

## Problem Sheet 5 (Data Modeling)

### Engineering Computation

1. Derive the *normal equations* for the approximation of a function

$$f(x, a_i) = \sum_{k=1}^m a_k Y_k(x)$$

over the data points  $(x_1, y_1), \dots, (x_n, y_n)$  where the basis functions  $Y_k(x)$  are known. Find the best *quadratic* fit to the data set,

$$\begin{array}{c|ccccc} x & -2 & -1 & 0 & 1 & 2 \\ \hline y & -5 & -3 & 0 & 4 & 4 \end{array}$$

2. Develop the method of least squares for the approximation of a straight line

$$f(x, a_1, a_2) = a_1 + a_2 x$$

over the data points  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$  with errors  $\sigma_1, \sigma_2, \dots, \sigma_n$  associated with each y-value. Find the linear least squares approximation to the data

$$\begin{array}{c|ccccc} x & 1 & 2 & 3 & 4 & 5 \\ \hline y & 0 & 1.5 & 2.8 & 3.9 & 5.3 \end{array}$$

where you may assume that the errors are all zero.

3. It is theoretically predicted that the rate of flow from a fire hose is proportional to some power of the pressure at the nozzle. Do the following data confirm the speculation? What is the least-squares value of the exponent?

$$\begin{array}{c|ccccc} \text{Flow} & 94 & 118 & 147 & 180 & 230 \\ \hline \text{Pressure} & 10 & 16 & 25 & 40 & 60 \end{array}$$

4. Using the data of question (3) find the least-squares values for the constants in

$$\text{Flow} = aP^2 + bP + c$$

where  $P = \text{Pressure}$ .

5. Find the least-square values for the parameters of  $y = ae^{bx}$  where

$$\begin{array}{c|ccccc} x \text{ (Temperature)} & 77 & 100 & 185 & 239 & 285 \\ \hline y \text{ (Solubility)} & 2.4 & 3.4 & 7.0 & 11.1 & 19.6 \end{array}$$