University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 2 EXAMINATION 2013/2014

ADEDEX428

Mathematics for Engineering

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Time Allowed: 3 hours

Instructions for Candidates

Candidates should attempt all questions.
Note that not all questions are allocated the same number of marks.

Notes for Invigilators

Non programmable calculators are permitted.
The statistical tables provided are permitted.
The formula sheet provided is permitted.

1. (i) (a) Determine if the following matrix operations can be performed and if so perform them.

$$3\begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix},$$

$$2\begin{pmatrix} 1 & -2 & 0 \\ -2 & -3 & 6 \end{pmatrix} - 3\begin{pmatrix} 2 & 4 & -1 \\ 3 & -3 & 2 \end{pmatrix}$$

$$\text{and} \quad 3\begin{pmatrix} -3 & 1 \\ 2 & -1 \\ -1 & 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}.$$

[6]

(b) Find the angle (in radians to 2 decimal places) between the vectors $(1, 2)$ and $(4, 1)$. **[2]**

(ii) (a) Determine if the following matrix operations can be performed and if so perform them.

$$\begin{pmatrix} 1 & -2 \\ -2 & -3 \end{pmatrix}\begin{pmatrix} -1 & 3 & 2 \\ 2 & -4 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 & 0 & 9 \end{pmatrix}^T \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}.$$

[6]

(b) Find the determinant of $\begin{pmatrix} 1 & -2 & 6 \\ -4 & 9 & -23 \\ -1 & 2 & -5 \end{pmatrix}$. **[4]**

(iii) (a) Using row reduction, determine if the following system of linear equations has a solution and give the solution if it has.

$$\begin{aligned} 3x + 5y - 12z &= 4 \\ x + y &= 2 \\ 2x + 3y - 4z &= 5 \end{aligned}$$

[6]

(b) Find the eigenvalues and corresponding eigenvectors of the matrix $\begin{pmatrix} 3 & 6 \\ -2 & -4 \end{pmatrix}$.

[6]

2. (i) For $z = 1 + 3i$ and $w = 2 - i$, calculate
$|z|$, $\overline{z}$, $\mathrm{Re}(z)$, $\mathrm{Im}(z)$, $z + w$, $z - w$, zw and $\dfrac{z}{w}$. **[5]**

(ii) Convert $-1 + i$ into polar form and hence calculate $(-1 + i)^6$, expressing your final answer both in exact polar form and in Cartesian form correct to three decimal places. **[5]**

(iii) Given that $3\sqrt{3} - 3i = 6\left(\cos\left(-\dfrac{\pi}{6}\right) + i\sin\left(-\dfrac{\pi}{6}\right)\right)$, calculate the five fifth roots of $3\sqrt{3} - 3i$, leaving your answers in polar form. **[5]**

3. (i) Differentiate the functions

$$f(x) = 3x^{-5} - 2x^{\frac{5}{3}},$$
$$g(x) = 3\sin(2x) - \cos(-4x)$$
$$\text{and} \quad h(x) = \ln\left(\frac{5}{3}x\right) - 2e^{\frac{4}{3}x}.$$

[6]

(ii) (a) Starting with the initial guess $x_0 = 3$, apply two iterations of the Newton-Raphson method to obtain an approximate solution of the equation

$$x^3 - 7x^2 + 6x + 10 = 0.$$

[4]

(b) Find the points where the global maximum and minimum of the function

$$f\colon [1, 6] \to \mathbb{R}$$
$$x \mapsto x^3 - 9x^2 + 24x - 15$$

occur. **[4]**

(iii) Differentiate the functions

$$f(x) = \frac{x^3\sin(2x)}{e^{-x}\cos(x)}$$
$$\text{and} \quad g(x) = \sin(x^4 + 2x^2 + 2).$$

[6]

4. (i) (a) Find

$$\int 3x^{-7} - 2x^{\frac{2}{3}}\,dx$$
$$\text{and} \int -2\cos(-2x) + \sin(3x)\,dx.$$

[4]

(b) Evaluate

$$\int_1^2 \frac{3}{x} - 3e^{2x}\,dx.$$

[2]

(ii) (a) Find the area lying between the graph of $f(x) = x^3 - 3x^2 - 10x + 24$ and the x-axis between the points $x = 1$ and $x = 3$. Hint: The graph of this function only crosses the x-axis at $x = 2$ in the interval $[1, 3]$. **[3]**

(b) Find the volume of revolution of the function $f(x) = \sqrt{x + 2x^3}$ about the x-axis between $x = 0$ and $x = 2$. **[3]**

(iii) (a) Find

$$\int 3xe^{-2x}\,dx.$$

[4]

(b) Evaluate

$$\int_{-1}^{1} x^2(3x^3 + 5)^5\,dx.$$

[4]

5. (i) (a) In a hospital wing, the medical staff consists of five doctors and eleven nurses. Two of the doctors are male and seven of the nurses are female. What is the probability of a staff member picked at random being either a nurse or a male? **[3]**

(b) A physics lecturer gave his class two tests. Eleven percent of the class failed the first test and five percent of the class failed both tests. If we pick a student who failed the first test at random, then what is the probability that they also failed the second test? **[3]**

(ii) The average number of cyclists arriving at UCD between 9am and 10am on Mondays is 5 per minute. What is the probability that at least 11 cyclists arrive at UCD between 9am and 9.03am next Monday? **[4]**

(iii) Suppose that the heights of adult females in Ireland are normally distributed with mean 171cm and standard deviation 10cm. What is the probability of an adult female chosen at random in Ireland being both taller than 169cm and shorter than 184cm? **[5]**

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