983*a*), Hawawini (1980*b*) and Scholes and Williams (1977)) and determine whether co-skewness presents a similar phenomena. If co-skewness is horizon dependent, both the non-linear pricing kernel or portfolio selection with skewness may also be affected by the choice of the investment horizon.

2.2. Estimation of Co-skewness

We follow the definition of Harvey and Siddique (2000) for co-skewness. To estimate the degree of horizon-dependent co-skewness, we repeat the estimation procedure using returns measured at different horizons, and thus have several samples of co-skewness estimates. More precisely, we first employ the CAPM regression using a rolling window of 5-year excess returns for share i:[6]

$$R_{iT,t} - R_{fT,t} = \alpha_{iT,t} + \beta_{iT,t}(R_{mT,t} - R_{fT,t}) + \varepsilon_{iT,t}, \tag{4}$$

to extract the residuals $\varepsilon_{iT,t}$, which are, by definition, orthogonal to the excess market returns. $R_{iT,t}$ and $R_{mT,t}$ represent T-day asset returns and market returns, respectively. Therefore, these residuals are net of the covariance (beta) risk, but still incorporate co-skewness risk.

Using the residuals from Equation 4, Harvey and Siddique (2000) estimate standardized co-skewness for share i at horizon T, namely $\gamma_{iT,t}$, using a 5-year window up to time t as:

$$\gamma_{iT,t} = \frac{E[\varepsilon_{iT,t}\varepsilon^2_{mT,t}]}{\sqrt{E[\varepsilon^2_{iT,t}]}E[\varepsilon^2_{mT,t}]}, \tag{5}$$

where $\varepsilon_{iT,t} = [R_{iT,t} - R_{fT,t}] - [\alpha_{iT,t} + \beta_{iT,t}(R_{mT,t} - R_{fT,t})]$ is the residual from Equation 4 and $\varepsilon_{mT,t}$ is the deviation in the excess market return for month t from the average value over the corresponding window.

2.3. Decomposition of Co-skewness: the Sum of Intertemporal Cross-covariances

A substantial literature has documented the horizon effect on both security alphas (Boguth et al., 2016) and betas (see, for example, Handa et al. (1989) and Hawawini (1983)), but the impact on co-skewness remains an open question. Given that residuals $\varepsilon_{iT,t}$ are a function of both α and β, as shown in Equation 5, this, in isolation, should lead to co-skewness being horizon-dependent. In this paper, however, we wish to demonstrate the horizon dependence of co-skewness, itself related to intertemporal characteristics associated with return time series, independent of the horizon effect on alphas and betas. Similar to Cohen et al. (1983*a*), we develop a model to relate the horizon effect of co-skewness to characteristics of autocovariance and intertemporal cross-covariance, in light of the frequent documentation of autocorrelation and intertemporal cross-correlation of financial time series in the literature (Cohen and Frazz-

[6]The existing literature has also documented the possible impact on estimating financial parameters by the choice of the size of the window used in estimation. Our focus, in contrast, is the sensitivity to the sampling horizon, for example daily, weekly or monthly. We do, however, confirm the robustness of the horizon effect using a 15-year window.

ini, 2008; Lo and MacKinlay, 1990).[7] We acknowledge that there may be multiple fundamental sources of such horizon effects such as clientele, investment horizon preferences and estimation bias. Many of these suggested sources may manifest as autocorrelation and intertemporal correlation in the underlying data and, which, in turn, impact the estimation of co-skewness. Our approach has the benefit of allowing us to garner further insight into the estimation of co-skewness at various horizons in a frictionless environment. To this end the following proposition details the relationship between long-run co-skewness and characteristics estimated at short-run horizons.

Proposition 1. *Co-skewness at horizon T is a function of the length of the horizon, the intertemporal auto-covariance of asset returns and of market returns, the intertemporal cross-covariance between returns of the asset and squared market returns, and the estimated betas at horizon T, which is given by:*

$$\gamma_{iT,t} = \frac{cov(\varepsilon_{iT,t}, \varepsilon^2_{mT,t})}{\sigma(\varepsilon_{iT,t}) \cdot var(\varepsilon_{mT,t})}$$
$$= \frac{\sum_{j=0}^{T-1}\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{i1,t\text{-}j}, R_{m1,(t\text{-}k)} \cdot R_{m1,(t\text{-}l)}) - \hat{\beta}_{iT,t} \sum_{j=0}^{T-1}\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{m1,t\text{-}j}, R_{m1,(t\text{-}k)} \cdot R_{m1,(t\text{-}l)})}{\sqrt{\sum_{k=0}^{T-1}\sum_{u=0}^{T-1} [cov(R_{i1,(t\text{-}k)}, R_{i1,(t\text{-}u)}) - \hat{\beta}_{iT,t} cov(R_{i1,(t\text{-}k)}, R_{m1,(t\text{-}u)})]} \cdot \sum_{k=0}^{T-1}\sum_{u=0}^{T-1} cov(R_{m1,(t\text{-}k)}, R_{m1,(t\text{-}u)})} \quad (6)$$

Proof: See Appendix A.

Equation 6 demonstrates that co-skewness at horizon T can be expressed as a function of short-run returns, $R_{i,1}$ and $R_{m,1}$, accounting for characteristics of auto-covariance and intertemporal cross-covariance. In Appendix A we show that α has no impact on the estimation of co-skewness and $\hat{\beta}_{iT,t}$ can be expressed as a function of short-run characteristics using the fol-

[7] While our model can be expressed in terms of intertemporal cross-correlation terms, as exposition in terms of intertemporal cross-covariance provides equivalent insights, we follow this route for brevity.

lowing term, as suggested in the literature:

$$\hat{\beta}_{iT,t} = \frac{cov(R_{iT,t}, R_{mT,t})}{var(R_{mT,t})} = \frac{\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{i1,(t\text{-}k)}, R_{m1,(t\text{-}u)})}{\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{m1,(t\text{-}k)}, R_{m1,(t\text{-}u)})}. \tag{7}$$

2.4. The Long-Run Horizon Effect

Hawawini (1980*b*) derives a covariance-time function, and links long-horizon covariance to short-horizon estimated covariance, as:

$$\sigma_{im,t}(T) = \sigma_{im,t}(1)[T + \lambda_{im}(T)] \tag{8}$$

where $\lambda_{im}(T) = \sum_{s=1}^{T-1}(T-s)\frac{\rho_{im}^{-s}+\rho_{im}^{+s}}{\rho_{im}} = \sum_{s=1}^{T-1}(T-s)q_{im}^{s}$; ρ_{im}^{+s} is the lead and ρ_{im}^{-s} the lagging intertemporal cross-correlation coefficients of order s in unit returns, ρ_{im} is the contemporaneous cross-correlation coefficient.

Cohen et al. (1983*a*) documents that the estimated beta would converge to a "true" beta when the horizon is sufficiently lengthened, since returns measured using longer-horizons are less affected by serial correlations. Hawawini (1980*a*) suggests that if there is neither autocorrelation in market returns nor intertemporal cross-correlation between securities' returns and the market returns, beta is invariant to the length of the investment horizon. In a similar sense, we derive the following proposition to understand the long-horizon implications for co-skewness in a frictionless environment:

Proposition 2. *For unconditional estimates, with stationary returns, if individual assets' intertemporal auto-covariances and intertemporal cross-covariances among asset returns are not present, intertemporal variation in unconditional co-skewness still exists and is inversely related to the square-root of the horizon T, as follows:*

$$\gamma_{iT,t} = \frac{\gamma_{i1,t}}{\sqrt{T}}. \tag{9}$$

Proof: See Appendix B.

Equation 9 is analogous to the square root of time scaling law associated with risk and the related rule proposed for skewness (Lau and Wingender, 1989). The former has also been shown to be perturbed by serial dependence in the underlying time series (Wang et al., 2011).

In Equation 9, we demonstrate that the horizon effect on co-skewness may exist even in an efficient market with no price delay. Furthermore, Equation 9 shows the horizon effect on co-skewness is not merely a consequence of the horizon effect associated with beta and alpha, as might be expected from their individual horizon-dependence. Instead, co-skewness has an inherent sensitivity to the investment horizon, but without a frictionless market co-skewness may be perturbed by autocorrelation and intertemporal cross-correlations.

3. Data and Empirical Facts

3.1. Data

To empirically examine the horizon effect on co-skewness, our initial sample consists of all NYSE/AMEX/NASDAQ-listed stocks with available data from the Center for Research in Security Prices (CRSP). Alongside prices, the total market capitalization, trading volumes of outstanding shares and adjusted dividends are also collected. We impose several screening criteria to our initial sample as suggested in the literature: we use stocks with share codes 10 or 11, but exclude firms with market value less than $5 million and firms with price less than $5 at the beginning of each month, and firms without a minimum of 1 year of return data. We include both listed and dead firms. Thus, our data set is free of any potential survivorship bias. Our sample period is August 1, 1962 to December 31, 2015.

Non-overlapping logarithmic returns are calculated for seven sampling horizons: 1, 2, 5, 10, 20, 40 and 65 business days. These horizons encompass the majority of regular investment horizons used both in practice and in the literature: daily, bi-daily, weekly, bi-weekly, monthly, bi-monthly and quarterly. For all horizons, we primarily employ self-calculated value-weighted returns across all stocks as the market index.[8] The risk-free rate is the (compounded) three-month T-bill rate, and we scale the rate for each sampling horizon.

[8]We do not use the CRSP index as our market index because its returns are not logarithmic, and therefore are not additive. Our daily market returns are reasonable and closely track the daily CRSP index having a correlation coefficient of 0.994.

Following previous research examining the horizon effect, we detail our empirical results by forming portfolios based on the market capitalization of securities. As mentioned above, Theobald and Yallup (2004) suggest that the speed of pricing adjustment for larger firms is greater than that for smaller counterparts. Therefore, we hypothesize that larger firms have a relatively smaller horizon effect on co-skewness. We sort all stocks into 20 equal-sized portfolios by market capitalizations (*MC* portfolios). We reassign securities on an annual basis into these 20 portfolios according to their average market capitalization ranks.[9] The first, *MC*1, contains the smallest 5% of firms and the last, *MC*20, contains the largest 5%.

Table 1 documents some intertemporal cross-correlations between each MC portfolio and the market portfolio. The "1-lag" ("1-lead") cross-correlation coefficient indicates the cross-correlation between the MC portfolio returns and market returns lagged by (leading by) one period. For all portfolio sizes, we observe an increase in 1-lag serial cross-correlations and autocorrelation between daily data and monthly data. This increase in serial cross-correlation will impact upon the estimation of co-skewness, as demonstrated in our discussions on the long-run horizon effect in Section 2.4. Moreover, returns of smaller firms have greater cross-correlations to market returns. This supports the evidence detailed by previous literature that the speed of pricing adjustment for larger firms is relatively more rapid. Finally, we observe some evidence for significant serial cross-correlation and autocorrelation for daily data for a lag of 5 days.

TABLE 1 ABOUT HERE

3.2. Sensitivity of Estimated Co-skewness to the Investment Horizon

In this subsection, we provide empirical evidence for the existence of a horizon effect on portfolio co-skewness. Co-skewness is calculated for each portfolio using a 5-year moving window. Sorted on the basis of securities' market capitalizations, Table 2 details the magnitude of co-skewness averaged over all 5-year windows and associated test statistics. In each case, co-skewness is estimated relative to the value-weighted market index.

[9] Our set-up for portfolios is similar to those in Perron et al. (2013) and Handa et al. (1989). Our portfolio composition changes each year, since firms' average market capitalizations are re-ranked and firms enter and leave the sample as a result of listing, delisting, mergers, and the like. Moreover, only the NYSE security market cap is used for setting breakpoints.

TABLE 2 ABOUT HERE

Panel A documents the average co-skewness for each market capitalization (*MC*) formed portfolio across horizons, showing clear evidence of the horizon effect on co-skewness. First, we find evidence that the estimated level of co-skewness differs according to the horizon considered. This is particularly evident for portfolios consisting of small cap stocks. For example, at a 1-day horizon co-skewness for portfolio MC1 is -0.425, increasing to -0.067 and 0.123 for 20-Day and 65-Day horizons, respectively. This also highlights a second novel finding. For all of the portfolios MC1-MC15, the sign of co-skewness also reverses as we move from short to long-horizons. While $MC16-MC19$ display negative co-skewness at all horizons, $MC20$ is the only portfolio for which co-skewness is estimated to be consistently positive. Given that firm size is a proxy for price adjustment delay, we conclude that the possibility of sign reversal may be influenced by the speed of stock price reaction to market wide information.

Our findings also relate to previous work, which demonstrates that betas of smaller firms tend to increase, while those for the largest firms decline as the investment horizon lengthens, also linked with price adjustment delay (Cohen et al., 1983*a*,*b*). Analogously, we find that co-skewness estimates for the first 17 portfolios ($MC1-17$) in Table 2 are broadly monotonically increasing as the sampling horizon is lengthened. The trend in co-skewness estimates for the portfolio comprised of stocks with the largest market capitalizations ($MC19$ and $MC20$) is, in contrast, downward sloping as the sampling horizon increases from one day to a quarter. These findings for a firm-size related effect are in agreement with the behaviour of security betas documented in the extant literature.

Panel B summarizes three measures through which we capture the relative size and significance of the horizon effect in co-skewness. The extant literature considering the horizon effect has not specified a standard test to measure the strength of the horizon effect. The standard deviations of estimates for the same security over horizons and the F-statistics of one-way analysis of variance (ANOVA1) are suggested as indicators (Corhay, 1992). We first use the standard deviation as one measure for the magnitudes of the horizon effect. As our second measure, we augment the use of ANOVA1, with a One-Way Repeated Measures ANOVA[10],

[10]The one-way repeated measures ANOVA (also known as a within-subjects ANOVA) is ideal in testing the

since tests of significance for ANOVA1 are known to be valid only if the samples are independent and the variance of each sample is equal. Therefore, in our tests, if there is no horizon effect, the standard deviation and augmented F-statistics should be zero, while larger values indicate a stronger effect. Moreover, considering that the degree of standard deviation of any sample depends on the magnitudes of the average of the sample, we define an intuitive variable, referred to as Estimation Bias or $EB_{\gamma,i}$, as the sum of the absolute differences between other horizon estimates and monthly (20-Day) estimates:

$$EB_{\gamma,i} = \sum_{n} \left| \overline{\gamma}_{in,t} - \overline{\gamma}_{iT_m,t} \right| \qquad n \in \{1,2,5,10,20,40,65\} \tag{10}$$

where $\overline{\gamma}_{in,t}$ are the averaged co-skewness for an $n-day$ horizon and T_m stands for the length of a business month. EB quantitatively captures the aggregate tendency of co-skewness estimates to vary from that measured at a monthly (20-Day) horizon, given monthly data is mostly employed in the existing literature. This measure of estimation bias is zero if there is no horizon effect.

We reject the null hypothesis that co-skewness is equal to that estimated at a monthly horizon at a 1% level for the three measures shown in Panel B. In more detail, for all securities with smaller market capitalization than that of the averaged market capitalizations of firms constituting the market portfolio, the magnitudes of the horizon effect on their co-skewness are inversely related to their market capitalizations. For example, $MC17$ and $MC18$ have the smallest standard deviations of 0.037 and 0.030 respectively. Companies in the first 16 MV portfolios all have lower market capitalizations than the market average and their standard deviations are found to be inversely related to the market capitalization. In contrast, for portfolios which have greater-than-average firm size, the sensitivity of co-skewness estimates to the investment horizon is positively related to their market capitalizations. The standard deviations of $MC18$, $MC19$ and $MC20$ in Panel A of Table 2, for example, are 0.030, 0.107 and 0.067, respectively. Therefore, we could conclude that for any security, the greater the deviation in the firm's size relative to the average size of companies in the market index, the more sensitive the co-moments will be to the selection of the investment horizon.

horizon effect, since it is used to determine whether three or more group means are different where the participants are the same in each group.

Avoidance of data problems due to the listing and delisting of securities is important in estimating and analyzing systematic risks. Moreover, by using non-overlapping returns over a fixed time window (5 years in the analysis just presented) the number of observations may be very small for longer investment horizons. For example, we measure co-skewness using approximately 1250 observations when using 1-Day returns, but only 20 observations when using 65-Day returns. To provide a larger sample at longer horizons, we re-examine co-skewness of securities during the whole period from August 1962 to December 2015, in addition to two sub-periods from August 1962 to December 1989 and from January 1990 to December 2015, respectively. Findings, shown in Table 3, provide further evidence for the horizon-dependency of co-skewness. A strong horizon effect is detailed for companies in the portfolios consisting of the smallest stocks, in keeping with the idea of price transmission delays in market-wide information. The change in sign from 1-Day to 2-Day for small stocks in the 1990 – 2015 subperiod is notable. In unreported analysis, we exclude the two-day period around the 'Black-Monday' crash of October 1987, with the result that 2-Day estimates of co-skewness are negative in this sub-period. For the largest stocks, co-skewness is predominantly found to be positive in keeping with our earlier findings.

TABLE 3 ABOUT HERE

3.3. Model of Horizon-Dependent Co-skewness

We now test if the theoretical decomposition, Equation 6, of co-skewness using short-horizon returns while accounting for intertemporal cross-covariance provides an adequate description of the properties of co-skewness estimates. In Table 4, we illustrate the estimates of co-skewness originating from our horizon-dependent model for portfolios $MC1$ and $MC20$, using the same set-up over the entire period provided in Panel A of Table 3.

TABLE 4 ABOUT HERE

To calibrate Equation 6, we require estimates of $\varepsilon_{iT,t}$ and $\varepsilon_{mT,t}$ at the shortest horizon under consideration (1-Day). Equation 6 also requires estimates of long-horizon beta. These, in turn can be either empirically estimated using long horizon data or model-generated using Equation

7. In Table 4, beta is empirically estimated using long horizon data, but results are consistent for model generated betas.

Panel A of Table 4 details estimates of co-skewness by incorporating different orders of serial-covariance and intertemporal cross-covariance. The first row lists the simulated results if there is no serial covariance terms (0 order). Moving from short to long horizons, a clear monotonic increase in co-skewness is observed, with the long horizon theoretical estimate converging towards zero. These findings help to validate the $1/\sqrt{T}$ rule, described in Proposition 2. In the absence of serial-covariance and intertemporal cross-covariance co-skewness estimates converge towards zero. The results, however, present a clear difference between modelled co-skewness with no serial covariance and estimates found using long-horizon data. For example, for $MC1$ the model suggests a value of -0.0373 for 65-Day co-skewness, while the empirical estimate is 0.265. These differences can be attributed to a presence of intertemporal covariance terms for the estimated value.

Including intertemporal covariance terms of order 1, we see that the 2-Day theoretical estimate of -0.0501 is now in keeping with that of the 2-Day empirical estimate. Similar findings are evident at higher orders. At the limit, we consider up to 64 intertemporal covariance terms and we observe that 65-Day theoretical co-skewness increases from -0.0373 to 0.2670 as the number of intertemporal covariance terms is increased, where the latter is in keeping with the estimate from long horizon data. Similar findings are evident for the largest portfolio $MC20$. This highlights the importance of intertemporal covariance terms in the estimation of long-horizon co-skewness. Furthermore, these results illustrate why empirical co-skewness estimates do not follow the $1/\sqrt{T}$ rule described in Proposition 2, as they are perturbed by intertemporal covariance in the underlying time series from the theoretical level.

In panel B, we further examine the importance of intemporal relations in the underlying time-series on co-skewness estimates at different horizons. The objective here is to determine the extent of the impact of serial- and cross-serial correlation on co-skewness. To this end, we add increments to the serial- and cross-serial correlation terms and examine whether this changes the magnitude and sign of co-skewness. 1^{st} and 4^{th} order intertemporal relations are considered as examples. Our findings indicate that adding larger increments in absolute value terms (e.g. +0.03 or -0.03) further biases the estimation of co-skewness.

For example, considering just 1^{st} order intertemporal terms, the model in Panel A estimates a co-skewness of -0.1517 at a 5-Day horizon. Including an increment of -0.03 (0.03) to the serial correlation and cross-correlation terms, we see that the magnitude of co-skewness changes to -0.2373 (-0.0864). Similarly, for $MC20$ with 4^{th} order intertemporal terms, co-skewness is 0.273 using 5-Day data. Incrementing the intertemporal correlations, both serial and cross-correlations, by terms ranging from -0.03 to 0.03 results in co-skewness estimates ranging from -0.199 to 0.522. This highlights how intertemporal relations in the underlying time series may not only alter the magnitude of co-skewness but also the associated sign.

Our model of co-skewness horizon dependence shows a clear relationship with intertemporal covariance terms. Without such terms the scaling law derived in Equation 9 is found to hold, with co-skewness converging to zero at long horizons. In other words, co-skewness has an inherent horizon dependence, but this may be masked in long horizon co-skewness estimates due to intertemporal covariance. Incorporating various lags of interdepdence we provide evidence that co-skewness estimates at different horizons are impacted by underlying time series characteristics. These findings and those detailed earlier may have implications for higher-order asset pricing, a question we address next.

4. Higher-order Asset Pricing and Horizon-Dependent Co-skewness

We next explore implications for higher-order asset pricing. The pricing of the co-skewness risk factor and related research on the three-moment asset pricing model (3M-CAPM) may also be sensitive to the horizon. In particular, we examine whether the cross-sectional variation in asset returns can be explained by exposure to co-skewness using the Fama and MacBeth (1973) two-step method at different investment horizons. Given co-skewness in each month t for asset i in horizon T (denoted as gamma or $\gamma_{it,T}$)[11], estimated using Equation 5, the first-stage regression

[11] Harvey and Siddique (2000) apply this conditional co-skewness in asset pricing tests and Doan et al. (2014) also use the standardized conditional co-skewness measure and check the performance both using time-series and Fama-Macbeth cross-sectional regressions. Harvey and Siddique (2000) and Kostakis et al. (2012), among other studies, have also constructed the co-skewness risk factor in a similar way to the Fama and French (1993) factors by using return spreads between portfolios with lowest and highest co-skewness. We have also tested this alternative approach and found a similar horizon effect on pricing of co-skewness.

involves estimating the factor loading for $\gamma_{it,T}$ through the 3M-CAPM,

$$\overline{R}^{i}_{t,T} - R^{f}_{t,T} = \beta^{Mkt}_{t,T}(\overline{R}^{M}_{t,T} - R^{f}_{t,T}) + \beta^{\gamma_{it,T}}_{it,T}\gamma_{it,T}, \qquad t = 1,2,3,4...,T \;\; for \;\; each \;\; i. \tag{11}$$

Following Harvey and Siddique (2000) we use individual securities in the Fama-MacBeth second-stage regression. To control for the variations, we weight the securities using $1/\sigma(\hat{\varepsilon})_{it,T}$, where $\sigma(\hat{\varepsilon})_{it,T}$ is the standard deviation of the residuals from the beta estimation. We also consider the approach suggested by Kamara et al. (2016) to better compare our results across horizons, by using overlapping returns and adjusting factor loadings ($\beta^{Mkt}_{t,T}$ and $\beta^{\gamma_{it,T}}_{it,T}$) standardized to a mean of zero and a standard deviation of one in each month.

These standardized factor loadings are used in the second-stage regression to estimate the corresponding market and co-skewness risk premia coefficients, using the cross-sectional regression,

$$\overline{R}^{i}_{t,T} - R^{f}_{t,T} = \lambda_{t,T} + \lambda^{Mkt}_{t,T}\hat{\beta}^{Mkt}_{t,T} + \lambda^{\gamma_{it,T}}\hat{\beta}^{\gamma_{it,T}}_{t,T}, \qquad i = 1,2,3,4...,N \;\; for \;\; each \;\; t. \tag{12}$$

The slope coefficients are generated as average values across all test months, as shown in Table 5. The t-statistics (in brackets) of cross-sectional regression coefficients are calculated using Newey-West standard errors with five lags to reduce the impact of overlapping returns and the Shanken correction is also employed to help alleviate the errors-in-variables problem.

TABLE 5 ABOUT HERE

Panel A illustrates results using returns collected from the CRSP daily tape. Co-skewness is consistently priced in most horizons and the lambda coefficients are all negative, except using 65-D (quarterly) returns. Harvey and Siddique (2000) also find a significant negative premium associated with β_{γ} when monthly returns are considered. It is reasonable to expect that the pricing role of co-skewness is diminished for longer horizons, since long-horizon asset returns tend to be more normally distributed. Moreover, we have found in Table 2 that the sign of co-skewness changes systematically as the horizon lengthens. Although the coefficient is insignificant, the sign associated with 65-day lambda of co-skewness is found to be positive.

Thus, to validate our findings over longer horizons, in Panel B we repeat our tests by using returns data from the CRSP Monthly tape. We find that co-skewness is significantly priced only for horizons of three months or less, in keeping with findings detailed using daily data.

To summarize, co-skewness risk is only found to be priced at short horizons. Our results contribute to the literature by considering the impact of investment horizons and the changing pricing roles of higher-order moment factors.

5. Discussion and Concluding Remarks

Co-skewness extends the Sharpe-Lintner CAPM, allowing for more detailed characterization of individual asset risk. While the extant literature has highlighted the importance of co-skewness in explaining the cross-section of stock returns, the choice and implications of the return horizon selected have not been considered. Based on an extensive data sample of stocks from the CRSP database for the period 1962 to 2015, this paper details the sensitivity of co-skewness estimates to the return horizon. Developing a system of consistent sequential tests, we also measure the strength of the horizon effect for co-skewness estimation. The magnitude of the effect is shown to be linked with firm characteristics and relates to price adjustment delay, as represented by firm size. In order to develop an understanding as to the drivers of horizon-dependent co-skewness, we propose a model of long-horizon co-skewness which employs only data at the shortest horizon. Using this, we provide evidence that auto- and intertemporal cross-correlations influence the estimation of co-skewness, but that co-skewness also has an inherent horizon dependence.

Accordingly, this study sheds new light on research considering higher-order asset pricing and portfolio selection. First, we show evidence that the magnitudes of the estimates of third moment-related pricing coefficients (gamma) are significantly influenced by sampling horizon. Moreover, the sign of gamma may reverse for longer horizons. Second, we refine our estimation of gammas by using returns of market-capitalization sorted portfolios measured over horizons from one day to one quarter. Firm market capitalization is used as a positive proxy for price adjustment delay. We find evidence of significance changes in co-skewness estimates as the return horizon is lengthened. Furthermore, we provide evidence that the horizon effect on co-skewness is strongest for the smallest and largest companies, in keeping with the premise of price adjustment delays. We also propose a "scaling law" for co-skewness, highlighting an inherent horizon dependency for co-skewness. Finally, we examine implications for asset pricing and show that co-skewness is priced at short horizons ranging from one-day to one-quarter.

We conclude with a word of caution to empirical researchers who use higher-order asset pricing or portfolio selection theories in their empirical work. First, the literature documents some return anomalies such as the Size Effect (Banz, 1981) and Liquidity Effect (Amihud, 2002; Amihud and Mendelson, 1986). Investors may pursue stocks with smaller capitalizations or less liquidity while seeking out opportunities for extra returns. Our results suggest that investors in smaller or less liquid firms should pay greater attention to the choice of the investment horizon in analysing the higher-order systematic risk exposure of such securities. Second, recent portfolio selection theory suggests that risk-averse investors are attracted to securities with positive co-skewness estimates. One implication of our findings, however, is that the relative ranks of portfolio co-skewness alter when the sampling horizon is lengthened. In particular, co-skewness is significantly sensitive to the investment horizon, since not only the magnitudes but also the signs of co-skewness change. An asset that is selected based on its positive co-skewness using a particular sampling horizon may have negative co-skewness using another horizon. To sum up, the horizon effect presents potential ambiguity in pricing and the selection of assets for different situations.

References

Aït-Sahalia, Y. and Brandt, M. W. (2001), 'Variable selection for portfolio choice', *The Journal of Finance* **56**(4), 1297–1351.

Amihud, Y. (2002), 'Illiquidity and stock returns: cross-section and time-series effects', *Journal of Financial Markets* **5**(1), 31–56.

Amihud, Y. and Mendelson, H. (1986), 'Asset pricing and the bid-ask spread', *Journal of Financial Economics* **17**(2), 223–249.

Andries, M., Eisenbach, T. M. and Schmalz, M. C. (2015), Asset pricing with horizon-dependent risk aversion, *in* '2015 Meeting Papers', number 1069, Society for Economic Dynamics.

Arrow, K. J. (1964), 'The role of securities in the optimal allocation of risk-bearing', *The Review of Economic Studies* **31**(2), 91–96.

Badrinath, S. G., Kale, J. R. and Noe, T. H. (1995), 'Of shepherds, sheep, and the cross-autocorrelations in equity returns', *Review of Financial Studies* **8**(2), 401–430.

Banz, R. W. (1981), 'The relationship between return and market value of common stocks', *Journal of Financial Economics* **9**(1), 3–18.

Boguth, O., Carlson, M., Fisher, A. J. and Simutin, M. (2016), 'Horizon effects in average returns: The role of slow information diffusion', *The Review of Financial Studies* **29**(8), 2241–2281.

Brailsford, T. J. and Faff, R. W. (1997), 'Testing the conditional capm and the effect of intervaling: a note', *Pacific-Basin Finance Journal* **5**(5), 527–537.

Brennan, M. J., Jegadeesh, N. and Swaminathan, B. (1993), 'Investment analysis and the adjustment of stock prices to common information', *Review of Financial Studies* **6**(4), 799–824.

Brennan, M. and Zhang, Y. (2019), 'Capital asset pricing with a stochastic horizon', *Journal of Financial and Quantitative Analysis,* **Forthcoming**.

Chan, K. and Chen, N.-F. (1988), 'An unconditional asset-pricing test and the role of firm size as an instrumental variable for risk', *The Journal of Finance* **43**(2), 309–325.

Chung, Y. P. and Schill, M. J. (2006), 'Asset pricing when returns are nonnormal: Fama-French factors versus higher-order systematic comoments', *The Journal of Business* **79**(2), 923–940.

Cohen, K. J., Hawawini, G. A., Maier, S. F., Schwartz, R. A. and Whitcomb, D. K. (1980), 'Implications of microstructure theory for empirical research on stock price behavior', *Journal of Finance* **35**(2), 249–257.

Cohen, K. J., Hawawini, G. A., Maier, S. F., Schwartz, R. A. and Whitcomb, D. K. (1983*a*), 'Estimating and adjusting for the intervalling-effect bias in beta', *Management Science* **29**(1), 135–148.

Cohen, K. J., Hawawini, G. A., Maier, S. F., Schwartz, R. A. and Whitcomb, D. K. (1983*b*), 'Friction in the trading process and the estimation of systematic risk', *Journal of Financial Economics* **12**(2), 263–278.

Cohen, L. and Frazzini, A. (2008), 'Economic links and predictable returns', *The Journal of Finance* **63**(4), 1977–2011.

Conrad, J., Dittmar, R. F. and Ghysels, E. (2013), 'Ex ante skewness and expected stock returns', *The Journal of Finance* **68**(1), 85–124.

Corhay, A. (1992), 'The intervalling effect bias in beta: A note', *Journal of Banking and Finance* **16**(1), 61–73.

Dierkes, M., Erner, C. and Zeisberger, S. (2010), 'Investment horizon and the attractiveness of investment strategies: A behavioral approach', *Journal of Banking & Finance* **34**(5), 1032–1046.

Dimson, E. (1979), 'Risk measurement when shares are subject to infrequent trading', *Journal of Financial Economics* **7**(2), 197–226.

Dittmar, R. F. (2002), 'Nonlinear pricing kernels, kurtosis preference, and evidence from the cross section of equity returns', *The Journal of Finance* **57**(1), 369–403.

Doan, M. P., Lin, C.-T. and Chng, M. (2014), ‘Higher moments and beta asymmetry: evidence from australia’, *Accounting & Finance* **54**(3), 779–807.

Fama, E. F. and French, K. R. (1993), ‘Common risk factors in the returns on stocks and bonds’, *Journal of Financial Economics* **33**(1), 3–56.

Fama, E. F. and MacBeth, J. D. (1973), ‘Risk, return, and equilibrium: Empirical tests’, *The Journal of Political Economy* pp. 607–636.

Fang, H. and Lai, T.-Y. (1997), ‘Co-kurtosis and capital asset pricing’, *Financial Review* **32**(2), 293–307.

Gençay, R., Selçuk, F. and Whitcher, B. (2005), ‘Multiscale systematic risk’, *Journal of International Money and Finance* **24**(1), 55–70.

Guidolin, M. and Timmermann, A. (2008), ‘International asset allocation under regime switching, skew, and kurtosis preferences’, *The Review of Financial Studies* **21**(2), 889–935.

Handa, P., Kothari, S. and Wasley, C. (1989), ‘The relation between the return interval and betas: Implications for the size effect’, *Journal of Financial Economics* **23**(1), 79–100.

Harvey, C. R. and Siddique, A. (2000), ‘Conditional skewness in asset pricing tests’, *The Journal of Finance* **55**(3), 1263–1295.

Hawawini, G. (1983), ‘Why beta shifts as the return interval changes’, *Financial analysts journal* **39**(3), 73–77.

Hawawini, G. A. (1980*a*), ‘An analytical examination of the intervaling effect on skewness and other moments’, *Journal of Financial and Quantitative Analysis* **15**(5), 1121–1127.

Hawawini, G. A. (1980*b*), ‘Intertemporal cross-dependence in securities daily returns and the short-run intervaling effect on systematic risk’, *Journal of Financial and Quantitative Analysis* **15**(1), 139–149.

Hur, J. and Singh, V. (2016), ‘Cross-section of expected returns and extreme returns: The role of investor attention and risk preferences’, *Financial Management* .

Jegadeesh, N. and Titman, S. (1995), ‘Overreaction, delayed reaction, and contrarian profits’, *Review of Financial Studies* **8**(4), 973–993.

Jondeau, E. and Rockinger, M. (2012), ‘On the importance of time variability in higher moments for asset allocation’, *Journal of Financial Econometrics* **10**(1), 84–123.

Kamara, A., Korajczyk, R. A., Lou, X. and Sadka, R. (2016), ‘Horizon pricing’, *Journal of Financial and Quantitative Analysis* **51**(6), 1769–1793.

Kimball, M. S. (1990), ‘Precautionary saving in the small and in the large’, *Econometrica* **58**(1), 53–73.

Kostakis, A., Muhammad, K. and Siganos, A. (2012), ‘Higher co-moments and asset pricing on london stock exchange’, *Journal of Banking and Finance* **36**(3), 913 – 922.
URL: *http://www.sciencedirect.com/science/article/pii/S0378426611002792*

Kraus, A. and Litzenberger, R. (1983), ‘On the distributional conditions for a consumption-oriented three moment capm’, *The Journal of Finance* **38**(5), 1381–1391.

Kraus, A. and Litzenberger, R. H. (1976), ‘Skewness preference and the valuation of risk assets’, *The Journal of Finance* **31**(4), 1085–1100.

Lau, H.-S. and Wingender, J. R. (1989), ‘The analytics of the intervaling effect on skewness and kurtosis of stock returns’, *Financial Review* **24**(2), 215–233.

Levhari, D. and Levy, H. (1977), ‘The capital asset pricing model and the investment horizon’, *The Review of Economics and Statistics* **59**(1), 92–104.

Lintner, J. (1965), ‘The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets’, *The Review of Economics and Statistics* pp. 13–37.

Lo, A. W. and MacKinlay, A. C. (1990), ‘When are contrarian profits due to stock market overreaction?’, *Review of Financial Studies* **3**(2), 175–205.

Martellini, L. and Ziemann, V. (2010), ‘Improved estimates of higher-order comoments and implications for portfolio selection’, *Review of Financial Studies* **23**(4), 1467–1502.

Mitton, T. and Vorkink, K. (2007), 'Equilibrium underdiversification and the preference for skewness', *Review of Financial Studies* **20**(4), 1255–1288.

Parhizgari, A., Dandapani, K. and Prakash, A. (1993), 'Arbitrage pricing theory and the investment horizon', *Journal of Business Finance and Accounting* **20**(1), 27–40.
URL: *http://dx.doi.org/10.1111/j.1468-5957.1993.tb00248.x*

Perron, P., Chun, S. and Vodounou, C. (2013), 'Sampling interval and estimated betas: Implications for the presence of transitory components in stock prices', *Journal of Empirical Finance* **20**, 42–62.

Scholes, M. and Williams, J. (1977), 'Estimating betas from nonsynchronous data', *Journal of financial economics* **5**(3), 309–327.

Scott, R. C. and Horvath, P. A. (1980), 'On the direction of preference for moments of higher order than the variance', *The Journal of Finance* **35**(4), 915–919.

Sharpe, W. F. (1964), 'Capital asset prices: A theory of market equilibrium under conditions of risk', *The Journal of Finance* **19**(3), 425–442.

Theobald, M. and Yallup, P. (2004), 'Determining security speed of adjustment coefficients', *Journal of Financial Markets* **7**(1), 75–96.

Wang, J.-N., Yeh, J.-H. and Cheng, N.-P. (2011), 'How accurate is the square-root-of-time rule in scaling tail risk: A global study', *Journal of Banking & Finance* **35**(5), 1158–1169.

Zhang, X. (2006), 'Information uncertainty and stock returns', *The Journal of Finance* **61**(1), 105–137.

Table 1:
Intertemporal Correlation Tests among Market Capitalization formed Portfolios and the Market Index across Horizons, CRSP 1962-2015.

	Panel A: Intertemporal Correlations with Daily Returns							Panel B: Intertemporal Correlations with Monthly Returns						
	Mkt. Cap	Cross-correlations to Market Returns				Autocorrelations		Mkt. Cap	Cross-correlations to Market Returns				Autocorrelations	
	(Million $)	1-lag	5-lag	1-lead	5-lead	1-lag	5-lag	(Million $)	1-lag	5-lag	1-lead	5-lead	1-lag	5-lag
MC1	12.29	0.1579*	0.0290*	0.0217*	-0.0233*	0.1462*	0.0404*	14.83	0.2315*	0.0447	0.0541	0.0403	0.2393*	0.0644
MC2	20.87	0.1221*	0.0101	0.0337*	-0.0235*	0.1138*	0.0087	32.94	0.2305*	0.0126	0.0633	0.0405	0.2306*	0.0354
MC3	29.86	0.1190*	0.0067	0.0301*	-0.0152	0.0995*	0.0118	47.52	0.2245*	0.0186	0.0678	0.0386	0.2110*	0.0243
MC4	40.31	0.1086*	0.0020	0.0378*	-0.0215*	0.0940*	0.004	60.19	0.2113*	0.0059	0.0793*	0.0336	0.2075*	0.014
MC5	52.79	0.1108*	-0.0013	0.0328*	-0.0200*	0.0932*	-0.0023	82.17	0.1852*	0.0228	0.0605	0.0471	0.1558*	0.0219
MC6	67.23	0.1140*	-0.0043	0.0280*	-0.0155	0.0911*	-0.0017	88.95	0.1800*	0.0094	0.0622	0.0427	0.1667*	0.0142
MC7	85.16	0.1160*	0.0026	0.0341*	-0.0166	0.1019*	0.0021	106.03	0.1834*	-0.0012	0.0779*	0.0526	0.1665*	0.022
MC8	105.94	0.1157*	-0.0007	0.0344*	-0.0212*	0.0927*	-0.0064	127.49	0.1845*	0.0049	0.0822*	0.0413	0.1699*	0.0138
MC9	132.07	0.1184*	-0.0082	0.0354*	-0.0120	0.0991*	-0.0059	138.56	0.1675*	0.0113	0.0741	0.0479	0.1550*	0.0234
MC10	164.16	0.1424*	-0.0053	0.0334*	-0.0202*	0.1193*	-0.006	153.33	0.1588*	-0.0020	0.0860*	0.0342	0.1531*	-0.0045
MC11	205.23	0.1281*	-0.0040	0.0368*	-0.0192*	0.1152*	-0.0061	168.82	0.1510*	0.0089	0.0917*	0.0428	0.1501*	0.0149
MC12	258.89	0.1250*	-0.0089	0.0378*	-0.0167	0.1091*	-0.0093	235.44	0.1462*	0.0085	0.1048*	0.0327	0.1445*	0.0028
MC13	329.72	0.1172*	-0.0036	0.0417*	-0.0143	0.1103*	-0.003	289.92	0.1455*	0.0162	0.1073*	0.0430	0.1529*	0.0157
MC14	423.35	0.1092*	-0.0061	0.0376*	-0.0211*	0.0900*	-0.0137	321.26	0.1299*	0.0111	0.0994*	0.0368	0.1323*	0.0061
MC15	563.07	0.1117*	-0.0032	0.0378*	-0.0185*	0.0975*	-0.0076	353.74	0.1236*	0.0112	0.0991*	0.0289	0.1283*	-0.0128
MC16	777.46	0.0939*	-0.0179*	0.0497*	-0.0153	0.0922*	-0.0158	765.05	0.1240*	0.0370	0.1254*	0.0444	0.1599*	0.0287
MC17	1120.91	0.0738*	-0.0145	0.0431*	-0.0130	0.0670*	-0.0116	1130.63	0.1092*	0.0392	0.1122*	0.0525	0.1243*	0.0329
MC18	1790.30	0.0687*	-0.0177*	0.0506*	-0.0137	0.0719*	-0.0146	1991.70	0.0970*	0.0405	0.1028*	0.0361	0.1107*	0.0136
MC19	3568.77	0.0428*	-0.0233*	0.0595*	-0.0120	0.0525*	-0.0180*	3460.58	0.0895*	0.0557	0.0974*	0.0518	0.0957*	0.0461
MC20	42100.00	-0.0217*	-0.0211*	0.0560*	-0.0154	-0.0158	-0.0173*	40560.00	0.0530	0.0877*	0.0725	0.0872*	0.0381	0.1085*

Note: All stocks during 1962 to 2015 on the CRSP daily tape are used. For each MC portfolio and market index, the intertemporal cross-correlations between lead and lag market portfolio and MC portfolios are also calculated. The "1-lag" cross-correlation coefficient indicates the cross-correlation between the MC portfolio returns and 1-lag market returns. * indicates that we reject the null hypothesis at a 5% level.

Table 2:
Co-skewness of Market-Capitalization Portfolios Estimated using Value-weighted Market Index, CRSP 1962-2015.

		Panel A: Mean Co-skewness of Each Portfolio							Panel B: Measures of Estimation Variations		
Portfolio	Mkt. Cap (Million $)	1-Day	2-Day	5-Day	10-Day	20-Day	40-Day	65-Day	Standard Deviation	Repeated Measures ANOVA	Estimation Bias
MC1	12.29	-0.425	-0.420	-0.313	-0.204	-0.067	0.142	0.123	0.221***	395.420***	1.493***
MC2	20.83	-0.420	-0.421	-0.357	-0.184	-0.014	0.173	0.171	0.244***	372.340***	1.698***
MC3	29.77	-0.446	-0.425	-0.343	-0.207	-0.019	0.215	0.277	0.276***	413.170***	1.874***
MC4	40.26	-0.328	-0.365	-0.242	-0.082	0.051	0.225	0.239	0.233***	354.320***	1.583***
MC5	52.51	-0.281	-0.322	-0.181	-0.060	0.021	0.208	0.278	0.215***	369.650***	1.371***
MC6	66.95	-0.275	-0.246	-0.164	-0.123	0.032	0.167	0.290	0.199***	341.430***	1.329***
MC7	84.65	-0.250	-0.200	-0.125	-0.095	0.024	0.184	0.288	0.185***	339.510***	1.191***
MC8	105.41	-0.218	-0.163	-0.119	-0.096	0.047	0.219	0.307	0.186***	338.060***	1.216***
MC9	131.55	-0.177	-0.180	-0.096	-0.106	0.020	0.192	0.272	0.167***	308.000***	1.063***
MC10	162.83	-0.159	-0.157	-0.082	-0.075	0.024	0.152	0.261	0.148***	216.050***	0.934***
MC11	203.47	-0.173	-0.160	-0.049	-0.057	0.053	0.195	0.247	0.152***	219.400***	0.988***
MC12	256.48	-0.186	-0.168	-0.076	-0.127	-0.064	0.047	0.160	0.115***	157.000***	0.637***
MC13	326.53	-0.131	-0.132	-0.017	-0.065	0.042	0.137	0.192	0.117***	161.550***	0.758***
MC14	419.37	-0.171	-0.155	-0.061	-0.110	0.023	0.139	0.166	0.127***	178.550***	0.848***
MC15	557.17	-0.159	-0.134	0.003	-0.092	-0.003	0.081	0.121	0.099***	82.900***	0.591***
MC16	768.68	-0.160	-0.185	-0.119	-0.200	-0.130	-0.092	-0.078	0.042***	49.750***	0.255***
MC17	1108.56	-0.163	-0.188	-0.078	-0.127	-0.117	-0.091	-0.096	0.037***	49.350***	0.213***
MC18	1767.84	-0.135	-0.181	-0.109	-0.187	-0.165	-0.160	-0.206	0.030***	11.070***	0.170***
MC19	3522.59	-0.052	-0.104	-0.099	-0.209	-0.292	-0.290	-0.347	0.107***	27.850***	0.763***
MC20	41700.00	0.234	0.224	0.162	0.214	0.146	0.065	0.061	0.067***	73.200***	0.415***

Note: All stocks during 1962 to 2015 on the CRSP daily tape are used. Co-skewness estimated using value-weighted portfolios returns and market index are illustrated in Panel A. Stocks are sorted into 20 market capitalization equal-size portfolios. The first, $MC1$, contains the smallest 5% of firms and the last, $MC20$, contains the largest 5%. For each MC portfolio and each horizon, we compute co-skewness estimate at each month from August 1967 to December 2015 using the previous five-year periods. This table details the average across all estimated portfolio co-skewness estimates. In Panel B, the standard deviation, repeated measure ANOVA tests and estimation bias estimates (or $EB_{\gamma,i}$ as in Equation 10) are used in detecting the magnitudes of the horizon effect. *** indicate that the null hypothesis is rejected at a 5% level.

Table 3:
Testing the Horizon Effect on Co-skewness for Market Capitalization formed Portfolios in the Whole Period from 1962 to 2015 and Two Sub-periods

	Panel A: Co-skewness Estimated Using Whole Period 1962-2015							Panel B: Co-skewness Estimated Using First Sub-period 1962-1989							Panel C: Co-skewness Estimated Using Second Sub-period 1990-2015						
	1-Day	2-Day	5-Day	10-Day	20-Day	40-Day	65-Day	1-Day	2-Day	5-Day	10-Day	20-Day	40-Day	65-Day	1-Day	2-Day	5-Day	10-Day	20-Day	40-Day	65-Day
MC1	-0.302	-0.050	-0.253	-0.186	-0.052	0.187	0.265	0.236	-0.160	-0.163	-0.216	-0.166	0.074	0.114	-0.518	0.009	-0.393	-0.178	0.007	0.167	0.244
MC2	-0.076	-0.048	-0.122	-0.108	-0.049	0.074	0.114	0.176	-0.179	-0.148	-0.207	-0.183	0.053	0.074	-0.152	0.006	-0.108	-0.063	0.007	0.030	0.057
MC3	0.038	-0.133	-0.165	-0.182	-0.134	0.030	0.101	0.372	-0.147	-0.051	-0.142	-0.161	0.021	0.061	-0.264	-0.093	-0.303	-0.228	-0.088	0.021	0.144
MC4	0.139	-0.078	-0.117	-0.124	-0.075	0.075	0.108	0.439	-0.104	-0.059	-0.107	-0.094	0.063	0.052	-0.172	-0.040	-0.193	-0.148	-0.034	0.081	0.174
MC5	0.103	-0.100	-0.149	-0.190	-0.101	0.005	0.097	0.331	-0.173	-0.147	-0.256	-0.175	-0.039	0.057	-0.140	-0.010	-0.166	-0.126	-0.010	0.026	0.116
MC6	0.173	-0.053	0.020	-0.036	-0.043	0.077	0.162	0.330	-0.051	0.143	0.037	-0.042	0.047	0.089	-0.199	-0.056	-0.221	-0.194	-0.041	0.122	0.267
MC7	0.038	-0.070	-0.171	-0.190	-0.071	0.106	0.211	0.234	-0.150	-0.191	-0.227	-0.150	-0.101	-0.068	-0.238	0.010	-0.184	-0.172	0.006	0.279	0.403
MC8	0.107	-0.080	-0.079	-0.131	-0.086	0.092	0.235	0.312	-0.178	-0.060	-0.186	-0.179	-0.024	0.079	-0.213	0.045	-0.116	-0.085	0.032	0.229	0.415
MC9	0.120	-0.002	-0.027	-0.101	-0.007	0.068	0.093	0.149	-0.464	-0.421	-0.614	-0.458	-0.162	-0.044	-0.063	0.208	0.092	0.087	0.204	0.165	0.132
MC10	0.145	-0.041	-0.108	-0.137	-0.048	0.199	0.295	0.348	-0.183	-0.182	-0.268	-0.178	0.061	0.045	-0.187	0.124	-0.058	-0.050	0.109	0.388	0.585
MC11	0.190	-0.010	-0.092	-0.065	-0.007	0.111	0.139	0.153	-0.071	-0.104	-0.129	-0.061	0.134	0.193	-0.089	-0.048	-0.163	-0.138	-0.046	-0.028	0.000
MC12	0.026	-0.129	-0.131	-0.178	-0.132	-0.019	0.037	0.181	-0.163	-0.153	-0.182	-0.164	-0.038	-0.013	-0.213	-0.070	-0.116	-0.173	-0.069	0.052	0.175
MC13	-0.046	-0.058	-0.109	-0.119	-0.055	0.010	0.023	-0.070	0.004	-0.118	-0.090	0.016	0.116	0.156	-0.288	-0.216	-0.172	-0.258	-0.219	-0.239	-0.205
MC14	-0.138	-0.081	-0.110	-0.106	-0.073	-0.016	0.022	-0.079	-0.049	0.024	0.007	-0.040	0.015	0.008	-0.129	-0.046	-0.122	-0.120	-0.040	0.037	0.138
MC15	-0.122	-0.183	-0.055	-0.133	-0.181	-0.167	-0.093	-0.097	-0.095	-0.072	-0.093	-0.085	0.052	0.124	-0.313	-0.228	-0.055	-0.181	-0.234	-0.277	-0.169
MC16	-0.046	-0.161	-0.108	-0.152	-0.158	-0.106	-0.177	0.027	-0.179	-0.169	-0.229	-0.171	0.037	0.186	-0.154	-0.140	-0.080	-0.103	-0.142	-0.169	-0.319
MC17	-0.115	-0.139	-0.048	-0.119	-0.140	-0.144	-0.111	-0.302	0.079	0.049	0.017	0.079	0.134	0.237	-0.151	-0.325	-0.144	-0.270	-0.319	-0.325	-0.298
MC18	-0.144	-0.270	-0.155	-0.215	-0.261	-0.326	-0.318	-0.232	-0.078	-0.140	-0.097	-0.076	-0.051	-0.044	-0.145	-0.460	-0.175	-0.344	-0.448	-0.568	-0.513
MC19	-0.019	0.031	0.153	0.155	0.033	-0.093	-0.138	0.024	0.458	0.376	0.507	0.452	0.264	0.228	0.048	-0.584	-0.105	-0.330	-0.570	-0.626	-0.577
MC20	-0.036	0.352	0.273	0.425	0.354	0.257	0.215	-0.159	0.290	0.340	0.489	0.289	0.131	0.058	0.246	0.419	0.217	0.355	0.409	0.379	0.323

Note: All stocks from 1962 to 2015 on the CRSP daily tape are used. Co-skewness estimated using value-weighted portfolios returns and market index are illustrated in Panel A. Stocks are sorted into 20 market capitalization equal-size portfolios. The first, $MC1$, contains the smallest 5% of firms and the last, $MC20$, contains the largest 5%. For each MC portfolio and each horizon, we compute single co-skewness for the whole period from 1962 to 2015 and two sub-periods (1962-1989 and 1990-2015).

Table 4:
Modelled Co-skewness Estimates Using Value-weighted Market Index, CRSP 1962-2015.

	Section A: Simulated Results for Smallest-Size Portfolios (MC1)							Section B: Simulated Results for Largest-Size Portfolio (MC20)						
	Panel A: Modeling Non-overlapping Co-skewness with Corrections in Different Degrees of Orders													
Orders	1-Day	2-Day	5-Day	10-Day	20-Day	40-Day	65-Day	1-Day	2-Day	5-Day	10-Day	20-Day	40-Day	65-Day
0	-0.3020	-0.2134	-0.1347	-0.0953	-0.0670	-0.0474	-0.0373	-0.0361	-0.0255	-0.0162	-0.0113	-0.0077	-0.0051	-0.0044
1		-0.0501	-0.1517	-0.0913	-0.0576	-0.0384	-0.0300		0.3515	0.0262	-0.0108	-0.0178	-0.0164	-0.0136
2			-0.2396	-0.1812	-0.1301	-0.0925	-0.0731			0.2189	0.1955	0.1527	0.1131	0.0902
4			-0.2528	-0.2172	-0.1554	-0.1087	-0.0846			0.2728	0.4467	0.4148	0.3308	0.2665
9				-0.1857	-0.0358	0.0135	0.0237				0.4245	0.3015	0.2073	0.1570
19					-0.0521	0.0007	0.0135					0.3535	0.2516	0.1887
39						0.1872	0.2929						0.2563	0.2247
64							0.2670							0.2134
Estimated	-0.3020	-0.0502	-0.2529	-0.1861	-0.0524	0.1867	0.2652	-0.0361	0.3517	0.2729	0.4246	0.3537	0.2570	0.2150
Modeled	-0.3020	-0.0501	-0.2528	-0.1857	-0.0521	0.1872	0.2670	-0.0361	0.3525	0.2728	0.4245	0.3535	0.2563	0.2134
$1/\sqrt{T}$	-0.3020	-0.2136	-0.1351	-0.096	-0.0675	-0.0478	-0.0390	-0.0361	-0.0255	-0.0161	-0.0114	-0.0081	-0.0057	-0.0047

Panel B: Modeling Non-overlapping Co-skewness with Corrections Using Different Increments

Orders	Increments	2-Day	5-Day	10-Day	20-Day	40-Day	65-Day	Increments	2-Day	5-Day	10-Day	20-Day	40-Day	65-Day
1	-0.03	-0.4759	-0.2373	-0.1488	-0.0959	-0.0642	-0.0496	-0.03	0.1573	-0.0839	-0.1050	-0.0901	-0.0679	-0.0545
	-0.02	-0.4312	-0.2061	-0.1277	-0.0818	-0.0547	-0.0424	-0.02	0.1846	-0.0434	-0.0702	-0.0633	-0.0488	-0.0393
	-0.01	-0.3895	-0.1777	-0.1086	-0.0691	-0.0461	-0.0359	-0.01	0.2096	-0.0069	-0.0390	-0.0393	-0.0318	-0.0258
	+0.01	-0.3146	-0.1280	-0.0756	-0.0473	-0.0315	-0.0247	+0.01	0.2536	0.0561	0.0145	0.0016	-0.0027	-0.0026
	+0.02	-0.2810	-0.1063	-0.0612	-0.0378	-0.0252	-0.0199	+0.02	0.2729	0.0832	0.0373	0.0190	0.0097	0.0072
	+0.03	-0.2495	-0.0864	-0.0482	-0.0293	-0.0194	-0.0155	+0.03	0.2907	0.1077	0.0579	0.0348	0.0209	0.0161
4	-0.03		-0.4080	-0.2646	-0.1186	-0.0507	-0.0319	-0.03		-0.1988	-0.4056	-0.4615	-0.3691	-0.3090
	-0.02		-0.3471	-0.2469	-0.1374	-0.0788	-0.0575	-0.02		-0.0061	-0.0247	-0.0520	-0.0346	-0.0322
	-0.01		-0.2960	-0.2312	-0.1488	-0.0970	-0.0740	-0.01		0.1483	0.2484	0.2246	0.1844	0.1474
	+0.01		-0.2159	-0.2047	-0.1587	-0.1161	-0.0912	+0.01		0.3737	0.5919	0.5472	0.4298	0.3465
	+0.02		-0.1844	-0.1936	-0.1600	-0.1206	-0.0953	+0.02		0.4556	0.6988	0.6397	0.4970	0.4004
	+0.03		-0.1571	-0.1835	-0.1598	-0.1232	-0.0975	+0.03		0.5222	0.7773	0.7041	0.5422	0.4365

Note: The value-weighted returns of all stocks during 1962 to 2015 on CRSP daily tape are used as the market index. Single-period estimation from 1962 to 2015 is used. Modelled results for $MC1$ and $MC20$ are illustrated as representative. Panel A details the modelled co-skewness for differing orders of intertemporal covariance. The first row lists the simulated results if there is no serial-relationships (0 order), the second row with 1-order intertemporal relationships, etc. The row title "Estimated" corresponds to the co-skewness estimated using data returns with the relevant horzions. The row named "Modeled" shows the modelled co-skewness taking into account all relevant intertemporal relationships. The row titled $1/\sqrt{N}$ highlights the expected co-skewness estimate in the absence of intertemporal relationships, in keeping with Proposition 2. Panel B further lists modelled results with fixed 1 or 4-order intertemporal relationships but with increments for serial- and cross-serial correlation of between -0.03 and 0.03.

Table 5:
Horizon Pricing Using Co-skewness (SKD) in 3M-CAPM.

Panel A: Using CRSP Daily Tape			Panel B: Using CRSP Monthly Tape		
Horizon	λ^{MKT}	λ^{γ}	Horizon	λ^{MKT}	λ^{γ}
1-Day	0.0314	-0.0263**	1-Month	0.3179	-0.1588**
	(0.0318)	(0.0056)		(0.1965)	(0.0513)
2-Day	0.0083	-0.0923**	2-Month	0.3243	-0.1806**
	(0.0279)	(0.0221)		(0.1991)	(0.0485)
5-Day	0.1486	-0.2139**	3-Month	0.2178*	-0.1841*
	(0.0759)	(0.0836)		(0.1185)	(0.1085)
10-Day	0.1784	-0.5189**	4-Month	0.2378**	-0.0178
	(0.1735)	(0.2651)		(0.0456)	(0.0131)
20-Day	0.2011*	-0.2066**	5-Month	0.2087**	-0.0294
	(0.1372)	(0.0363)		(0.0470)	(0.0280)
40-Day	0.3649*	-0.2906**	6-Month	0.1475*	-0.0300
	(0.2044)	(0.0314)		(0.0779)	(0.0405)
65-Day	0.1891**	0.0206	12-Month	0.3535**	-0.0258
	(0.0355)	(0.5010)		(0.0266)	(0.1372)

Note: This table reports the results of the second stage of the Fama-MacBeth regressions. Given the co-skewness at each month t of asset i in horizon T ($\gamma_{it,T}$) estimated using Equation 5, the first-stage regression involves estimating the factor loading for $\gamma_{it,T}$ through the 3M-CAPM. We weigh the securities using $1/\sigma(\hat{\varepsilon})_{it,T}$, where $\sigma(\hat{\varepsilon})_{it,T}$ is the standard deviation of the residuals from the beta estimation. We also use overlapping returns and adjusting factor loadings ($\beta_{t,T}^{Mkt}$ and $\beta_{it,T}^{\gamma_{it,T}}$) standardized to a mean of zero and a standard deviation of one in each month. Panel A in this table reports coefficients of cross-sectional regressions that are generated as average values across all test months. T-statistics (in brackets) of cross-sectional regression coefficients are calculated using Newey-West standard errors with five lags to reduce the impact of overlapping returns and also corrected by Shanken standard errors for Error-in-Variable issue. Shares listed on the CRSP Daily Tape are considered. Panel B repeats the tests by using the CRSP Monthly Tape. ** and * stand for 95% and 90% significance level.

APPENDIX A:

Proof of Proposition 1: The Horizon Effect on Co-skewness (Equation 6)

Proof. Given that $\varepsilon_{iT,t}$ in Equation 6 is the residual previously extracted from the CAPM regression and $\varepsilon_{mT,t}$ is the deviation of the excess market return in month t from the average value over the corresponding window, it follows:

$$\begin{cases} \overline{\varepsilon}_{iT,t} = 0 \;\; using \;\; OLS \;\; regression \\ \overline{\varepsilon}_{mT,t} = \overline{R_{mT,t} - \overline{R}_{mT,t}} = 0 \end{cases}$$

Thus, co-skewness as in Equation 5 can be re-written as:

$$\gamma_{iT,t} = \frac{E[\varepsilon_{iT,t}\varepsilon^2_{mT,t}]}{\sqrt{E[\varepsilon^2_{iT,t}]E[\varepsilon^2_{mT,t}]}} = \frac{E[(\varepsilon_{iT,t} - \overline{\varepsilon}_{iT,t})(\varepsilon^2_{mT,t} - \overline{\varepsilon}^2_{mT,t})]}{\sqrt{E[(\varepsilon^2_{iT,t} - \overline{\varepsilon}^2_{iT,t})]E[(\varepsilon^2_{mT,t} - \overline{\varepsilon}^2_{mT,t})]}}. \tag{A.1}$$

Given that $E[(\varepsilon_{iT,t} - \overline{\varepsilon}_{iT,t})] = \overline{\varepsilon}_{iT,t} - \overline{\varepsilon}_{iT,t} = 0$, the numerator can be derived as

$$\begin{aligned} E[(\varepsilon_{iT,t} - \overline{\varepsilon}_{iT,t})(\varepsilon^2_{mT,t} - \overline{\varepsilon}^2_{mT,t})] &= E[(\varepsilon_{iT,t} - \overline{\varepsilon}_{iT,t})(\varepsilon^2_{mT,t} - \overline{\varepsilon}^2_{mT,t} + \overline{\varepsilon^2_{mT,t}} - \overline{\varepsilon^2_{mT,t}})] \\ &= E[(\varepsilon_{iT,t} - \overline{\varepsilon}_{iT,t})(\varepsilon^2_{mT,t} - \overline{\varepsilon^2_{mT,t}})] + E[(\varepsilon_{iT,t} - \overline{\varepsilon}_{iT,t})(\overline{\varepsilon^2_{mT,t}} - \overline{\varepsilon}^2_{mT,t})] \\ &= E[(\varepsilon_{iT,t} - \overline{\varepsilon}_{iT,t})(\varepsilon^2_{mT,t} - \overline{\varepsilon^2_{mT,t}})] + E[\varepsilon_{iT,t} - \overline{\varepsilon}_{iT,t}] \cdot [\overline{\varepsilon^2_{mT,t}} - \overline{\varepsilon}^2_{mT,t}] \\ &= E[(\varepsilon_{iT,t} - \overline{\varepsilon}_{iT,t})(\varepsilon^2_{mT,t} - \overline{\varepsilon^2_{mT,t}})] + 0 \cdot [\overline{\varepsilon^2_{mT,t}} - \overline{\varepsilon}^2_{mT,t}] \\ &= cov(\varepsilon_{iT,t}, \varepsilon^2_{mT,t}). \end{aligned} \tag{A.2}$$

Similarly, $\sqrt{E[(\varepsilon^2_{iT,t} - \overline{\varepsilon}^2_{iT,t})]} = \sigma(\varepsilon_{iT,t})$ and $E[(\varepsilon^2_{mT,t} - \overline{\varepsilon}^2_{mT,t})] = var(\varepsilon_{mT,t})$.

Therefore, with $\varepsilon_{iT,t} = R_{iT,t} - \hat{\beta}_{iT,t}R_{mT,t} - \hat{\alpha}_{iT,t}$ and $\varepsilon_{mT,t} = R_{mT,t} - \overline{R}_{mT,t}$, co-skewness can be further refined as follows:

$$\gamma_{iT,t} = \frac{cov(\varepsilon_{iT,t}, \varepsilon^2_{mT,t})}{\sigma(\varepsilon_{iT,t}) \cdot var(\varepsilon_{mT,t})} = \frac{cov(R_{iT,t} - \hat{\beta}_{iT,t}R_{mT,t} - \hat{\alpha}_{iT,t}, R^2_{mT,t} - 2\overline{R}_{mT,t}R_{mT,t} + \overline{R}^2_{mT,t})}{\sqrt{var(R_{iT,t} - \hat{\beta}_{iT,t}R_{mT,t} - \hat{\alpha}_{iT,t}) \cdot var(R_{mT,t} - \overline{R}_{mT,t})}}. \tag{A.3}$$

We refer the returns of shortest horizon (1-Day returns in this study) as unit returns. Using logarithmic returns, it follows that $R_{iT,t} = \sum_{j=0}^{T-1} R_{i1,(t\text{-}j)}$, i.e., any return $R_{iT,t}$ can be expressed as the sum of unit returns. This also applies to market returns, $R_{mT,t}$. That is, $R_{mT,t} = \sum_{j=0}^{T-1} R_{m1,(t\text{-}j)}$. Accordingly, $R^2_{mT,t} = (\sum_{j=0}^{T-1} R_{m1,(t\text{-}j)})^2 = \sum_{k=0}^{T-1}\sum_{l=0}^{T-1} R_{m1,(t\text{-}k)} \cdot R_{m1,(t\text{-}l)}$.

Given that the covariance between a constant and a variable is zero and

$$
\begin{aligned}
&cov(R_{iT,t}, R_{mT,t}) - \hat{\beta}_{iT,t} cov(R_{mT,t}, R_{mT,t}) \\
=&cov(R_{iT,t}, R_{mT,t}) - \frac{cov(R_{iT,t}, R_{mT,t})}{cov(R_{mT,t}, R_{mT,t})} \cdot cov(R_{mT,t}, R_{mT,t}) = 0,
\end{aligned}
$$

we can now rewrite the numerator of Equation A.3 as:

$$
\begin{aligned}
&cov(R_{iT,t} - \hat{\beta}_{iT,t} R_{mT,t} - \hat{\alpha}_{iT,t}, R^2_{mT,t} - 2\overline{R}_{mT,t} R_{mT,t} + \overline{R}^2_{mT,t}) \\
&= cov(R_{iT,t} - \hat{\beta}_{iT,t} R_{mT,t}, R^2_{mT,t} - 2\overline{R}_{mT,t} R_{mT,t}) \\
&= cov(R_{iT,t}, R^2_{mT,t}) - \hat{\beta}_{iT,t} cov(R_{mT,t}, R^2_{mT,t}) \\
&\qquad - 2 \cdot \overline{R}_{mT,t} [cov(R_{iT,t}, R_{mT,t}) - \hat{\beta}_{iT,t} cov(R_{mT,t}, R_{mT,t})] \\
&= cov(R_{iT,t}, R^2_{mT,t}) - \hat{\beta}_{iT,t} cov(R_{mT,t}, R^2_{mT,t}) \\
&= \sum_{j=0}^{T-1}\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{i1,t\text{-}j}, R_{m1,(t\text{-}k)} \cdot R_{m1,(t\text{-}l)}) \\
&\qquad - \hat{\beta}_{iT,t} \sum_{j=0}^{T-1}\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{m1,t\text{-}j}, R_{m1,(t\text{-}k)} \cdot R_{m1,(t\text{-}l)}).
\end{aligned}
\tag{A.4}
$$

where $\hat{\beta}_{iT,t}$ can also be written as a function of unit returns,

$$
\hat{\beta}_{iT,t} = \frac{cov(R_{iT,t}, R_{mT,t})}{var(R_{mT,t})} = \frac{\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{i1,(t\text{-}k)}, R_{m1,(t\text{-}u)})}{\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{m1,(t\text{-}k)}, R_{m1,(t\text{-}u)})}
$$

Similarly, for the two terms in the denominator, we have:

$$\begin{aligned}
&\sqrt{var(R_{iT,t}-\hat{\beta}_{iT,t}R_{mT,t}-\hat{\alpha}_{iT,t})}\\
&=\sqrt{cov(R_{iT,t}-\hat{\beta}_{iT,t}R_{mT,t}-\hat{\alpha}_{iT,t},R_{iT,t}-\hat{\beta}_{iT,t}R_{mT,t}-\hat{\alpha}_{iT,t})}\\
&=\sqrt{cov(R_{iT,t},R_{iT,t})-\hat{\beta}_{iT,t}cov(R_{iT,t},R_{mT,t})-\hat{\beta}_{iT,t}[cov(R_{iT,t},R_{mT,t})-\hat{\beta}_{iT,t}cov(R_{mT,t},R_{mT,t})]}\\
&=\sqrt{\sum_{k=0}^{T-1}\sum_{u=0}^{T-1}[cov(R_{i1,(t\text{-}k)},R_{i1,(t\text{-}u)})-\hat{\beta}_{iT,t}cov(R_{i1,(t\text{-}k)},R_{m1,(t\text{-}u)})]}. \qquad \text{(A.5)}
\end{aligned}$$

and

$$var(R_{mT,t}-\overline{R}_{mT,t})=cov(R_{mT,t},R_{mT,t})=\sum_{k=0}^{T-1}\sum_{u=0}^{T-1}cov(R_{m1,(t\text{-}k)},R_{m1,(t\text{-}u)}). \qquad \text{(A.6)}$$

Therefore, co-skewness can be decomposed into sums of intertemporal cross-covariance formed using unit-returns, by using A.4 in the numerator, and A.5 and A.6 in the denominator. That is, co-skewness measured with longer-horizon returns is a function of the length of the horizon T, the intertemporal auto-covariance of asset unit returns and of market unit returns, the intertemporal cross-covariance between returns of the asset and the second-order of market returns, and the estimated betas at horizon T:

$$\begin{aligned}
&\gamma_{iT,t}=\frac{cov(\varepsilon_{iT,t},\varepsilon^2_{mT,t})}{\sigma(\varepsilon_{iT,t})\cdot var(\varepsilon_{mT,t})}\\
&=\frac{\sum_{j=0}^{T-1}\sum_{k=0}^{T-1}\sum_{l=0}^{T-1}cov(R_{i1,t\text{-}j},R_{m1,(t\text{-}k)}\cdot R_{m1,(t\text{-}l)})-\hat{\beta}_{iT,t}\sum_{j=0}^{T-1}\sum_{k=0}^{T-1}\sum_{l=0}^{T-1}cov(R_{m1,t\text{-}j},R_{m1,(t\text{-}k)}\cdot R_{m1,(t\text{-}l)})}{\sqrt{\sum_{k=0}^{T-1}\sum_{u=0}^{T-1}[cov(R_{i1,(t\text{-}k)},R_{i1,(t\text{-}u)})-\hat{\beta}_{iT,t}cov(R_{i1,(t\text{-}k)},R_{m1,(t\text{-}u)})]}\cdot\sum_{k=0}^{T-1}\sum_{u=0}^{T-1}cov(R_{m1,(t\text{-}k)},R_{m1,(t\text{-}u)})}.
\end{aligned}$$

(A.7)

□

APPENDIX B:

Proof of Proposition 2: the "Scaling Law" (Equation 9)

Proof. There are four sums of covariance terms and a horizon-dependent beta estimate in the decomposition of co-skewness into terms relating to unit returns, according to Equation A.7:

1. the sum of auto-covariances of asset returns,

$$\sum_{k=0}^{T-1}\sum_{u=0}^{T-1} cov(R_{i1,(t\text{-}k)}, R_{i1,(t\text{-}u)}),$$

   which includes T contemporaneous auto-covariances (corresponding to $k = u$) and $T(T-1)$ intertemporal auto-covariances (when $k \neq u$) of asset returns.

2. the sum of auto-covariances of market returns,

$$\sum_{k=0}^{T-1}\sum_{u=0}^{T-1} cov(R_{m1,(t\text{-}k)}, R_{m1,(t\text{-}u)})$$

   which includes T contemporaneous auto-covariances (corresponding to $k = u$) and $T(T-1)$ intertemporal auto-covariances of market returns.

3. the sum of cross-covariances between returns of asset i and a second-order term using market returns,

$$\sum_{k=0}^{T-1}\sum_{u=1}^{T-1}\sum_{j=0}^{T-1} cov(R_{i1,t\text{-}k}, R_{m1,(t\text{-}u)} \cdot R_{m1,(t\text{-}j)})$$

   which includes T contemporaneous cross-covariances (corresponding to $k = u = j$), and $T * T(T-1)$ intertemporal cross-covariances.

4. the sum of cross-covariances between market returns and a second-order term using market returns,

$$\sum_{k=0}^{T-1}\sum_{u=1}^{T-1}\sum_{j=0}^{T-1} cov(R_{m1,t\text{-}k}, R_{m1,(t\text{-}u)} \cdot R_{m1,(t\text{-}j)})$$

   which includes T contemporaneous cross-covariances (corresponding to $k = u = j$), and $T * T(T-1)$ intertemporal cross-covariances.

5. the beta estimate

$$\hat{\beta}_{iT,t} = \frac{cov(R_{iT,t}, R_{mT,t})}{var(R_{mT,t})} = \frac{\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{i1,(t\text{-}k)}, R_{m1,(t\text{-}u)})}{\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{m1,(t\text{-}k)}, R_{m1,(t\text{-}u)})}$$

Given stationary returns and for unconditional estimates, the four contemporaneous covariance terms equal to $cov(R_{i1,t}, R_{i1,t})$, $cov(R_{m1,t}, R_{m1,t})$, $cov(R_{i1,t}, R^2_{m1,t})$ and $cov(R_{m1,t}, R^2_{m1,t})$, respectively and there are T of each one of the these.

In the "perfect market" proposed by Hawawini (1980*b*), securities display no intertemporal cross- and auto-correlations. In this case, beta is constant over all horizons, as documented by Hawawini (1980*b*),

$$\beta_{iT,t} = \frac{cov(R_{iT,t}, R_{mT,t})}{var(R_{mT,t})} = \beta_{i1,t} \cdot \frac{T + \sum_{s=1}^{T-1}(T-s)\frac{\rho^{+s}_{im,t} + \rho^{-s}_{im,t}}{\rho_{im,t}}}{T + 2\sum_{s=1}^{T-1}(T-s)\rho^{s}_{mm,t}} = \beta_{i1,t}. \tag{B.1}$$

Similarly, in the absence of the intertemporal cross- and auto-covariance terms described above, Equation A.7 can be rewritten as:

$$\begin{aligned} \gamma_{iT,t} &= \frac{cov(\varepsilon_{iT,t}, \varepsilon^2_{mT,t})}{\sigma(\varepsilon_{iT,t}) \cdot var(\varepsilon_{mT,t})} \\ &= \frac{T \cdot cov(R_{i1,t}, R_{m1,t} \cdot R_{m1,t}) - \hat{\beta}_{i1,t} T \cdot cov(R_{m1,t}, R_{m1,t} \cdot R_{m1,t})}{\sqrt{T \cdot cov(R_{i1,t}, R_{i1,t}) - \hat{\beta}_{i1,t} cov(R_{i1,t}, R_{m1,t})} \cdot T \cdot cov(R_{m1,t}, R_{m1,t})} \\ &= \frac{1}{\sqrt{T}} \cdot \gamma_{i1,t}. \end{aligned} \tag{B.2}$$

That is, in contrast to the earlier findings of the horizon effect on betas in the literature, in a market where intertemporal cross- and auto-covariance terms are equal to zero, coskewness is still horizon dependent. In this extreme case, with the lengthening of the measurement horizon, unconditional coskewness converges to zero, highlighting a "scaling law" of co-skewness.

□

APPENDIX C:

Robustness Check on Co-skewness Estimated Using a 15-year Window

We use non-overlapping returns over a 5-year period to estimate parameters for our main tests, Table 2. There is no agreement in the literature regarding the choice of the length of the estimation window and of using non-overlapping returns versus overlapping returns. Using a 5-year estimation window is a common choice in the literature because of concerns about parameter stationarity.

There is an active debate on the impact of the length of the estimation window on the horizon effect. Longer estimation periods are also widely used in the research on the horizon effect or when low frequency data is tested. Perron et al. (2013) and Handa et al. (1989) use returns over a 15-year period to estimate betas and in analysing the sensitivity of betas to the investment horizon. Chan and Chen (1988) suggests that betas estimated using an estimation period as long as a 30-year window are greater than those estimated from returns over 5-year periods.

Thus, in this section, we check whether the horizon effect on co-skewness is robust to the choice of estimation window. We use a 15-year window, as suggested by Perron et al. (2013) and Handa et al. (1989). This allows more observations in testing our estimates across longer return horizons, but not at the expense of the accuracy of estimation. Empirical results are illustrated in Table C.1.

TABLE C.1 ABOUT HERE

Overall, our results are found to be consistent. Considering market capitalization as a proxy for price delay, the magnitude of the horizon effect for smaller securities is inversely related to company size, and vice versa. Co-skewness estimates for firms in portfolios $MC18$ are still least affected (with the smallest estimation variation). Table 2 and Table C.1 are not strictly comparable because the estimation period are different. Both tables suggest, however, that the horizon effect is significant across a large range of investment horizons and highly related to friction in the trading process, highlighted by the dependence upon firm market capitalization.

APPENDIX D:

Robustness Check on Co-kurtosis

In this paper, we focus on testing the sensitivity of co-skewness to the investment horizon. It is also interesting to test the horizon effect on higher-than-3^{rd} order co-moments. Building on the 2-moment and 3-moment asset pricing models, Fang and Lai (1997) and Dittmar (2002) suggest a four-moment capital asset pricing model by including co-kurtosis. Co-kurtosis is defined as the component of an asset's kurtosis that is related to the market portfolio's kurtosis. Thus, given our findings for co-skewness and previous work on betas, it is reasonable to expect that co-kurtosis may also be sensitive to the horizon.

Similar to Kostakis et al. (2012), we estimate co-kurtosis as an extension to the co-skewness measure of Harvey and Siddique (2000):

$$\delta_{iT,t} = \frac{E[\varepsilon_{iT,t}\varepsilon^3_{mT,t}]}{\sqrt{E[\varepsilon^2_{iT,t}]}E[\varepsilon^3_{mT,t}]} \tag{D.1}$$

where $\varepsilon_{iT,t} = [R_{iT,t} - R_{fT,t}] - [\alpha_{iT,t} + \beta_{iT,t}(R_{mT,t} - R_{fT,t})]$ is the residual previously extracted in Equation 4 from a CAPM regression and $\varepsilon_{mT,t}$ is the deviation of the excess market return in month t from the average over the corresponding window.

Empirical results are shown in Table D.1. Panel A shows the estimated co-kurtosis for MC portfolios, which indicates the existence of an horizon effect on co-kurtosis. Moreover, the relationship between the horizon effect and firm size holds when co-kurtosis is tested. All three tests in Panel B confirm that the magnitudes of the horizon effect are also significant. The size related U-shape pattern, as shown for co-skewness estimates, is not obvious, although generally portfolio $MC18$ is least affected by the choice of investment horizon.

TABLE D.1 ABOUT HERE

These findings highlight the importance and potential implications of our detailed investigation on co-skewness in the main body above. The horizon effect on co-skewness is not just a replication or sequel to those on alpha and beta. Instead, the horizon effect is a pervasive concern with implications throughout many aspects of asset pricing.

Table C.1:
Appendix: Testing the Horizon Effect on Co-skewness of Portfolios Using a 15-year Estimation Window.

Portfolio	Mkt. Cap (Million $)	Panel A: Mean Co-skewness of Each Portfolio							Panel B: Measures of Estimation Variations		
		1-Day	2-Day	5-Day	10-Day	20-Day	40-Day	65-Day	Standard Deviation	Repeated Measures ANOVA	Estimation Bias
MC1	9.0	-0.362	-0.421	-0.322	-0.257	-0.148	0.091	0.064	0.189***	478.27***	1.219***
MC2	14.3	-0.338	-0.427	-0.349	-0.277	-0.082	0.138	0.107	0.213***	444.49***	1.471***
MC3	19.7	-0.437	-0.534	-0.387	-0.321	-0.130	0.113	0.122	0.245***	466.73***	1.654***
MC4	25.9	-0.338	-0.464	-0.321	-0.215	-0.053	0.173	0.175	0.235***	440.24***	1.580***
MC5	32.9	-0.297	-0.454	-0.305	-0.222	-0.093	0.147	0.187	0.223***	419.18***	1.425***
MC6	41.1	-0.280	-0.405	-0.308	-0.254	-0.047	0.170	0.243	0.233***	433.10***	1.565***
MC7	50.9	-0.269	-0.401	-0.310	-0.284	-0.107	0.155	0.220	0.224***	436.21***	1.424***
MC8	62.8	-0.200	-0.335	-0.259	-0.231	-0.047	0.209	0.235	0.213***	364.33***	1.375***
MC9	78.0	-0.120	-0.315	-0.227	-0.194	-0.014	0.201	0.209	0.191***	327.86***	1.237***
MC10	97.1	-0.065	-0.299	-0.228	-0.246	-0.104	0.090	0.148	0.159***	175.21***	0.946***
MC11	120.5	-0.037	-0.244	-0.159	-0.146	-0.015	0.179	0.179	0.153***	169.77***	0.913***
MC12	151.2	-0.041	-0.279	-0.194	-0.231	-0.119	0.085	0.136	0.147***	164.17***	0.883***
MC13	192.9	0.000	-0.245	-0.133	-0.161	-0.038	0.119	0.143	0.134***	131.87***	0.801***
MC14	248.4	-0.077	-0.287	-0.193	-0.228	-0.062	0.108	0.105	0.145***	171.00***	0.873***
MC15	330.1	0.019	-0.204	-0.120	-0.192	-0.076	0.033	0.056	0.100***	74.54***	0.623***
MC16	458.2	0.073	-0.194	-0.125	-0.224	-0.140	-0.056	-0.055	0.093***	73.67***	0.533***
MC17	666.2	0.036	-0.199	-0.081	-0.114	-0.092	-0.009	-0.014	0.073***	59.89***	0.428***
MC18	1054.8	-0.004	-0.182	-0.092	-0.189	-0.153	-0.101	-0.182	0.063***	35.67***	0.356***
MC19	1938.1	-0.019	-0.121	-0.035	-0.162	-0.227	-0.279	-0.327	0.109***	52.66***	0.722***
MC20	24500.0	0.316	0.284	0.223	0.189	0.173	0.102	0.095	0.078***	86.22***	0.470***

Note: This table illustrates the robustness of results for co-skewness estimates of securities estimated using a 15-year estimation window. The value-weighted returns of these stocks are used as the market index. Value-weighted returns are calculated for 20 market capitalization sorted portfolios. The first, $MC1$, contains the smallest 5% of firms and the last, $MC20$, contains the largest 5%. For each MC portfolio and each horizon, we compute co-skewness at each month from August 1977 to December 2015 using previous 15-year periods. This table illustrates the averages of 462 estimated portfolio co-skewness estimates. In Panel B, the standard deviation, repeated measure ANOVA tests and estimation bias estimates (or $EB_{\gamma,i}$ as in Equation 10) are used in detecting the magnitudes of the horizon effect. *** indicate the rejection of the null hypothesis at a 1% level.

Table D.1:
Co-Kurtosis of Market-Capitalization Portfolios Estimated using Value-weighted Market Index, CRSP 1962-2015.

Portfolio	Mkt. Cap (Million $)	Panel A: Mean Co-skewness of Each Portfolio							Panel B: Measures of Estimation Variations		
		1-Day	2-Day	5-Day	10-Day	20-Day	40-Day	65-Day	Standard Deviation	Repeated Measures ANOVA	Estimation Bias
MC1	12.29	-0.975	-0.492	-1.079	-0.458	-0.181	-0.339	-0.603	0.337***	518.26***	4.630***
MC2	20.83	-1.218	-0.657	-1.233	-0.286	0.014	-0.358	-0.506	0.532***	519.45***	7.349***
MC3	29.77	0.141	-0.123	-1.090	-0.302	0.054	-0.039	-0.472	0.499***	568.25***	4.730***
MC4	40.26	-0.640	-0.079	-0.748	-0.120	0.295	0.119	-0.530	0.444***	521.09***	6.259***
MC5	52.51	0.587	0.530	-0.540	-0.138	0.268	0.300	-0.171	0.425***	530.07***	3.647***
MC6	66.95	1.711	0.593	-0.569	0.003	0.401	0.522	-0.039	0.706***	523.34***	5.329***
MC7	84.65	-0.913	-0.608	-1.018	-0.283	0.097	0.184	-0.247	0.497***	479.22***	6.109***
MC8	105.41	-3.562	-1.657	-1.927	-0.717	-0.074	0.127	-0.271	1.153***	381.23***	12.110***
MC9	131.55	-1.773	-1.031	-1.417	-0.645	-0.079	0.469	0.001	0.803***	455.29***	8.453***
MC10	162.83	-0.629	-0.088	-0.771	-0.284	0.284	0.737	0.170	0.590***	423.24***	6.139***
MC11	203.47	-0.096	0.380	-0.849	-0.381	0.061	0.618	0.301	0.556***	446.17***	4.553***
MC12	256.48	2.009	1.368	-0.194	-0.325	-0.087	0.476	0.139	0.758***	373.21***	4.866***
MC13	326.53	1.633	0.972	-0.194	-0.133	0.128	0.671	0.022	0.610***	378.57***	4.168***
MC14	419.37	1.404	0.849	-0.512	-0.494	-0.387	0.424	0.169	0.684***	377.83***	4.984***
MC15	557.17	1.748	0.918	-0.236	-0.434	-0.268	0.356	0.075	0.688***	242.16***	4.446***
MC16	768.68	1.829	0.960	0.003	-0.343	-0.428	-0.307	-0.316	0.721***	111.51***	4.959***
MC17	1108.56	1.565	1.261	0.174	-0.149	-0.374	0.294	0.071	0.608***	77.87***	6.415***
MC18	1767.84	-3.165	-1.159	-1.595	-1.519	-1.793	-1.596	-1.031	0.589***	76.76***	4.182***
MC19	3522.59	-0.743	-0.140	-0.862	-1.249	-1.978	-1.792	-1.501	0.583***	104.52***	8.133***
MC20	41700	-1.380	-1.068	0.130	0.353	0.624	0.470	0.317	0.660***	53.44***	5.904***

Note: All stocks during 1962 to 2015 on the CRSP daily tape are used. Co-kurtosis estimated using value-weighted portfolios returns and market index, and using value-weighted returns are illustrated in Panel A and Panel B, respectively. Stocks are sorted into 20 market capitalization equal-size portfolios. The first, $MC1$, contains the smallest 5% of firms and the last, $MC20$, contains the largest 5%. For each MC portfolio and each horizon, we compute the co-kurtosis at each month from August 1967 to December 2015 using previous five-year periods. This table illustrates the averages of all estimated portfolio co-kurtosis estimates. In Panel B, the standard deviation, repeated measure ANOVA tests and estimation bias estimates (or $EB_{\gamma,i}$ as in Equation 10) are used in detecting the magnitudes of the horizon effect. *** indicate the rejection of the null hypothesis at a 1% level.