Some thoughts on the Draft Background Paper and Brief for the Review of Applied Mathematics

Ted Cox, Russell Higgs, Peter Lynch, Maria Meehan, Aoibhinn Ní Shúilleabháin, Lennon Ó Náraigh,* Adrian Ottewill, Conor Sweeney, and James Sweeney

School of Mathematical Sciences, University College Dublin, Belfield, Dublin 4, Ireland

(Dated: January 20, 2015)

EXECUTIVE SUMMARY

The School of Mathematical Sciences in UCD convened an ad-hoc committee to study the draft paper from the NCCA and to formulate a school opinion on the paper. This document reports on these discussions. The study was held under the Chatham House rule, which allows comments to be reported without attribution to encourage a frank exchange of views. The school is delighted to see that the NCCA is engaging with the thorny question of reform of the Applied Mathematics syllabus and we welcome the ideas contained in the draft paper. The main proposal in our submission is the refinement of the *core and option* structure proposed in the NCCA draft paper (Section 3, Table 1) possibly to be replaced instead by a core component together with a number of *case studies of Mathematics and its applications*, to be delivered ideally in a partnership between teachers in the classroom industry, third-level institutions.

I. OVERVIEW

We started our discussion from a principles-based position, focused on the following particular questions:

- What is Applied Mathematics?
- Should Applied Mathematics be taught in isolation from other kinds of Mathematics?
- What is the best way to teach the applications of Mathematics?
- What are the practical constraints that have hindered the takeup of the subject "Applied Mathematics" in the past? Can a new programme in Mathematics and its applications be designed in such a way to kickstart a widespread engagement by second-level students in advanced Mathematics and its applications?

These discussion points are not new, and neither are our answers below. Indeed, the presentation of some of our answers below is based on a previous submission to the Teaching Council [1].

* onaraigh@maths.ucd.ie

A. What is Applied Mathematics?

Historically, there has been much overlap between the terms "Applied Mathematics", and "Mathematical Physics". This overlap is exemplified by the research tradition in Cambridge University [2], where Newton, Clerk Maxwell, Larmor, Stokes, Rayleigh, Eddington, and Dirac developed Mathematical Physics into a scientific discipline with immense predictive power. Here, "Mathematical Physics" refers to the application and development of mathematical techniques to solve problems in mechanics (particles, solids, and fluids), gravitation, electromagnetism, relativity, and quantum theory. This tradition has heavily influenced the existing Leaving Certificate Applied Mathematics, with its strong emphasis on problem-solving in classical mechanics.

Although many universities maintain a sharp focus on Mathematical Physics in their Applied Mathematics research divisions (the exemplar being Cambridge-DAMTP), the modern trend is for diversity in the range of topics to which mathematical methods are applied. Sample topics outside of Mathematical Physics researched by leading international universities include computational mathematics, image processing, networks, pattern formation, mathematical biology, financial mathematics, and dynamical systems [3]. This same diversity is also visible in the research interests of academics based in Irish universities. This trend is also reflected in modern second-level Mathematics Syllabi, as evidenced by the survey in the NCCA draft paper, as well as by our own perusal of syllabi [1]: the Applied Mathematics syllabus of Hong Kong includes statistical techniques and numerical methods [4]. The core Mathematics syllabus in California includes a strand on mathematical modelling [5]. The International Baccalaureate contains options involving financial mathematics, graph theory, and statistical methods [6]. In summary, the modern notion of Applied Mathematics is ably given more eloquently by others [7], as follows:

> Applied Mathematics concerns the application of Mathematics in a wide range of disciplines in various areas such as science, technology, business and commerce. Applied mathematicians are engaged in the creation, study and application of advanced mathematical methods relevant to specific problems. Once this referred mainly to the application of Mathematics to such disciplines as mechanics and fluid dynamics but currently, Applied Mathematics has assumed a much broader meaning and embraces such diverse fields as communication theory, theory of optimization, theory of games and numerical analysis. Indeed, today there is a remarkable range and variety of applications of Mathematics in industry and government, involving important real-world problems such as materials processing, design, medical diagnosis, development of financial products, network management and weather prediction.

B. Should Applied Mathematics be taught in isolation from other kinds of Mathematics?

The resounding answer in our discussion was 'no': Applied Mathematics cannot be separated from Mathematics in its wider context. This chimes nicely with the consensus by a more esteemed teacher and scholar than ourselves, who maintained that a deep understanding of Core Mathematics (or "Pure Mathematics") is crucial to the success of Applied Mathematics (Lin, Reference [8]). Here, Core Mathematics is understood as basic mathematical principles in analysis, algebra, probability, and geometry. The programme of Applied-Mathematics education recommended by Lin recognises the importance of these areas, and prescribed the following education (taken verbatim from [8]):

1. An education in the attitude of an applied mathematician, with emphasis on solving real-world problems;
2. An education in the usual working tools (methods of Applied Mathematics);
3. A survey in Applied Mathematics with emphasis on the total picture;
4. A basic education in Pure Mathematics;
5. An in-depth education in at least one branch of science

For these reasons, **we agree with the proposal in Section 3 of the draft background paper to integrate education in "core mathematics" with education in application areas.** Indeed, we agree furthermore with the proposal of the draft background paper to rename the programme in Leaving Certificate Applied Mathematics as "Further Mathematics with Applications" (or some similar title), as this not only highlights the indivisibility of Mathematics and Applied Mathematics as outlined by Lin [8], but also aligns with international practice in the design of syllabi, as outlined in the draft discussion paper (Section 2 therein). Note that a title along the particular lines of "Further Mathematics with Applications" would also make it clearer that (after negotiation), maths teachers could teach the subject, not just teachers of Applied Mathematics. Therefore, throughout the remainder of this paper, we refer to the proposed new programme in Mathematics and its applications as 'the new specification'.

C. What is the best way to teach the applications of Mathematics?

Arguably, this question is ill-posed, as the answer will depend on a specific national context, informed by international developments, as well as developments in the relevant academic disciplines. However, starting with the proposed structure in the draft background paper, as well as the prescriptions in Section I B, we present below a structure that might be appropriate in the Irish context, taking into account the existing Applied Mathematics syllabus, as well as the developments brought about by the Project Maths syllabus. The basic structure is summarized in Table I and discussed further below:

	Units	Weighting
Core Mathematics	Matrices and Vectors Further Calculus Ordinary Differential Equations Computational Science with programming Mathematical treatment of Mechanics **Other essential elements, to be established by determining precisely what is 'missing' from the Project Maths syllabus compared to the old Leaving Cert. Syllabus**	75%, 135 hours
Case studies in application areas	Students choose 5 *case studies* from a list. Each case study to comprise 9 hours, for a total of 45 hours.	25%, 45 hours.

TABLE I. Proposed structure on the foot of discussions in UCD

- **Matrices and Vectors:** It is appropriate to re-introduce these two topics to Leaving Certificate students. This is already recommended in the Draft Background Paper and agrees with our recent experiences in teaching Science students in UCD in a first-year module (ACM 10080). In ACM 10080 relevant students in the Science Programme who have not done the current Applied Mathematics course at Leaving Certificate Level (or who have not attained a sufficiently high grade therein) are required to take a 'catch-up' module in elementary Classical Mechanics. Our experience in teaching this module is that the students rapidly develop understanding of the concepts in Mechanics and Kinematics, but struggle with the tools needed to describe these subjects in a mathematical framework, i.e. Matrices and Vectors (especially Vectors). This experience is echoed by colleagues in Engineering and Physics. Spending time on these mathematical techniques at second level would build the students' confidence, increase their skills in doing actual calculations (this being essential part of education in the Physical Sciences and Engineering) and therefore ease the transition to Third Level.

  We have combined together the topics of Matrices and Vectors here: it would seem natural to introduce vectors first as they can be linked in an intuitive way to physically understandable things such as displacement and velocity. On a higher level of abstraction, matrices can then be introduced as linear operators acting on vectors. The more abstract ideas in Vector Algebra can then be introduced in the context of 2×2 matrices (matrix multiplication, determinant, inverse etc), which in turn could pave the way for the introduction of further abstract ideas (e.g. possible inclusion of some Group Theory in the new specification).

- **Further Calculus:** Again, it is appropriate to re-introduce this topic to Leaving Certificate students, as recommended in the Draft Background paper. Again, this agrees with our recent experiences in teaching students in UCD. For example, in a first-year module (ACM 10070), Actuarial Science students (and other Science students) are taught some mathematical modelling (with some applications to Financial Mathematics) based on Differential Equations. The students take their first Calculus module in the following semester (although it is conceivable that the could take a Calculus module concurrently with ACM 10070). Even in a concurrent structure, it is extremely difficult to teach the mathematical modelling without basic notions such as Integration by Parts, Integration by Substitution, Maclaurin Series, graphing continuous functions, etc. Again, the transition to Third Level could be eased by the introduction of further topics in Calculus to the new specification, while presenting the second-level students with some interesting and challenging ideas and mathematical methods.

- **Ordinary Differential Equations:** It is appropriate to retain this aspect of the existing Leaving Certificate Applied Mathematics syllabus (and possibly to expand it), as it underpins third-level Applied Mathematics, Physics, and Engineering, but also because it complements the topics in the proposed Further Calculus strand. Also, it would underpin the proposed case studies described below.

- **Computational Science with Programming:** This suggestion relates to the suggestion of including Computer Programming as a potential 'option' in the proposed new specification. We thought that this idea should be modified. First, Computer Programming is a subject in its own right, and not an addendum to Mathematics. Indeed, because of the current debate about the teaching of programming in secondary schools (and the rise of the 'Coder Dojo' movement), it seems quite likely that Computer Programming might in the future become a subject in its own right in the Leaving Certificate, whereupon its appearance in a small 'option' of the proposed new specification would appear inappropriate and redundant. In order to 'future-proof' the proposed new specification, **we think that the inclusion of computer programming per se is not a good idea. However, Computational Science is a the heart of modern applications of Mathematics, and for that reason, the inclusion of Computational Science in the core of the proposed new specification is appropriate.**

  Here, by Computational Science we mean the formulation and implementation of numerical algorithms to solve equations arising from mathematical modelling in the sciences. Examples include the numerical solution of ordinary differential equations, numerical quadrature, and numerical linear algebra. Algorithms should not however be taught without an eye to their practical implementation and thus, as a part of a training in Computational Science students should be introduced to a programming language (and the basic programming constructs) so as to implement algorithms and present solutions of the associated mathematical models in a graphical way.

  A basic principle in the teaching of Computational Science (and computer programming more generally) should be the **open-source principle.** Open-source software refers to software whose source code is available free of charge, and can be modified at will by end-users, such that the source code can be continuously improved and further functionality added and redistributed. A further positive aspect of this model is that it leads to a vibrant software development community. This is an essential feature of modern Computational Science, and is a philosophy that is fervently followed by many critical pieces of software, from operating systems (e.g. Linux and its derivatives), interpreter-level programming languages (e.g. Python), visualization software (e.g. Paraview), and computational fluid dynamics (CFD) software with full industry-ready capabilities (e.g. OpenFoam), as well as software developed for academic research (e.g Gerris CFD solver).

Open-source software is valuable for all of the reasons listed above, and is a key paradigm in academia and in wider contexts. Introducing Computational Science to second-level students in this context would be a good idea because it would expose the students to cutting-edge practice in software development. Crucially, by avoiding the procurement of expensive proprietary software on a national level, the roll-out of this part of the syllabus would come at minimal cost to the Exchequer and to parents of second-level students, thereby making the proposed new specification even more attractive to second-level students. The same principle could be extended to the hardware side, for example, by using the *Raspberry Pi* computer [9].

- **Mathematical treatment of Mechanics:** It is appropriate to retain a certain amount of Mechanics in the core of the proposed new specification, not only to maintain some continuity between the existing Leaving Certificate Applied Mathematics Syllabus and the proposed replacement subject (thereby gaining some 'buy-in' from the existing Leaving Certificate Applied Mathematics community), but more pertinently because the development of mathematics and mechanics are closely intertwined. Newton devised calculus to solve problems in celestial mechanics - to model the universe! Mechanics also provides a valuable means of applying mathematics to practical problems. It is the most natural and obvious way of motivating and using vectors, and an ideal framework for using differential equations. This gives students a real sense of the usefulness of the mathematics that they are learning.

  Mechanics is also exciting. There are new opportunities all the time for motivating it. Recently, the Rosetta space-probe went into orbit around the comet 67P, and dropped a landing craft onto the surface of the comet. The basic dynamics can be described in simple mechanical terms. the comet and orbiter will continue their journey together. Who knows what will happen?

- **Other essential elements:** We propose to carry out a comprehensive survey of the old Leaving Cert Mathematics syllabus and to determine aspects (other than those mentioned above) that have been effectively 'dropped' from the Project Maths syllabus, with a view possibly into reinstating these to the proposed new specification.

  A possibility is also to introduce some entirely new topics, with suggestions from the following list:

  - Group Theory
  - Discrete Mathematics
  - Graph Theory

  - Elementary Number theory (including for example the Euclidean Algorithm), which is a great example of an "algorithm" and could open doors to other algorithms. The whole concept of an algorithm is something that could be added to modernize the syllabus.

  In deciding what essential topics would be included in a new core syllabus, **a key criterion here would be to select those topics that would feed into the case studies in a natural and unforced way** (e.g. one could imagine learning about Group Theory and Finite Fields, which would then lead into a case study in Coding Theory).

- **Case studies in application areas:** We propose to modify the proposed 'option' structure, for a number of reasons. The first reason is a little cynical, but nonetheless, we place it here first: the mere proposal of a list of 'options' will result in an unseemly squabble by academics, teachers, and other (rightly) interested parties for the inclusion of 'their' option in the new specification. Further along these lines, the inclusion of a finite and static list of options could lead students to the mistaken conclusion that there is a finite list of canonical topical areas comprising the whole of Applied Mathematics. This would be similar to the trap inherent in the current Leaving Certificate structure, wherein students confuse 'Applied Mathematics' with 'Mechanics', and it is difficult to disabuse students of this notion upon arrival at third level. For these reasons (and for others listed below) we propose instead **that 30% of the specification be comprised of a non-exhaustive and evolving list of case studies in application areas, to be determined (and updated) by a consultation with industry, academia, government, and educational stakeholders.**

  The proposed structure is that a number of case studies in application areas would be available to students in a given year. From this list, the students in a particular class would choose 5 case studies, and would spend 9 hours on each case study (for a total of 45 hours, i.e. 30% of the total syllabus). In each case study the students would be presented with a problem from industry or some other academic discipline. The problem would not be contrived, rather it would be a simplified version of a real problem in industry or science, amenable to mathematical modelling. The students would then learn how to translate the industrial 'word problem' into a mathematical model, to solve the model (solution to include parameter estimation) and hence to compare the model with industry-supplied data or experimental data. The students would be assessed on this aspect by a mixture of project work (with computational science at the heart of this project work), together with a final-examination component. More detail of the mode of delivery of the case studies is explained in Section II below, together with a sample case study (background lecture notes, project-type continuous assessment and final-exam-style assessment) delivered in UCD as part of ACM 10070.

  The 'case studies' approach shares a philosophy with the 'study group with industry' movement:

  > Initiated in Oxford in 1968, Study Groups with Industry provide a forum for industrial scientists to work alongside academic mathematicians on problems of direct industrial relevance. They are an internationally recognised method of technology and knowledge transfer between academic mathematicians and industry, usually lasting one week.

> The success of the (now called) ESGI: European Study Groups with Industry, in which problems presented by industry are used as a basis for mathematical research, is demonstrated by the extent to which the unique format has been copied around the world and is now extending into other areas where mathematics may be applied [10].

This approach therefore underscores the connections between Mathematics and industry, and will demonstrate to students the many career opportunities available through Mathematics, and will support the efforts of our people and society to "generate the capable and flexible workforce needed to compete in a global marketplace," as outlined in the Draft Background Paper.

D. Removing the practical constraints that have hindered the takeup of "Applied Mathematics" in the past – a new opportunity?

The draft background paper rightly highlights the unhealthy state of Applied Mathematics as a current Leaving Certificate subject, including the low takeup rate of the subject, and the clear evidence from the pattern of examination questions answered that the whole syllabus is not being covered in schools. Another concern is the gender gap in the subject: the male-female ratio at higher level is 80-20 [11]: mathematical talent is going unspotted and is possibly lost at the early stage of second-level education. A final concern not mentioned in the draft background paper (but related to the low takeup rate) is the lack of dedicated Applied Mathematics teaching in schools: the subject seems to be taught either 'out of hours' or in fee-paying schools as a way for students taking both Physics and higher-level Mathematics 'pick up' some 'extra' CAO points. If implemented properly, the proposed new specification could address all of these issues:

- Introducing a new specification wherein the core (counting for 70% of the syllabus) would contain material familiar to existing second-level Mathematics teachers means that there would be a pool of experienced and knowledgeable teachers able to teach the proposed new specification. Indeed, the new specification should be structured in such a way teachers already accredited by the Teaching Council to teach Mathematics would also be accredited to teach the new subject.

- By retaining mechanics as part of the core of the new specification (and by including additional case studies based on mechanics), the proposed new specification would be teachable by and supported by existing teachers of Applied Mathematics.

The aim here is to devise a new syllabus that is **deliverable in the vast majority of secondary schools**. Furthermore,

- The proposed new specification would be aimed at a wide cohort of students, namely those wishing to pursue a STEM subject at third level. **If the new specification is promoted properly in schools and with parents and guidance councillors, this would create a large demand for the new subject.**

- The new specification would also be supported by third-level institutions, and the quantitative disciplines therein, since these disciplines have struggled in recent years to teach incoming students the mathematics removed from the Leaving Certificate

Mathematics syllabus, **in addition to teaching standard first-year course material appropriate to third level**. For this reason, the new specification would be promoted vigorously by third-level institutions as well.

The aim therefore is to **design and promote the new specification to ensure a wide take-up of the subject in secondary schools**: the new specification should be available to and taken by all secondary-school students who are planning to do a STEM subject at third level.

The draft background paper discusses the idea of offering bonus CAO points for the new specification, much like the bonus CAO points already available for the Mathematics paper. We would add a further cautionary note to the discussion in the draft consultation paper: until the new specification is widely available in secondary schools, and until the gender gap in the uptake of the subject is addressed, the offer of bonus points in the new specification would be strongly discriminatory against female students and students in non-fee-paying schools.

II. CASE STUDIES IN APPLICATION AREAS – MORE DETAIL AND EXAMPLES

In this section we describe some of our experiences in UCD in teaching a first-year module in Applied Mathematics (ACM 10070) based on a core component and on subsequent case studies. This could be a model for the proposed new specification. However, in the following we outline how the new specification could transcend the current model to involve input from industry, academia, and other stakeholders, and to be studied in a *blended learning* framework, with assessment to be carried out possibly by a combination of project work and a final exam.

A. ACM 10070 in UCD – Mathematical Modelling in the Sciences

ACM 10070 is a first-year first-semester module taught to Science and Actuarial Science students in UCD (there is a further cohort of second-year Mathematics and Science education students in the class). The aim of the module is to showcase a range of scientific contexts in which mathematical modelling with differential equations is appropriate. To do this in a self-consistent manner, the students are first of all exposed to a core content involving the following topics:

- First-order linear ODEs, quantitative methods – solution by ansatze, separable ODEs, linear inhomogeneous ODEs
- First-order nonlinear ODEs, quantitative methods – Bernoulli's ODE.
- Second-order linear ODEs, quantitative methods.
- First-order nonlinear ODEs, qualitative methods – fixed points, one-dimensional vector fields, stability of fixed points, bifurcation analysis
- Discrete dynamical systems, quantitative methods: definition of discrete dynamical system, solution of linear difference equations.

- Discrete dynamical systems, qualitative methods: fixed points, stability, bifurcation diagrams, periodic orbits, period doubling. Chaos in the logistic map.
- A brief discussion of the scientific method: the definition of a theory and the difference between a theory and a model. Examining the limitations of mathematical modelling.

Thereafter, the students study a range of case studies, taken from the following (evolving, non-exhaustive list):

- Mechanical Oscillations: Hooke's law. Damping. Solution method for the linear damped unforced oscillator. The damped driven oscillator. Asymptotic solution after the decay of the transience.
- Electrical Circuits: basic circuit elements: resistors, capacitors, inductors. Using Kirchoff's laws to write down an equation for a circuit. Solution methods. Nonlinear circuits: the Van der Pol oscillator and solutions of the associated equation using numerical methods.
- Population dynamics: exponential, logistic, Gompertz models. Analytical solutions. Parameter estimation via least squares. Mathematical biology of population: study of various models (including harvesting), rescaling of models, dimensionless groups, fixed points, bifurcation analysis.
- Epidemic modelling: SIR model, reduced Kermack and McKendrick model. The threshold parameter R_0. Numerical solution of epidemic modelling and parametric study investigating the effect of variations in R_0. 'Epidemics' versus 'Endemics'. Further models that support endemic diseases. Sensitivity analysis examining the effect of the model parameters on the equilibrium population in an endemic scenario. Modelling fictional on-screen zombie outbreaks using SIR models (!!)
- Drug delivery: the notion of drug half life. Calculating the drug concentration in the body as a function of time given a schedule of repeated doses. Calculating the correct drug dose as a function of upper and lower concentration limits. Repeating the procedure for different models of drug decay and quantifying the sensitivity of the whole approach to the particular decay model used.
- Financial Mathematics: Continuous versus discrete compounding of interest and the connection to ODEs. Annuity mortgages. 'Exotic' mortgages. Interest-only mortgages as a proxy for the fundamental value of an asset. The risk-free rate of interest. European call and put options. Pricing European options using a one-step binomial tree model. The notion of a risk-neutral probability. The notion of arbitrage. Using the no-arbitrage assumption to derive *a priori* bounds on the price of the option.
- Discrete Mathematical Biology: iterative maps applied to problems in Mathematical Biology, numerical investigation of chaos, study of relevant accessible scholarly literature on the subject

ACM 10070 is assessed by a means of continuous assessment and final exam. For the continuous assessment, the students are given 10 assignments throughout the 12-week semester. The students submit the even-numbered assignments and receive credit for these. These assignments contain a mixture of procedural exercises and more challenging aspects that comprise complex calculations involving a number of interconnected steps, and some computational work. The odd-numbered assignments are used by the students as ‘extra practice’ and are designed again to be challenging and to involve complex calculations and computational work. Additionally, a strong emphasis is placed in the module on *tutorials*, and a graduate teaching assistant works with small groups (20-25) on the odd-numbered assignments. In the final examination, the questions are based on case studies and are similar to the work done in the class (but sufficiently different to challenge the students). In the Annexe to this document we give an example of a case study from ACM 10070 together with continuous assessment and a sample question from the final exam, to demonstrate the feasibility of this approach.

B. Industrial and Scientific Case Studies: a vision for second level

The Core+(Case Studies) structure proposed by the UCD discussion group aims to be far more radical than the approach described above in relation to ACM 10070, in the following ways:

- A set of case studies will be prescribed for students at the beginning of their two-year period of study. The list will not be definitive or static: future students will be presented with a different list. This will highlight to students the dynamic and evolving nature of Applied Mathematics. Crucially, it will remove some predictability from the exam!
- The case studies will be selected in consultation with industrial and academic partners, with industry ‘buy-in’ and support to be encouraged. For example, in a particular Leaving Certificate session, pharmaceutical company X might present a case study in drug delivery, and electronics company Y might present a case study in electronics and electrical circuits.
- The written material for the case studies will be developed in partnership with academics, industry and the NCCA. To support teachers in schools, all materials will be made available on a website which will act as a portal for the new specification and a showcase more generally for Mathematics and its applications. A ‘blended learning’ approach to the teaching of the case studies might be appropriate where academics and industrial R&D partners develop video lectures as part of the programme, to be made available on the subject website. The video series could be studied in class under guidance from the teacher, or used as supplementary material to aid the students’ study.

This approach might seem radical, and the transition to this new approach might be difficult for existing teachers. However, by delivering the case studies in partnership with academia and industry, using a blended-learning approach, the number of new duties placed on teachers would be minimized. Some upskilling of existing teachers might however still be required (similar to the current Professional Diploma in Mathematics for Teaching). This could be done in partnership with third-level institutions: given the pressing need for reform of the Applied Mathematics syllabus, it is likely that NCCA will experience significant goodwill should such an upskilling programme be necessary, and the School of Mathematical Sciences in UCD would be very happy to contribute here.

III. CONCLUSIONS

In conclusion, we are pleased with the approach taken in the NCCA review document, although we propose a slightly different Core+(Case Studies) structure, which would have the following additional benefits:

- Strengthen the students' skills in core mathematics
- Present the application of Mathematics to industrial and scientific problems in a non-contrived way. The same approach would underscore the dynamism and 'open' nature of the dialogue between Mathematics and societal problems.
- Address the mathematical needs of industry, and maintain an appropriate interaction between industry and education.
- Reduce the predictability of the examination, while still making the subject attractive for students interested in a STEM career
- Minimize the number of new burdens to be placed on existing teachers of Mathematics and Applied Mathematics.

Moreover, this approach is **feasible**, as our own evidence from teaching in UCD suggests (e.g. ACM 10070) although the proposed new specification should be more ambitious in the use of technology (especially open-source Computational Science), assessment, and the use of a variety of different appropriate modes of learning.

---

[1] Maria Meehan and Lennon Ó Náraigh. "Background to the proposed guidelines for the teaching of Applied Mathematics", 2012. Discussion paper, part of UCD's (successful) application for Teaching Council approval for the programme "Mathematics and Applied Mathematics Education".

[2] DAMTP website. `http://www.damtp.cam.ac.uk/about/history.html`. Accessed: 08-12-2014.

[3] Survey of websites of Imperial College London (UK), Brown University (RI, USA), UCLA (CA, USA), and EPFL, Switzerland.

[4] Education bureau of Hong Kong website. `http://www.edb.gov.hk/index.aspx?nodeID=4901&langno=1`. Accessed: 15-12-2014.

[5] California state board of education website. `http://www.cde.ca.gov/be/st/ss/`. Accessed: 15-12-2014.

[6] National Council of Teachers of Mathematics website (USA). `http://www.nctm.org/resources/content.aspx?id=1778`. Accessed: 15-12-2014.

[7] Mathematics department website, wichita state university (ka, usa). `http://kirk.math.twsu.edu/appliedmath.html`. Accessed: 16-05-2012.

[8] CC Lin. Education of applied mathematicians. *SIAM Review*, 20(4):838–845, 1978.

[9] Website of the raspberry pi computer. `http://www.raspberrypi.org/`. Accessed: 10-05-2012.

[10] University of Oxford, Mathematics in Industry website. `http://miis.maths.ox.ac.uk/how/`. Accessed: 12-12-2014.

[11] Chief examiner's reports, applied mathematics, 2012,2013.