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Monotonicity and the Roy Model.

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Abstract:

In this note we study the implications on a bivariate normal Roy Model of two sets of monotonicity hypotheses proposed recently by Manski and Pepper (2000). In that simple context, we show that these hypotheses imply strong restrictions on the correlations structure between the decision and the rewards.

Keyword: Roy Model; Monotonicity Conditions.

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Introduction

For the past forty years, labour economists have been studying the relationship between schooling and earnings (see Card, 1999 for a recent review). Universally, researchers observe that average individual earnings increase with years of schooling. However, the causality of this association as well as its magnitude is a subject of debate. On the one hand, Human Capital Theory predicts a positive and direct association between schooling and earnings, while on the other hand distinct theoretical arguments (for example signalling, Spence 1973) suggest that an unobserved factor, *i.e.* ability, is positively correlated with both education and earnings. The latter point has important consequences in terms of empirical methodology. Indeed, estimates obtained from least squares estimates are plagued by ability and endogeneity biases and by measurement error (Griliches, 1977).

Despite the diversity of estimation methods used to provide unbiased estimates (twin studies, Instrumental Variable, propensity score matching), Ashenfelter *et al.* (1999) report in their meta-analysis that results are not sensitive to the technique used. Moreover, these methods are contentious; see Neumark (1999) for criticisms on the twin literature, or Bound *et al.* (1995) and Angrist *et al.* (1996) on instrumental variables estimators and their interpretation. Identification of the schooling effect crucially relies on the assumptions researchers impose on the data.

Recently, several authors have attempted to characterise the magnitude of the returns to education without imposing such stringent constraints. For example, Manski (1990) relies solely on the distributions of the treatment and outcome variables to estimate bounds rather than point estimates. However, without further restrictions these “worst-case” bounds are not precise enough to provide much information on the “true” returns to education (Ginther, 2000).

In order to tighten the bounds, exclusion and/or monotonicity restrictions can be imposed. Manski and Pepper (2000) show that by imposing the following two restrictions, *i.e.* monotonicity in the selection (MTS) and monotonicity in the response to the treatment (MTR), the initial “worst-case” bounds can be tightened. MTR is equivalent to recognising that a higher level of treatment cannot have a negative effect on any individual’s outcome. MTS implies that individuals who choose a higher level of education level would receive above average rewards if they were otherwise reassigned to lower education levels.

This note compares the restrictions imposed by MTS and a weaker version of MTR on a selection model *a la* Roy (1951), where individuals choose the treatment maximising their expected outcome. We limit ourselves to the bivariate case and we find that in general the MTS-MTR assumptions impose strong restrictions on the structure of the correlation between treatment decisions and rewards. Far from being a “low-cost” estimation method in terms of restriction, the MTS-MTR bounds can be, on the contrary, based on stringent untestable constraints of the correlation structure of the model.

Monotone Instrumental Variables

Following Manski and Pepper (2000), we consider education as a treatment, which is possibly endogenously determined. The returns to education can be defined as the differences between the population means for earnings, $Y(t)$, associated with t years of schooling and for earnings, $Y(s)$, associated with s years of schooling,

$$D(t,s) = E [Y(t)] - E[Y(s)], \quad (1)$$

with $s < t$. When earnings are measured in logarithm, this difference is equivalent to a rate of return and is therefore directly comparable to estimates obtained by conventional methods.

As in any analysis of the effect of treatment, the difficulty is to compare the different groups, as individuals are only observed in one state and no information on the outcomes in the counterfactual states is available. The analysis supposes then that the treatment chosen, measured in years of education, Z , is observed. Some additional information is available in a monotone instrumental variable V . V is a monotone instrumental variable in the sense that for any two different values in its range, say v_1 and v_2 , with $v_1 < v_2$,

$$E[Y(t) | V = v_1] < E[Y(t) | V = v_2] \quad (2)$$

Note that Z can be a particularly convenient choice of such a monotone instrument variable, since it provides some hope that an estimate of the return can be obtained with a limited amount of extra information.

Manski and Pepper (2000) show that imposing some further structure on the latent distribution of rewards can lead to relatively tight intervals. Two sets of assumptions taken together proved to have some power in that respect. The first set of assumptions demands that the rewards depend in a monotonous fashion on the amount of education acquired (monotonicity in the response, MTR). On the other hand, the second set of assumptions requires that more able individuals would be rewarded better than less able individuals at any level of education (monotonicity in the selection, MTS).

MTS is based on the assumption that more able individuals earn more than less able individuals of the same educational level. As education attained is a function of unobserved ability, MTS is expressed in terms of education attained; an individual A with a higher level of education than an individual B, is thought to be more able. Hence, at all levels of education, A would have had higher returns than B. Formally, the MTS assumption states that:

$$u_2 \geq u_1 \Rightarrow E(y(t)/z = u_2) \geq E(y(t)/z = u_1) \quad \forall t \in T \quad (3)$$

for any possible value of t .

MTR assumes that for any given realisation of the couples, $(Y(t_2), Y(t_1))$, $(y_w(t_2), y_w(t_1))$ say, we have

$$t_2 \geq t_1 \Rightarrow y_j(t_2) \geq y_j(t_1) \quad (4)$$

i.e. in our schooling context, more schooling has a non negative effect on earnings.

At first glance, these two assumptions do not appear to be too restrictive and appear rather plausible. Furthermore imposing them allows us to obtain bounds on $E[Y(t)]$, in terms of quantities easily measured with data. We have:

$$\begin{aligned} \sum_{u < t} E(y/z = u) \times \Pr(z = u) + E(y/z = t) \times \Pr(z = t) \\ \leq E(y(t)) \leq \\ E(y/z = t) \times \Pr(z = t) + \sum_{u > t} E(y/z = u) \times \Pr(z = u) \end{aligned} \quad (5)$$

where Y stands for the observed reward.

For every treatment level, the expected wage for all individuals in the population can be bounded above and below. Hence the difference in earnings between two levels of

educational attainment is simply equal to the difference between the upper bound for the higher level of schooling and the lower bound for the lower level of schooling¹. In particular, it can then be shown that:

$$\begin{aligned}
D(s, t) \leq & \sum_{u < s} [E(y/z = t) - E(y/z = u)] \times \Pr(z = u) \\
& + [E(y/z = t) - E(y/z = u)] \times \Pr(s \leq z \leq t) \quad (6) \\
& + \sum_{u > t} [E(y/z = u) - E(y/z = s)] \times \Pr(z = u)
\end{aligned}$$

which bounds the return from above. Such a bound is useful in practice since an estimate can be readily calculated from observed data. Indeed estimates for all the quantities on the right-hand side can be obtained as empirical means of observed quantities.

Monotonicity and the Roy Model: the Bivariate Normal Case

The MTR and MTS assumptions are not innocuous. In particular given the implied decision process that determines individual actions, these two assumptions limit the nature of the joint latent reward process in general. This is easily seen in the context of a simple Roy model with normal distribution of skills. Consider the simple case where individuals have the choice between two education levels 0 and 1. Without loss of generality we can describe the latent wages as follows:

$$\begin{aligned}
Y(0) &= b_0 + e_0, \\
Y(1) &= b_1 + e_1,
\end{aligned} \tag{7}$$

where b_0 and b_1 are two constants, and where e_0 and e_1 are random variables which describe the heterogeneity in the population. In what follows we assume a weaker version of MTR (WMTR), which requires only that:

$$E[Y(0)] < E[Y(1)] \tag{8}$$

¹ Similarly, the lower bound can be defined as the difference between the lower bound for the higher level of schooling and the higher bound for the lower level of schooling.

The decision between the two education decisions, the binary indicator Z , is therefore such that:

$$\begin{aligned} Z = 0 & \text{ if } Y(0) > Y(1) \Rightarrow e_0 - e_1 > b_1 - b_0 \equiv \gamma_{10} \\ Z = 1 & \text{ if } Y(1) > Y(0) \Rightarrow e_1 - e_0 < \gamma_{10} \end{aligned} \quad (9)$$

We assume further that the heterogeneous returns to education for each education level are jointly normally distributed such that:

$$\begin{bmatrix} e_0 \\ e_1 \\ e_0 - e_1 \end{bmatrix} \sim N \left[\mathbf{0}, \begin{bmatrix} \sigma_0^2 & \rho\sigma_0\sigma_1 & \sigma_0^2 - \rho\sigma_0\sigma_1 \\ & \sigma_1^2 & \rho\sigma_0\sigma_1 - \sigma_1^2 \\ & & \sigma_0^2 + \sigma_1^2 - 2\rho\sigma_0\sigma_1 \end{bmatrix} \right] \quad (10)$$

The relevant conditional moments are easily calculated. For $k=0$ or 1 , we have:

$$E[Y(k)|Z=0] = b_k + E[e_k | e_0 - e_1 > \gamma_{10}] = b_k + J_k \frac{\theta\left(\frac{\gamma_{10}}{\Sigma}\right)}{1 - \Phi\left(\frac{\gamma_{10}}{\Sigma}\right)}, \quad (11)$$

$$E[Y(k)|Z=1] = b_k + E[e_k | e_0 - e_1 < \gamma_{10}] = b_k - J_k \frac{\theta\left(\frac{\gamma_{10}}{\Sigma}\right)}{1 - \Phi\left(\frac{\gamma_{10}}{\Sigma}\right)}, \quad (12)$$

where $\gamma_{10} > 0$ under WMTR,

$$\begin{aligned} J_0 &= \frac{\sigma_0^2 - \rho\sigma_0\sigma_1}{\sqrt{\sigma_0^2 + \sigma_1^2 - 2\rho\sigma_0\sigma_1}} \\ J_1 &= \frac{\rho\sigma_0\sigma_1 - \sigma_1^2}{\sqrt{\sigma_0^2 + \sigma_1^2 - 2\rho\sigma_0\sigma_1}} \end{aligned} \quad (13)$$

and

$$\Sigma = \sqrt{\sigma_0^2 + \sigma_1^2 - 2\rho\sigma_0\sigma_1} \quad (14)$$

MTS implies that:

$$\begin{aligned}
E[Y(0) | Z = 0] &< E[Y(0) | Z = 1] \\
\Rightarrow J_0 < 0 &\Leftrightarrow \sigma_0^2 - \rho\sigma_0\sigma_1 < 0 \Leftrightarrow \rho > \frac{\sigma_0}{\sigma_1}
\end{aligned} \tag{15}$$

and

$$\begin{aligned}
E[Y(1) | Z = 0] &< E[Y(1) | Z = 1] \\
\Leftrightarrow J_1 < 0 &\Leftrightarrow \rho\sigma_0\sigma_1 - \sigma_1^2 < 0 \Leftrightarrow \rho < \frac{\sigma_1}{\sigma_0}
\end{aligned} \tag{16}$$

Thus MTS imposes: $0 < \frac{\sigma_0}{\sigma_1} < \rho < \min\left\{\frac{\sigma_1}{\sigma_0}, 1\right\}$. Hence MTS implies that the latent distribution of rewards without education must be less variable than the latent distribution of reward with some education, *i.e.* $\sigma_0 < \sigma_1$. Furthermore, in order to satisfy MTS the correlation between rewards must be strictly positive. Finally, in the limit as $\sigma_0 \rightarrow \sigma_1$ imposing MTS and WMTR implies that all individuals acquire some education almost surely².

This conclusion changes somewhat if we introduce some unobserved heterogeneity in the choices, say $k + h$, where k is some constant and h is a random variable. This can be understood as an individual specific fixed cost associated with the higher education level. Such an interpretation would lead to the following decision rule:

$$\begin{aligned}
Z = 0 & \text{ if } Y(0) > Y(1) - k - h, \\
Z = 1 & \text{ if } Y(0) < Y(1) - k - h,
\end{aligned} \tag{17}$$

where $h \sim N(0, \sigma_h)$.

² Formally, $\sigma_0 \rightarrow \sigma_1 \Rightarrow \rho \rightarrow 1 \Rightarrow \Sigma \rightarrow 0 \Rightarrow \frac{\gamma_{10}}{\Sigma} \rightarrow \infty \Rightarrow \begin{cases} \Phi\left(\frac{\gamma_{10}}{\Sigma}\right) \rightarrow 1 \\ 1 - \Phi\left(\frac{\gamma_{10}}{\Sigma}\right) \rightarrow 0 \end{cases}$

The joint distribution of the unobservable becomes now:

$$\begin{bmatrix} e_0 \\ e_1 \\ e_0 + h - e_1 \end{bmatrix} \sim N \left[0, \begin{bmatrix} \sigma_0^2 & \rho\sigma_0\sigma_1 & q_0\Sigma' \\ & \sigma_1^2 & q_1\Sigma' \\ & & \Sigma'^2 \end{bmatrix} \right] \quad (18)$$

where: $\delta_0 = \text{corr}(e_0, h)$ and $\delta_1 = \text{corr}(e_1, h)$ and

$$\Sigma'^2 = \sigma_0^2 + \sigma_1^2 + \sigma_h^2 + 2\delta_0\sigma_0\sigma_h - 2\rho\sigma_0\sigma_1 - 2\delta_1\sigma_1\sigma_h \quad (19)$$

$$q_0 = \frac{\sigma_0^2 + \delta_0\sigma_0\sigma_h - \rho\sigma_0\sigma_1}{\Sigma'} \quad (20)$$

$$q_1 = \frac{\delta_1\sigma_1\sigma_h + \rho\sigma_0\sigma_1 - \sigma_1^2}{\Sigma'} \quad (21)$$

And MTS implies that:

$$\begin{aligned} q_0 < 0 &\Rightarrow \rho > \frac{\sigma_0}{\sigma_1} + \delta_0 \frac{\sigma_h}{\sigma_1} \\ q_1 < 0 &\Rightarrow \rho < \frac{\sigma_1}{\sigma_0} - \delta_1 \frac{\sigma_h}{\sigma_0} \end{aligned} \quad (22)$$

Thus the correlation between the latent rewards is now bounded above and below by quantities that can be of different signs:

$$\max \left\{ \frac{\sigma_0}{\sigma_1} + \delta_0 \frac{\sigma_h}{\sigma_1}, -1 \right\} < \rho < \min \left\{ \frac{\sigma_1}{\sigma_0} - \delta_1 \frac{\sigma_h}{\sigma_0}, 1 \right\}. \quad (23)$$

In particular the correlation of importance here is $\delta_0 = \text{corr}(e_0, h)$, the correlation between the cost and the wage without education. Indeed for values of $\frac{\sigma_1}{\sigma_0}$ close to one, a

negative value for δ_1 does not modify the usual upper bound on a correlation, while a positive value for δ_1 does modify the upper bound and the existence of the correlation depends then on the value of δ_0 . For example $\frac{\sigma_1}{\sigma_0} = 1$, $\frac{\sigma_\eta}{\sigma_1} = \frac{\sigma_\eta}{\sigma_0} = 0.5$, and $\delta_0 = -0.5$ implies $\rho > 0.75$. For $\delta_1 > 0.5$ ρ is not defined, for $\delta_1 = 0.5$ ρ is exactly 0.75, for smaller positive values of δ_1 ρ is bounded below by 0.75 and above by a quantity less than 1, and for any negative δ_1 we have $0.75 \leq \rho \leq 1$.

Moreover, the restrictions given by Vijverberg (1993), which ensure that the variance covariance matrix of the unobservables is semi-definite positive, need to be verified, *i.e.*

$$\delta_0\delta_1 - \sqrt{(1 - \delta_0^2)(1 - \delta_1^2)} < \rho < \delta_0\delta_1 + \sqrt{(1 - \delta_0^2)(1 - \delta_1^2)} \quad (24)$$

For example when $\delta_1 = -0.5$, semi-positive definiteness requires that ρ belongs to the interval $[-0.5, 1]$ which contains $[0.75, 1]$ where MTS is true. While if $\delta_1 = 0.5$ ρ must belong to the interval $[-1, 0.5]$ and this does not include $\rho = 0.75$.

Furthermore if we believe that the latent rewards are uncorrelated, *i.e.* $\rho = 0$, for the extended Roy model to satisfy MTS, it is necessary for the correlations between the fixed costs and the latent rewards to be such that:

$$\delta_0 < -\frac{\sigma_0}{\sigma_1} \text{ and } \delta_1 < \min\left\{\frac{\sigma_1}{\sigma_h}, 1\right\} \quad (25)$$

and furthermore definiteness when $\rho = 0$ demands

$$\delta_0^2 + \delta_1^2 < 1 \quad (26)$$

However, this is satisfied when σ_h , the variance of the fixed costs, is large relative to the variance of the latent reward $Y(0)$, *i.e.* when $\sigma_h > \sigma_0$, and for fixed costs moderately correlated with $Y(0)$.

Concluding Remarks

This straightforward exercise clearly shows that the monotonicity assumptions that Manski imposes, are not compatible in general with the decision process assumed by the original Roy model and its extensions. The Roy model assumes that individuals decide on the level of education which leads to the highest level of earnings. In the normal case, for a range of parameter values, this is at odds with the MTS hypothesis. The MTS hypothesis requires that better able individuals are on average better rewarded whatever their educational attainment, *i.e.* not only do they choose more education but even if education was not available to them they would obtain a higher reward on average than the average individual choosing the lower education level. The Roy model does not impose such a requirement on the latent distribution of rewards, it only requires that individuals decide on the basis of the highest reward. Potentially, a better-educated individual, if denied the chance of an education may end up with a lower than average wage among the less educated group.

Hence the claims in Manski and Pepper (2000, Footnote 8) that the “MTS restriction is consistent with economic models of schooling choice” and that the “MTR restriction is consistent with economic models of the production of human capital through schooling” to be made more precise at least in the prototypical case of the bivariate Roy Model with normal unobservables. The measurements of the bounds which are obtained when both hypotheses are imposed, may not be as restriction free as one may believe at first glance.

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