

ISSC DISCUSSION PAPER SERIES

THE ROAD LESS TRAVELLED: OLIGOPOLY AND COMPETITION POLICY IN GENERAL EQUILIBRIUM

Prof. J. Peter Neary

Peter Neary is a member of the Economics Dept. in UCD, a Research Associate at the London School of Economics and a Research Fellow at the Centre for Economic Policy Research, London.

This paper is produced as part of the International Trade and Investment Programme at ISSC; however the views expressed here do not necessarily reflect those of ISSC. All errors and omissions remain those of the author.

**THE ROAD LESS TRAVELLED: OLIGOPOLY
AND COMPETITION POLICY IN GENERAL EQUILIBRIUM***

J. Peter Neary
University College Dublin and CEPR

First Draft July 2002
This version September 25, 2002

Abstract

I review previous approaches to modelling oligopoly in general equilibrium, and propose a new view which in principle overcomes their deficiencies: modelling firms as *large* in their own market but *small* in the economy as a whole. Implementing this approach requires a tractable specification of preferences. Dixit-Stiglitz preferences (which imply iso-elastic perceived demand functions) could be used, but "continuum-quadratic" preferences (which imply linear perceived demand functions) are more convenient. To illustrate their usefulness, I construct a simple closed-economy model of oligopoly in general equilibrium and derive some surprising implications for competition policy.

JEL: D50, L13, L40

Keywords: competition policy; Dixit-Stiglitz preferences; general equilibrium; GOLE (General Oligopolistic Equilibrium); oligopoly

* Prepared for *Imperfect Economics: Essays in Honor of Joseph Stiglitz*, edited by Richard Arnott, Bruce Greenwald, Ravi Kanbur and Barry Nalebuff, to be published by MIT Press. I am grateful to Joe Francois, Henrik Horn, Ravi Kanbur, Lars Persson, Johan Stennek and Jacques Thisse for valuable discussions and comments. This research is part of the Globalisation Programme of the Centre for Economic Performance at LSE, funded by the UK ESRC, and of the International Trade and Investment Programme of the Institute for the Study of Social Change at University College Dublin.

Address for Correspondence: Department of Economics, University College Dublin, Belfield, Dublin 4, Ireland; tel.: (+353) 1-716 8344; fax: (+353) 1-283 0068; e-mail: peter.neary@ucd.ie; home page: <http://www.ucd.ie/~economic/staff/pneary/neary.htm>.

1. Introduction

It is a truth universally acknowledged that product markets are often far from competitive. Economists have taken two main routes to addressing this fact. At the micro level, the field of industrial organisation has developed a sophisticated array of models which focus on strategic interactions between firms in a single market. At the aggregate level, many fields, including international trade, macroeconomics and growth, have used models of monopolistic competition to incorporate increasing returns to scale and product differentiation into general equilibrium. Each of these approaches has contributed enormously to our understanding of real-world economies. However, each ignores the insights of the other. Models of industrial organisation typically take factor prices and aggregate income as given, and pay little attention to interactions between markets. General-equilibrium models of monopolistic competition typically ignore strategic behaviour by incumbents, and assume a perfectly elastic supply of identical new firms, ready and able to enter in response to the smallest profit opportunity.

Many important issues could be illuminated by combining these two approaches, in other words, by modelling oligopoly in general equilibrium. However, progress in this direction has been held back by a number of related problems. The first is that, if firms are large in their own market, and if that market constitutes a significant segment of the economy, then the firms have direct influence on economy-wide variables. Assuming they behave rationally, they should exploit this influence, taking account of their effect on both aggregate income and economy-wide factor prices when choosing their output or price. Such behaviour is both implausible (apart from relatively rare cases where firms have monopsony power in local factor markets) and difficult to model. Second, large firms influence the cost of living, and rational shareholders should take this into account in choosing the profit-maximising level of output or price. This difficulty was first highlighted by Gabszewicz and Vial (1972), who interpreted it as implying that the predictions of general-equilibrium oligopoly models are

sensitive to the choice of numeraire. (See also Böhm (1994).) Finally, Roberts and Sonnenschein (1977) showed that oligopolistic firms in general equilibrium typically have reaction functions so badly-behaved that no equilibrium may exist.

These problems have held back the development of tractable models of oligopoly in general equilibrium. They have even found their way into textbooks. (See for example, Bhagwati et al. (1998, p. 382) and Kreps (1990, p. 727).) In this paper, I draw on recent work (Neary (2002a, b, c)) where I argue that they can all be avoided in a simple way. This is to allow firms to be "large" in their own sector, but to require them to be "small" in the economy as a whole. Technically, the key to doing this is to assume a continuum of sectors, each with a small number of firms. (In fact, any reasonably large number of sectors would suffice, but it is much easier to work with the continuum case than with a large finite number of sectors.) Just as in models of perfect or monopolistic competition, firms cannot influence national income or factor prices and so take them as given. Since the output of any one sector is infinitesimal relative to national output, maximisation of profits leads to the same real allocation irrespective of how they are deflated. Finally, with income effects absent for any one sector, reaction functions are more likely to be well-behaved, and existence of equilibrium is no more problematic than it is in partial equilibrium.

The purpose of this paper is to put this modelling strategy in context, and to consider its implications for competition policy in a closed economy. In Section 2 I review the literature on oligopoly in general equilibrium, a tiny one compared to the huge number of papers on either oligopoly in partial equilibrium or monopolistic competition in general equilibrium. Section 3 addresses a key element in implementing my approach, the specification of preferences. Section 4 presents a simple model of oligopoly in general equilibrium and derives some surprising implications for competition policy. Section 5 shows that analogous

results also apply in a free-entry model of monopolistic competition which is dual to the oligopoly model of Section 4.

2. Alternative Approaches

A variety of approaches has been taken to resolving the difficulties with embedding oligopoly in general equilibrium which I have highlighted. One strand of literature ignores them, and assumes that the partial equilibrium oligopoly first-order condition, $p_i(1-\alpha_i/\epsilon)=c_i$, continues to apply in general equilibrium, even though firms are large relative to the economy as a whole. (Here α_i is the market share of firm i under Cournot competition, and ϵ is the price-elasticity of demand.) For example, Batra (1972) explored the implications of monopoly for the two-sector growth model, while Melvin and Warne (1973) and Markusen (1981) introduced monopoly and oligopoly into the two-sector Heckscher-Ohlin trade model respectively. This tradition is continued by Ruffin (2002), though he explicitly describes the typical agent in his model as behaving "schizophrenically, changing price as a producer and accepting prices as given as a consumer."

One aspect of firms' schizophrenia in these models is that they ignore their ability to influence factor prices. A small number of papers has addressed this, explicitly assuming that firms exercise monopsony power: see Bishop (1966), Feenstra (1980), Markusen and Robson (1980), and McCulloch and Yellen (1980). Even in simple models, such as when output prices are fixed by the small open economy assumption, this leads to considerable complications without yielding much additional insight.

A different tradition is to assume that preferences are quasi-linear, so the utility function is $x_0+u(x)$, where x_0 and x are the consumption levels of goods produced under perfect and oligopolistic competition respectively. The inverse demand curve for x is then independent

of income. This approach has been widely used in industrial organisation and related fields. (See, for example, Dixit (1981), Vives (1985) and Ottaviano et al. (2002).) Models with quasi-linear preferences are sometimes defended as being general equilibrium. (See for example Brander and Spencer (1984, p. 198) and Konishi et al. (1990, p. 69).) But most authors concede that, since all income effects fall on the good produced in the perfectly competitive sector, and factor prices are typically assumed to be determined in that sector, quasi-linear preferences provide a secure foundation for partial equilibrium analysis but nothing more.

Finally, a number of writers face up to the numeraire problem, warts and all. Gabszewicz and Vial (1972) and Cordella and Gabszewicz (1997) assume that firms are owned by worker-producers, who maximise utility rather than profits. In the same vein, Dierker and Grodal (1998) assume that firms maximise shareholders' real wealth, taking account of how their decisions affect the deflator for nominal wealth. These papers avoid the problem of sensitivity to the choice of numeraire. However, their behavioral assumptions are not very plausible, and the analysis required for even the simplest models of this kind is complex.

Given these difficulties of modelling oligopoly in general equilibrium, much more effort has been devoted to modelling monopolistic competition.¹ This is sometimes defended on the grounds that models with a fixed number of firms are "short-run" (see for example, Ohyama (1999, p. 14)), suggesting that free entry is the more general "long-run" case. But a casual glance at many real-world markets shows that they continue to be dominated by a small number of firms for long periods of time. A different rationale for monopolistic competition is that, with profits driven to zero by free entry and exit, the problems noted above seem to disappear. But as d'Aspremont et al. (1996) pointed out, this is only strictly true if a continuum of monopolistically competitive firms is assumed.²

A number of authors have explored the implications of allowing firms to be large in their own market but small in the economy. Hart (1982) constructs an "island" economy in which firms on one island are owned by shareholders on other islands, so avoiding the problems of an endogenous deflator for profits. Shleifer (1986) and Murphy, Shleifer and Vishny (1989) assume a large number of sectors, in each of which a monopolist faces a competitive fringe. Rotemberg and Woodford (1992) construct a dynamic model of implicit collusion between firms in a large (though finite) number of sectors. However, these papers were not primarily focused on my goal of integrating the theory of industrial organisation with general equilibrium analysis. In the remainder of this paper I sketch how this can be done.

3. Preferences

The key technical step in operationalising the approach I advocated in Section 1 is to find a tractable specification of preferences. An obvious starting point is to assume that utility is an additively separable function of a continuum of goods:

$$U[\{x(z)\}] = g \left[\int_0^1 u\{x(z)\} dz \right], \quad g' > 0, \quad u' > 0, \quad u'' < 0 \quad (1)$$

This specification could be extended in a number of ways. The measure of integration could be endogenous rather than fixed, permitting a dynamic version of the model; the sub-utility functions could vary with z as well as $x(z)$; and they could allow for differentiated products produced by each firm in sector z . However, in the remainder of the paper it is convenient to concentrate on (1).

Assume therefore that a representative consumer maximises (1) subject to the budget constraint:

$$\int_0^1 p(z)x(z)dz \leq I \quad (2)$$

where I is aggregate income. The inverse demand functions are:

$$p(z) = \frac{1}{\lambda} u'[x(z)] \quad (3)$$

where λ is the marginal utility of income, the Lagrange multiplier attached to the budget constraint, normalised by the derivative g' . The key feature of (3) is that, apart from λ , the demand price of good z depends only on variables pertaining to sector z itself. The marginal utility of income serves as a "sufficient statistic" for the rest of the economy in each sector. It is this property of what Browning et al. (1985) call "Frisch demand functions", combined with the continuum assumption, which allows a consistent theory of oligopoly. In each sector z , firms take λ as given, but in general equilibrium it is determined endogenously. To solve for λ , multiply (3) by $p(z)$ and integrate, using (2), to obtain:

$$\lambda = \frac{1}{\sigma_p^2} \int_0^1 p(z) u'[x(z)] dz \quad \text{where:} \quad \sigma_p^2 \equiv \int_0^1 p(z)^2 dz \quad (4)$$

So λ equals a price-weighted mean of the marginal utilities of individual goods, divided by the (uncentred) variance of prices.

While any additively separable utility function gives the "sufficient statistic" property, the general form $u[x(z)]$ is not tractable, since the endogenous consumption levels appear on the right-hand side of (4). Hence we need to consider some special forms for the sub-utility function. The simplest case is where preferences are Cobb-Douglas:

$$g(h) = \exp h, \quad u[x(z)] = \beta(z) \ln x(z), \quad \int_0^1 \beta(z) dz = 1 \quad (5)$$

Here λ is the reciprocal of income, $\lambda=1/I$, and the inverse demand functions are unit-elastic, with $\beta(z)$ denoting the (infinitesimal) budget share of good z :

$$p(z) = \frac{1}{\lambda} \frac{\beta(z)}{x(z)} = \frac{\beta(z)I}{x(z)} \quad (6)$$

These demand functions are attractively simple or extremely restrictive, depending on your point of view. In any case, they are inconsistent with profit maximisation by a monopolist.

Alternatively, preferences may take the constant-elasticity-of-substitution form as in Dixit and Stiglitz (1977):

$$g(h) = h^{1/\theta}, \quad u[x(z)] = x(z)^\theta, \quad 0 < \theta < 1 \quad (7)$$

Now the inverse demand functions are iso-elastic:

$$p(z) = \frac{\theta}{\lambda} x(z)^{-\eta}, \quad \eta \equiv \frac{1}{1-\theta} > 1 \quad (8)$$

where the elasticity of demand η is also the elasticity of substitution between every pair of goods; while λ is an inverse Cobb-Douglas function of income and the true cost-of-living index P :

$$\lambda[\{p(z)\}, I] = \frac{\theta}{P^\theta I^{1-\theta}} \quad \text{where:} \quad P \equiv \left[\int_0^1 p(z)^{1-\eta} dz \right]^{\frac{1}{1-\eta}} \quad (9)$$

These demand functions allow convenient solutions in models of monopolistic competition, where they were first introduced. But in oligopoly they have unattractive implications: outputs are often strategic complements in Cournot competition, and reaction functions may be non-monotonic.

Finally, as in Neary (2002c), consider the case where each sub-utility function is quadratic:

$$g(h) = h, \quad u[x(z)] = ax(z) - \frac{1}{2}bx(z)^2 \quad (10)$$

Now the inverse demand functions and the marginal utility of income are:

$$p(z) = \frac{1}{\lambda}[a - bx(z)] \quad \text{and} \quad \lambda[\{p(z)\}, I] = \frac{a\mu_p - bI}{\sigma_p^2} \quad (11)$$

where μ_p is the mean of prices and we have already defined σ_p^2 as their (uncentred) variance. Hence, a rise in income, a rise in the (uncentred) variance of prices, or a *fall* in the mean of prices, all reduce λ and so shift the demand function for each good outwards. However, from the perspective of firms, λ is an exogenous variable over which they have no control. While (11) is the true demand curve, the *perceived* demand curve is linear, as in Negishi (1961), since firms treat λ as constant. Oligopoly models with linear demand functions are easy to solve in partial equilibrium, which allows us to construct a tractable but consistent model of oligopoly in general equilibrium.

4. Competition Policy in General Oligopolistic Equilibrium

The specification of preferences discussed in the last section could be combined with any one of a huge variety of assumptions about the supply side of the economy. For simplicity I adopt here probably the simplest possible approach. It combines Cournot competition and a given number n of firms in each sector with a Ricardian specification of technology and factor markets. The first-order condition for a typical firm in sector z is: $p(z) - c(z) = by(z)/\lambda$. Solving for equilibrium output:

$$y(z) = \frac{a - \lambda c(z)}{b(n+1)} \quad (12)$$

The unit cost of production in each sector, $c(z)$, equals the economy-wide wage rate, w , times the sector's labour requirement per unit output, denoted by $\alpha(z)$: $c(z) = w\alpha(z)$. The equilibrium condition in the labour market requires that the exogenous labour supply L equal the total demand for labour from all sectors:

$$L = \int_0^1 \alpha(z) n y(z) dz \quad (13)$$

Inspecting equations (12) and (13) shows that they are homogeneous of degree zero in two nominal variables, the wage rate w and the *inverse* of the marginal utility of income, λ^{-1} . As is normal in real models, the absolute values of these variables are indeterminate, and, indeed, uninteresting. Hence we can choose an arbitrary numeraire. (This will be a *true* numeraire: unlike the cases discussed in Section 2, the choice of numeraire has no implications for the model's behaviour.) It is most convenient to choose utility itself as numeraire, so that λ is unity by choice of units.

To solve the model, evaluate the integral in (13), using (12):

$$w = \left[a\mu - \frac{n+1}{n} bL \right] \frac{1}{\sigma^2} \quad (14)$$

μ and σ^2 denote the first and second moments of the technology distribution:

$$\mu \equiv \int_0^1 \alpha(z) dz \quad \text{and} \quad \sigma^2 \equiv \int_0^1 \alpha(z)^2 dz \quad (15)$$

For later use, note that the variance v^2 of the technology distribution is given by $v^2 = \sigma^2 - \mu^2$. A special case, which serves as a convenient benchmark, is the *featureless economy*, where all sectors have the same technology parameter α_0 : the variance v^2 is zero, and so $\sigma = \mu = \alpha_0$.

The first issue we want to examine is the effect of competition policy (in the sense of an increase in the number of firms n in all sectors) on the functional distribution of income. National income I , measured in utility units, equals the sum of wages wL and total profits Π :

$$I = wL + \Pi, \quad \Pi = \int_0^1 n\pi(z)dz \quad (16)$$

To evaluate total profits, note that profits in each sector $\pi(z)$ equal $[p(z)-c(z)]y(z)$. Using the first-order condition and equation (12) gives:

$$\Pi = bn \int_0^1 y(z)^2 dz = \frac{n}{b(n+1)^2} (a^2 - 2a\mu w + \sigma^2 w^2) \quad (17)$$

I show in the Appendix that this can be written in terms of parameters as follows:

$$\Pi = \frac{na^2}{b(n+1)^2} \frac{v^2}{\sigma^2} + \frac{bL^2}{n} \frac{1}{\sigma^2} \quad (18)$$

From (14), the wage rate w is strictly *increasing* in n ; while from (18) total profits are strictly *decreasing* in n . Hence it follows that:

Result 1: A rise in n raises the share of wages in national income.

Thus competition policy has the expected effect on income distribution.

Next, we want to determine the effects of competition policy on welfare. Substituting the direct demand functions implied by (11) into the utility function (1) and (10), gives the indirect utility function (ignoring a constant):

$$\tilde{U} = a^2 - \sigma_p^2 \quad (19)$$

To calculate the variance of the distribution of prices, we can use the Cournot equilibrium

price formula, $p(z)=[a+nc(z)]/(n+1)$. Squaring and integrating yields:

$$\sigma_p^2 = \frac{1}{(n+1)^2} (a^2 + 2an\mu w + n^2\sigma^2 w^2) \quad (20)$$

Substituting from (14) for w , this becomes (as shown in the Appendix):

$$\sigma_p^2 = \frac{a^2}{(n+1)^2} \frac{v^2}{\sigma^2} + \frac{(a\mu - bL)^2}{\sigma^2} \quad (21)$$

This is clearly decreasing, but not strictly so, in the number of firms. Hence:

Result 2: An increase in n raises welfare, strictly so if $v^2 > 0$.

Since there are no other distortions in the model, we would expect this to be so. However, the result is not strict, unlike Result 1, which has a surprising implication:

Corollary: In the featureless economy, competition policy has no effect on welfare:

$$d\tilde{U}/dn=0.$$

Inducing entry by more firms in all sectors raises the demand for labour. But since the aggregate labour supply constraint is binding, this merely redistributes income from profits to wages without any gains in efficiency.

This result should not be seen as an argument against activism in competition policy. The model is far too stylised to provide a basis for policy-making. In the realistic case where sectors are heterogeneous, the welfare costs of oligopoly implied by (21) may be greater than those implied by partial-equilibrium calculations in the tradition of Harberger (1954). Rather, the result should be seen as an extreme case which brings out the importance of a general-

equilibrium perspective. It was clearly stated by Lerner (1933-34), who also conjectured that a suitable measure of the economy-wide degree of monopoly is the standard deviation of the "degree of monopoly" (i.e., the price-cost margin) across all sectors. Equation (21) confirms Lerner's intuition, and shows how it relates to the underlying structural parameters in the economy.

Finally, consider the effects of an increase in the technology variance, v^2 , at constant μ . I show in the Appendix that equations (14), (18) and (21) imply:

Result 3: A mean-preserving spread in the technology distribution raises aggregate welfare but lowers the share of wages in national income.

So aggregate welfare and the share of wages need not move together.

5. Competition Policy in Monopolistic Competition

Competition policy was modelled in the previous section as an exogenous increase in the number of firms in each sector of the economy. In this section I show that the same results go through if instead the fixed costs of operating a firm are taken as parametric, and entry of new firms is assumed to be free. The model therefore becomes one of monopolistic competition rather than of oligopoly.

It would be possible to assume that firms' fixed costs vary in some systematic way with z . However, there is no basis for doing this within the model, so assume instead that they take a constant value f in all sectors. Hence the equilibrium condition for free entry in sector z is:

$$f = \pi(z) = by(z)^2 \quad (22)$$

We can solve this for the equilibrium output of each firm and use (12) to solve for the equilibrium number of firms in sector z (ignoring the integer constraint):

$$y(z) = \sqrt{f/b} \quad \text{and} \quad n(z) = \frac{a-w\alpha(z)}{\sqrt{bf}} - 1 \quad (23)$$

As in models of monopolistic competition based on Dixit-Stiglitz preferences, the equilibrium size of each firm depends only on cost and taste parameters. Adjustment to shocks in other parameters comes about solely through changes in the number of firms.

We can now solve for the equilibrium wage. As in the previous section, we integrate the labour-market equilibrium condition (13). The difference is that now the number of firms in each sector and not the output of each varies with z . This yields:

$$w = [(a - \sqrt{bf})\mu - bL] \frac{1}{\sigma^2} \quad (24)$$

Comparing this with equation (14) in the last section, it is clear that a reduction in fixed costs in monopolistic competition has a similar effect on the equilibrium wage as an increase in the number of firms in oligopoly: both tend to raise w . Moreover, the competitive limits of the two models (as f approaches zero and n approaches infinity, respectively) are identical.

To solve for welfare, use the first-order condition and the free-entry condition (22) to solve for the equilibrium price in sector z :

$$p(z) = \sqrt{bf} + w\alpha(z) \quad (25)$$

Note that this implies a fixed *absolute* price-cost margin, whereas in monopolistic competition with Dixit-Stiglitz preferences, the *relative* price-cost margin is fixed. From (25), I show in the Appendix that the variance of prices equals:

$$\sigma_p^2 = bf \frac{v^2}{\sigma^2} + \frac{(a\mu - bL)^2}{\sigma^2} \quad (26)$$

This is clearly increasing (but not strictly so) in the fixed cost of production. Hence we can conclude:

Result 4: A fall in f raises welfare, strictly so provided $v^2 > 0$.

The interpretation of this result is identical to that of Result 2 in the previous section. Competition policy has a similar effect on welfare under monopolistic competition as it has under oligopoly: welfare cannot fall, and must increase provided the variance of the technology distribution is strictly positive.

6. Conclusion

In this paper I reviewed previous approaches to modelling oligopoly in general equilibrium, and drew on Neary (2002c) to propose a new view which in principle overcomes their deficiencies: modelling firms as *large* in their own market but *small* in the economy as a whole. On the one hand, firms face a small number of local competitors, so they have an incentive to compete strategically against them in the manner familiar from the theory of industrial organisation. On the other hand, there is a continuum of sectors, so firms cannot influence economy-wide variables such as national income and factor prices. Furthermore, provided all profits are returned costlessly to the aggregate household, the implications of profit maximisation are independent of tastes and of the normalisation rule for prices. Hence, the approach avoids all the problems which have bedeviled previous studies of oligopoly in general equilibrium.

Implementing this approach requires a tractable specification of preferences. In principle, any form of additively separable preferences could be used, since they share the convenient property that the demand facing each sector depends only on variables pertaining to that sector and on the economy-wide marginal utility of income. For example, Dixit-Stiglitz preferences (which imply iso-elastic perceived demand functions) could be used. However, I have chosen to work instead with "continuum-quadratic" preferences (which imply linear perceived demand functions), since they guarantee existence and uniqueness of equilibrium at the sectoral level, and hence allow convenient aggregation across sectors.

To illustrate my proposed approach, I constructed a simple model of oligopoly in general equilibrium and used it to derive some surprising implications for competition policy. In particular, I showed that the desirability of increasing competition depends crucially on the distribution of what Lerner (1933-34) called the "degree of monopoly" across sectors. While some authors, such as Francois and Horn (1998), have recognised that competition policy should take account of general equilibrium constraints, a satisfactory analytic framework in which to do so has not been available so far.

Of course, competition policy is modelled here in an extremely simple way: a parametric increase in the number of firms in Section 4, or a parametric reduction in the fixed costs of operating a firm in Section 5. In other respects too this paper has clearly only scratched the surface of the new approach I have proposed. Even if continuum-quadratic preferences are retained, a wide range of alternative assumptions about market structure, firm behaviour and the workings of factor markets awaits exploration. For example, a richer theory of firm behaviour should allow for strategic entry and entry-deterrence, since the assumption in Section 4 that the number of firms in all sectors is exogenous is highly artificial. However, the standard assumption of completely free entry in all sectors is just as unrealistic. I hope

I have suggested the potential gains from exploring alternatives to the free-entry competitive paradigm, while still retaining a general-equilibrium perspective.

Appendix

Proof of Result 1: To evaluate total profits, substitute for w from (14) into the expression in brackets in (17) and rewrite as a difference in squares:

$$\begin{aligned}
 a^2 - 2a\mu w + \sigma^2 w^2 &= a^2 - w(2a\mu - \sigma^2 w) \\
 &= a^2 - \left(a\mu - \frac{n+1}{n}bL\right)\left(a\mu + \frac{n+1}{n}bL\right) \frac{1}{\sigma^2} \\
 &= a^2 \frac{v^2}{\sigma^2} + \left(\frac{n+1}{n}bL\right)^2 \frac{1}{\sigma^2}
 \end{aligned} \tag{27}$$

Substituting back into (17) gives the expression for profits in (18).

Proof of Result 2: To evaluate the level of welfare, proceed as in the proof of Result 1:

$$\begin{aligned}
 a^2 + 2an\mu w + n^2\sigma^2 w^2 &= a^2 + nw(2a\mu + n\sigma^2 w) \\
 &= a^2 + \left[an\mu - (n+1)bL\right]\left[a(n+2)\mu - (n+1)bL\right] \frac{1}{\sigma^2} \\
 &= a^2 + \left[(n+1)(a\mu - bL) - a\mu\right]\left[(n+1)(a\mu - bL) + a\mu\right] \frac{1}{\sigma^2} \\
 &= a^2 \frac{v^2}{\sigma^2} + (n+1)^2(a\mu - bL)^2 \frac{1}{\sigma^2}
 \end{aligned} \tag{28}$$

Substituting back into (20) gives (21).

Proof of Result 3: To establish how v^2 affects welfare, differentiate (21):

$$\begin{aligned}
 \frac{d\sigma_p^2}{dv^2} &\propto \frac{a^2}{(n+1)^2} \mu^2 - (a\mu - bL)^2 \\
 &= -\frac{n(n+2)}{(n+1)^2} \left(a\mu - \frac{n+1}{n+2}bL\right)\left(a\mu - \frac{n+1}{n}bL\right) < 0
 \end{aligned} \tag{29}$$

This is negative, and so welfare is increasing in v^2 , provided w in (14) is positive. The effect of v^2 on wages is negative, by inspection of (14). Finally, from (18), the effect on profits is:

$$\begin{aligned}
\frac{d\Pi}{dv^2} &\propto \frac{na^2}{b(n+1)^2} \mu^2 - \frac{bL^2}{n} \\
&= \frac{n}{b(n+1)^2} \left(a\mu + \frac{n+1}{n} bL \right) \left(a\mu - \frac{n+1}{n} bL \right) > 0
\end{aligned} \tag{30}$$

Proof of Result 4: To evaluate the variance of prices, square and integrate (25) and then proceed as in the proofs of Results 1 and 2:

$$\begin{aligned}
\sigma_p^2 &= bf + 2\sqrt{bf}\mu w + \sigma^2 w^2 \\
&= bf + w(2\sqrt{bf}\mu + \sigma^2 w) \\
&= bf + [(a\mu - bL) - \sqrt{bf}\mu][(a\mu - bL) + \sqrt{bf}\mu] \frac{1}{\sigma^2}
\end{aligned} \tag{31}$$

which leads to (26).

Endnotes

¹ I follow most of the literature in using "imperfect competition" to denote any type of market structure other than perfect competition, and "monopolistic competition" to refer to the Chamberlin large-group case, where entry and exit are free. Not all writers use the terms in this way. For example, "monopolistic competition" in Roberts and Sonnenschein (1977) refers to models with a fixed number of firms, which I prefer to describe as oligopoly models.

² Ironically, Dixit and Stiglitz adopted this assumption in an early draft of their 1977 paper, but dropped it from the published version at the request of a referee. See Brakman and Heijdra (2002).

References

- Batra, Raveendra. "Monopoly theory in general equilibrium and the two-sector model of economic growth." *Journal of Economic Theory* 4 (1972): 355-71
- Bhagwati, Jagdish N., Arvind Panagariya and T.N. Srinivasan. *Lectures on International Trade: Second Edition*. Cambridge: MIT Press, 1998
- Bishop, Robert L. "Monopoly under general equilibrium: Comment." *Quarterly Journal of Economics* 80 (1966): 652-9
- Brakman, Steven and Ben J. Heijdra (eds.). *The Monopolistic Competition Revolution After Twenty-Five Years*. Cambridge: Cambridge University Press, 2002
- Brander, James A. and Barbara Spencer. "Tariff protection and imperfect competition." in H. Kierzkowski (ed.) *Monopolistic Competition and International Trade*. Oxford: Clarendon Press, 1984, 194-206
- Browning, Martin, Angus Deaton and Margaret Irish. "A profitable approach to labor supply and commodity demands over the life-cycle." *Econometrica* 53 (1985): 503-43
- Cordella, Tito and Jean Jaskold Gabszewicz. "Comparative advantage under oligopoly." *Journal of International Economics* 43 (1997): 333-46
- d'Aspremont, Claude, Rodolphe Dos Santos Ferreira and Louis-André Gérard-Varet. "On the Dixit-Stiglitz model of monopolistic competition." *American Economic Review* 86 (1996): 623-29
- Dierker, Egbert and Birgit Grodal. "Modelling policy issues in a world of imperfect competition." *Scandinavian Journal of Economics* 100 (1998): 153-79
- Dixit, Avinash K. "The role of investment in entry deterrence." *Economic Journal* 90 (1981): 95-106
- _____ and Joseph E. Stiglitz. "Monopolistic competition and optimum product diversity."

American Economic Review 67 (1977): 297-308

Feenstra, Robert C. "Monopsony distortions in an open economy: A theoretical analysis."

Journal of International Economics 10 (1980): 213-35.

Francois, Joseph F. and Henrik Horn. "Competition policy in an open economy."

Discussion Paper 98-06, Center for International Economic Studies, University of Adelaide, 1998

Gabszewicz, Jean Jaskold and Jean-Philippe Vial. "Oligopoly à la Cournot in a general equilibrium analysis." *Journal of Economic Theory* 4 (1972): 381-400

Harberger, Arnold. "Monopoly and resource allocation." *American Economic Review (Papers and Proceedings)* 44 (1954): 77-87

Hart, Oliver. "A model of imperfect competition with Keynesian features." *Quarterly Journal of Economics* 97 (1982): 109-138

Konishi, Hideki, Masahiro Okuno-Fujiwara and Kotaro Suzumura. "Oligopolistic competition and economic welfare: A general equilibrium analysis of entry regulation and tax-subsidy schemes." *Journal of Public Economics* 42 (1990): 67-88

Kreps, David. *A Course in Microeconomic Theory*. Hemel Hempstead: Harvester Wheatsheaf, 1990

Lerner, Abba P. "The concept of monopoly and the measurement of monopoly power."

Review of Economic Studies 1 (1933-34): 157-75

Markusen James R. "Trade and the gains from trade with imperfect competition." *Journal of International Economics* 11 (1981): 531-51

_____ and Arthur Robson. "Simple general equilibrium and trade with a monopsonized sector." *Canadian Journal of Economics* 3 (1980): 389-402

McCulloch, Rachel and Janet L. Yellen. "Factor market monopsony and the allocation

of resources." *Journal of International Economics* 10 (1980): 237-48

Melvin, James R. and Robert D. Warne. "Monopoly and the theory of international trade." *Journal of International Economics* 3 (1973): 117-34

Murphy, Kevin M., Andrei Shleifer and Robert V. Vishny. "The big push." *Journal of Political Economy* 97 (1989): 1003-1026

Neary, J. Peter. "Competition, trade and wages." in D. Greenaway, R. Upward and K. Wakelin (eds.): *Trade, Migration, Investment and Labour Market Adjustment*, London: Macmillan, 2002a, 28-45

_____. "Foreign competition and wage inequality." *Review of International Economics* 10 (2002b): 680-93

_____. "International trade in general oligopolistic equilibrium." University College Dublin, mimeo., 2002c

Negishi, Takashi. "Monopolistic competition and general equilibrium." *Review of Economic Studies* 28 (1961): 196-201

Ohyama, Michihiro. "Market, trade and welfare in general equilibrium." *Japanese Economic Review* 50 (1999): 1-24

Ottaviano, Gianmarco, Takatoshi Tabuchi and Jacques-François Thisse. "Agglomeration and trade revisited." *International Economic Review* 43 (2002): 409-35

Roberts, John and Hugo Sonnenschein. "On the foundations of the theory of monopolistic competition." *Econometrica* 45 (1977): 101-113

Rotemberg, Jerome and Michael Woodford. "Oligopolistic pricing and the effects of aggregate demand on economic activity." *Journal of Political Economy* 100 (1992): 1153-1207

Ruffin, Roy J. "Oligopoly and trade: What, how much and for whom?" *Journal of*

International Economics (forthcoming), 2002

Shleifer, Andrei. "Implementation cycles." *Journal of Political Economy* 94 (1986): 1163-1190

Vives, Xavier. "On the efficiency of Bertrand and Cournot equilibria with product differentiation." *Journal of Economic Theory* 36 (1985): 166-75