ERGODIC THEORY AND COMPLEX DYNAMICS

The proposed thesis is in the field of Dynamical Systems, the qualitative study of deterministic systems which evolve with time. In interesting (*chaotic*) systems, there is sensitive dependence on initial conditions. A simple real-world example is the double-pendulum. Ergodic theory concerns measure-theoretical properties of such systems. Maybe we cannot predict what will happen for every (or for a given) initial condition, but we can still say what will happen for *almost every* initial condition (compare: almost every number is normal). Very simple models have rich properties. One of the most deeply studied is the discrete-time system obtained by iterating $z \mapsto z^2 + c$ in the complex plane, generating beautiful fractal objects (Julia sets) and an outstanding open question, The Fatou Conjecture.

Ergodic and geometric properties of quadratic maps of the complex plane and, more generally, of rational maps of the Riemann sphere, are very well-studied. Behaviour of transcendental dynamics (for example iterating $z \mapsto e^z + c$ in the complex plane) is a current area of active research with numerous open problems. Techniques used can be hands-on analysis or involve deep tools (quasiconformal surgery, renormalisation etc.).

The book *Complex Dynamics* by Lennart Carleson and Theodore Gamelin is recommended. See *Perturbing Misiurewicz parameters in the exponential family. Comm. Math. Phys. 335 (2015), no. 2, 571–608* for a recent result in the domain and further references.