Access to Engineering - Mathematics 2
ADEDEX428
Semester 2 2013-2014 Exam Solutions

1. (i) (a)

$$3\begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 6 & -12 \end{pmatrix}.$$

$$2\begin{pmatrix} 1 & -2 & 0 \\ -2 & -3 & 6 \end{pmatrix} - 3\begin{pmatrix} 2 & 4 & -1 \\ 3 & -3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -4 & 0 \\ -4 & -6 & 12 \end{pmatrix} + \begin{pmatrix} -6 & -12 & 3 \\ -9 & 9 & -6 \end{pmatrix}$$
$$= \begin{pmatrix} -4 & -16 & 3 \\ -13 & 3 & 6 \end{pmatrix}.$$

$$3\begin{pmatrix} -3 & 1 \\ 2 & -1 \\ -1 & 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \text{ can't be formed.}$$

(b) Using the formula $\theta = \cos^{-1}\left(\frac{\mathbf{x}\cdot\mathbf{y}}{\|\mathbf{x}\|\cdot\|\mathbf{y}\|}\right)$, where θ is the angle between the vectors $\mathbf{x}$ and $\mathbf{y}$, we have that the angle between $(1,2)$ and $(4,1)$ is

$$\begin{aligned}
\theta &= \cos^{-1}\left(\frac{(1,2)\cdot(4,1)}{\|(1,2)\|\cdot\|(4,1)\|}\right) \\
&= \cos^{-1}\left(\frac{(1)(4)+(2)(1)}{\sqrt{1^2+2^2}\cdot\sqrt{4^2+1^2}}\right) \\
&= \cos^{-1}\left(\frac{6}{\sqrt{5}\cdot\sqrt{17}}\right) \\
&= \cos^{-1}\left(\frac{6}{\sqrt{85}}\right) \\
&\simeq 0.86 \text{ to 2 d.p.}
\end{aligned}$$

(ii) (a)

$$\begin{pmatrix} 1 & -2 \\ -2 & -3 \end{pmatrix}\begin{pmatrix} -1 & 3 & 2 \\ 2 & -4 & 0 \end{pmatrix} = \begin{pmatrix} -5 & 11 & 2 \\ -4 & 6 & -4 \end{pmatrix}.$$

$$\begin{pmatrix} -1 & 0 & 9 \end{pmatrix}^T \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix} \text{ can't be formed.}$$

(b)

$$\det\begin{pmatrix} 1 & -2 & 6 \\ -4 & 9 & -23 \\ -1 & 2 & -5 \end{pmatrix} = 1\begin{vmatrix} 9 & -23 \\ 2 & -5 \end{vmatrix} - (-2)\begin{vmatrix} -4 & -23 \\ -1 & -5 \end{vmatrix} + 6\begin{vmatrix} -4 & 9 \\ -1 & 2 \end{vmatrix}$$

$$\begin{aligned} &= 1(9 \times (-5) - (-23) \times 2) \\ &\quad + 2(-4 \times (-5) - (-23) \times (-1)) \\ &\quad + 6(-4 \times 2 - 9 \times (-1)) \\ &= 1(1) + 2(-3) + 6(1) \\ &= 1 - 6 + 6 \\ &= 1. \end{aligned}$$

(iii) (a) We will row reduce the augmented matrix $\begin{pmatrix} 3 & 5 & -12 & 4 \\ 1 & 1 & 0 & 2 \\ 2 & 3 & -4 & 5 \end{pmatrix}$

$$R1 \leftrightarrow R2 \quad \begin{pmatrix} 1 & 1 & 0 & 2 \\ 3 & 5 & -12 & 4 \\ 2 & 3 & -4 & 5 \end{pmatrix}$$

$$\begin{aligned} R2 &\rightarrow R2 - 3R1 \\ R3 &\rightarrow R3 - 2R1 \end{aligned} \quad \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 2 & -12 & -2 \\ 0 & 1 & -4 & 1 \end{pmatrix}$$

$$R2 \rightarrow \tfrac{1}{2}R2 \quad \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & -6 & -1 \\ 0 & 1 & -4 & 1 \end{pmatrix}$$

$$R3 \rightarrow R3 - R2 \quad \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & -6 & -1 \\ 0 & 0 & 2 & 2 \end{pmatrix}$$

$$R3 \rightarrow \tfrac{1}{2}R3 \quad \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & -6 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$R2 \rightarrow R2 + 6R3 \quad \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$R1 \rightarrow R1 - R2 \quad \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Hence the solution is $x = -3$, $y = 5$, $z = 1$.

(b) We have

$$\det\left[\begin{pmatrix} 3 & 6 \\ -2 & -4 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right] = \det\begin{pmatrix} 3-\lambda & 6 \\ -2 & -4-\lambda \end{pmatrix}$$
$$= (3-\lambda)(-4-\lambda) - 6(-2)$$
$$= \lambda^2 + \lambda - 12 + 12$$
$$= \lambda^2 + \lambda$$

Hence the characteristic equation is $\lambda^2 + \lambda = 0$ or $\lambda(\lambda + 1) = 0$.
Thus the eigenvalues are $\lambda = 0$ and $\lambda = -1$.
We will now find the eigenvectors corresponding to these eigenvalues.

$\lambda = 0$:
We have $\begin{pmatrix} 3 & 6 \\ -2 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0\begin{pmatrix} x \\ y \end{pmatrix}$.
Thus $\begin{pmatrix} 3x+6y \\ -2x-4y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.
Hence we have the equations $3x + 6y = 0$ and $-2x - 4y = 0$. Both these equations reduce to $x = -2y$, so taking $y = 1$, say, we obtain the eigenvector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

$\lambda = -1$:
We have $\begin{pmatrix} 3 & 6 \\ -2 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -1\begin{pmatrix} x \\ y \end{pmatrix}$.
Thus $\begin{pmatrix} 3x+6y \\ -2x-4y \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}$.
Hence we have the equations $3x + 6y = -x$ and $-2x - 4y = -y$. Both these equations reduce to $2x = -3y$, so taking $y = 2$, say, we obtain the eigenvector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$.

2. (i)

$$|z| = |1+3i| = \sqrt{1^2+3^2} = \sqrt{1+9} = \sqrt{10}, \quad \overline{z} = \overline{1+3i} = 1-3i,$$
$$\mathrm{Re}(z) = \mathrm{Re}(1+3i) = 1, \quad \mathrm{Im}(z) = \mathrm{Im}(1+3i) = 3.$$

$$z + w = (1+3i) + (2-i) = (1+2) + (3-1)i = 3 + 2i$$
$$z - w = (1+3i) - (2-i) = (1-2) + (3-(-1))i = -1 + 4i$$
$$zw = (1+3i)(2-i) = ((1)(2) - (3)(-1)) + ((1)(-1) + (3)(2))i = 5 + 5i$$
$$\frac{z}{w} = \frac{1+3i}{2-i} = \frac{1+3i}{2-i}\cdot\frac{2+i}{2+i} = \frac{-1+7i}{5} = -\frac{1}{5} + \frac{7}{5}i$$

(ii) The real part of $-1 + i$ is negative and its imaginary part is positive, so we are in the situation of Figure 6 in the Complex Numbers notes.

Hence the argument of $-1+i$ is

$$\theta = \pi - \phi = \pi - \tan^{-1}\left(\left|\frac{1}{-1}\right|\right) = \pi - \tan^{-1}(1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}.$$

Also, the magnitude of $-1+i$ is $r = \sqrt{(-1)^2+1^2} = \sqrt{1+1} = \sqrt{2}$.
Hence, $-1+i$ in polar form is

$$-1+i = \sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right).$$

To calculate $(-1+i)^6$ we will use Corollary 2.3.9 from the Complex Numbers notes. That is we will use

$$(r(\cos(n\theta) + i\sin(n\theta)))^n = r^n(\cos(n\theta) + i\sin(n\theta)),$$

with $r = \sqrt{2}$, $\theta = \dfrac{3\pi}{4}$ and $n = 6$.
Hence

$$\begin{aligned}
(-1+i)^6 &= \left(\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)\right)^6 \\
&= (\sqrt{2})^6\left(\cos\left(\frac{18\pi}{4}\right) + i\sin\left(\frac{18\pi}{4}\right)\right) \\
&= 8\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right) \\
&= 8(0+i) \\
&= 8i.
\end{aligned}$$

(iii) We will use the fact (see P.12 of the Complex Numbers notes) that the nth roots are given by

$$z_k = r^{\frac{1}{n}}\left(\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)\right) \qquad k = 0, 1, \ldots, n-1.$$

In this case we have $r = 6$ and $\theta = -\dfrac{\pi}{6}$ and we are looking for the fifth roots, so we take $n = 5$.
Thus the roots are

$$z_k = 6^{\frac{1}{5}}\left(\cos\left(\frac{-\pi/6}{5} + \frac{2k\pi}{5}\right) + i\sin\left(\frac{-\pi/6}{5} + \frac{2k\pi}{5}\right)\right) \qquad k = 0, 1, 2, 3, 4.$$

That is

$$\begin{aligned}
z_0 &= 6^{\frac{1}{5}}\left(\cos\left(-\frac{\pi}{30}\right) + i\sin\left(-\frac{\pi}{30}\right)\right) \\
z_1 &= 6^{\frac{1}{5}}\left(\cos\left(\frac{11\pi}{30}\right) + i\sin\left(\frac{11\pi}{30}\right)\right) \\
z_2 &= 6^{\frac{1}{5}}\left(\cos\left(\frac{23\pi}{30}\right) + i\sin\left(\frac{23\pi}{30}\right)\right) \\
z_3 &= 6^{\frac{1}{5}}\left(\cos\left(\frac{35\pi}{30}\right) + i\sin\left(\frac{35\pi}{30}\right)\right) \\
z_4 &= 6^{\frac{1}{5}}\left(\cos\left(\frac{47\pi}{30}\right) + i\sin\left(\frac{47\pi}{30}\right)\right)
\end{aligned}$$

3. (i) For $f(x) = 3x^{-5} - 2x^{\frac{5}{3}}$ we have to use the sum and multiple rule, as well as the derivative of x^n with $n = -5$ and with $n = \frac{5}{3}$.

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(3x^{-5}\right) + \frac{d}{dx}\left(-2x^{\frac{5}{3}}\right) \\ &= 3\frac{d}{dx}\left(x^{-5}\right) - 2\frac{d}{dx}\left(x^{\frac{5}{3}}\right) \\ &= 3\left(-5x^{-6}\right) - 2\left(\frac{5}{3}x^{\frac{2}{3}}\right) \\ &= -15x^{-6} - \frac{10}{3}x^{\frac{2}{3}}. \end{aligned}$$

For $g(x) = 3\sin(2x) - \cos(-4x)$ we use the derivatives of $\sin(ax)$ and $\cos(ax)$ with $a = 2$ and $a = -4$ together with the sum and multiple rules.

$$g'(x) = 3(2\cos(2x)) - (-(-4)\sin(-4x)) = 6\cos(2x) - 4\sin(-4x).$$

For $h(x) = \ln\left(\frac{5}{3}x\right) - 2e^{\frac{4}{3}x}$ we use the derivatives of $\ln(ax)$ with $a = \frac{5}{3}$ and e^{ax} with $a = \frac{4}{3}$ together with the sum and multiple rules.

$$h'(x) = \frac{1}{x} - 2\cdot\frac{4}{3}e^{\frac{4}{3}x} = \frac{1}{x} - \frac{8}{3}e^{\frac{4}{3}x}.$$

(ii) (a) Since $f'(x) = x^3 - 7x^2 + 6x + 10$, using the formula $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$, we have $x_{n+1} = x_n - \dfrac{x^3 - 7x^2 + 6x + 10}{3x^2 - 14x + 6}$.
Since $x_0 = 3$,

$$x_1 = x_0 - \frac{x_0^3 - 7x_0^2 + 6x_0 + 10}{3x_0^2 - 14x_0 + 6} = 3 - \frac{3^3 - 7(3^2) + 6(3) + 10}{3(3^2) - 14(3) + 6} = \frac{19}{9}.$$

Then

$$\begin{aligned} x_2 &= x_1 - \frac{x_1^3 - 7x_1^2 + 6x_1 + 10}{3x_1^2 - 14x_1 + 6} \\ &= \frac{19}{9} - \frac{\left(\frac{19}{9}\right)^3 - 7\left(\frac{19}{9}\right)^2 + 6\left(\frac{19}{9}\right) + 10}{3\left(\frac{19}{9}\right)^2 - 14\left(\frac{19}{9}\right) + 6} \\ &= \frac{3263}{1485}. \end{aligned}$$

(b) We first have to find the critical points of f. To do this we will differentiate f and solve the equation $f'(x) = 0$.
However $f'(x) = 3x^2 - 18x + 24$, so we solve the equation $3x^2 - 18x + 24 = 0$.
Now

$$\begin{aligned} 3x^2 - 18x + 24 = 0 &\Leftrightarrow x^2 - 6x + 8 = 0 \\ &\Leftrightarrow (x-4)(x-2) = 0 \\ &\Leftrightarrow x = 2 \text{ or } x = 4. \end{aligned}$$

Thus the critical points are $x = 2$ and $x = 4$.
We can now find where the global maxima and minima of f occur by evaluating it at the endpoints of the domain and at critical points (since these both lie in the domain).
So we evaluate $f(x)$ at $x = 1, 2, 4, 6$.
$f(1) = 1$, $f(2) = 5$, $f(4) = 1$ and $f(6) = 21$.
Hence the global maximum of f is 21 attained at $x = 6$ and the global minimum of f is 1 attained at $x = 1$ and $x = 4$.

(iii) With $f(x) = \dfrac{x^3 \sin(2x)}{e^{-x}\cos(x)}$ we have a product in the numerator and the denominator, so we have to use the product rule twice before we use the quotient rule.
First let us differentiate $g(x) = x^3 \sin(2x)$.

$$\begin{aligned} g'(x) &= \frac{d}{dx}\left(x^3\right)\sin(2x) + x^3\frac{d}{dx}\left(\sin(2x)\right) \\ &= 3x^2\sin(2x) + x^3\left(2\cos(2x)\right) \\ &= 3x^2\sin(2x) + 2x^3\cos(2x). \end{aligned}$$

Next we differentiate $h(x) = e^{-x}\cos(x)$.

$$\begin{aligned} h'(x) &= \frac{d}{dx}\left(e^{-x}\right)\cos(x) + e^{-x}\frac{d}{dx}\left(\cos(x)\right) \\ &= -e^{-x}\cos(x) + e^{-x}(-\sin(x)) \\ &= -e^{-x}(\cos(x) + \sin(x)). \end{aligned}$$

We can now use the quotient rule.

$$\begin{aligned} f'(x) &= \frac{g'(x)h(x) - g(x)h'(x)}{h(x)^2} \\ &= \frac{\left(3x^2\sin(2x) + 2x^3\cos(2x)\right)e^{-x}\cos(x) - x^3\sin(2x)(-e^{-x}(\cos(x) + \sin(x)))}{\left(e^{-x}\cos(x)\right)^2}. \end{aligned}$$

With $g(x) = \sin(x^4 + 2x^2 + 2)$ we will use the chain rule with $u = x^4 + 2x^2 + 2$ and $y = \sin(u)$ (where we are letting $y = g(x)$).
Then

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \cos(u)\left(4x^3 + 4x\right) = \cos\left(x^4 + 2x^2 + 2\right)\left(4x^3 + 4x\right).$$

4. (i) (a) For $\int 3x^{-7} - 2x^{\frac{2}{3}}\,dx$, we have to use the sum and multiple rules as well as the integral of x^n with $n = -7$ and $n = \dfrac{2}{3}$.

$$\begin{aligned} \int 3x^{-7} - 2x^{\frac{2}{3}}\,dx &= \int 3x^{-7}\,dx + \int -2x^{\frac{2}{3}}\,dx \\ &= 3\int x^{-7}\,dx - 2\int x^{\frac{2}{3}}\,dx \\ &= 3\cdot\frac{1}{-6}x^{-6} - 2\cdot\frac{1}{5/3}x^{\frac{5}{3}} + c \\ &= -\frac{1}{2}x^{-6} - \frac{6}{5}x^{\frac{5}{3}} + c. \end{aligned}$$

For $\int -2\cos(-2x)+\sin(3x)\,dx$, we have to use the sum and multiple rules, but this time with the integral of $\cos(ax)$ and the integral of $\sin(ax)$, where we take $a=-2$ and $a=3$ respectively.

$$\begin{aligned}\int -2\cos(-2x)+\sin(3x)\,dx &= \int -2\cos(-2x)\,dx + \int \sin(3x)\,dx \\ &= -2\int \cos(-2x)\,dx + \int \sin(3x)\,dx \\ &= -2\cdot\frac{1}{-2}\sin(-2x) + \left(\frac{1}{3}\right)(-\cos(3x)) + c \\ &= \sin(-2x) - \frac{1}{3}\cos(3x) + c.\end{aligned}$$

(b)

$$\begin{aligned}\int_1^2 \frac{3}{x} - 3e^{2x}\,dx &= \left[3\ln(x) - \frac{3}{2}e^{2x}\right]_1^2 \\ &= 3\ln(2) - \frac{3}{2}e^4 - \left(3\ln(1) - \frac{3}{2}e^2\right) \\ &= 3\ln(2) + \frac{3}{2}\left(e^2 - e^4\right)\end{aligned}$$

(ii) (a) Since $f(1)=12$ and the graph of $f(x)=x^3-3x^2-10x+24$ only crosses the x-axis at $x=2$ between the points $x=1$ and $x=3$, it follows that the graph of $f(x)=x^3-3x^2-10x+24$ lies above the x-axis between the points $x=1$ and $x=2$ and below the x-axis between the points $x=2$ and $x=3$.
Hence the required area is

$$\begin{aligned}&\int_1^2 x^3-3x^2-10x+24\,dx - \int_2^3 x^3-3x^2-10x+24\,dx \\ &= \left[\frac{1}{4}x^4 - x^3 - 5x^2 + 24x\right]_1^2 - \left[\frac{1}{4}x^4 - x^3 - 5x^2 + 24x\right]_2^3 \\ &= \left[\left(\frac{1}{4}\cdot 2^4 - 2^3 - 5\cdot 2^2 + 24(2)\right) - \left(\frac{1}{4}\cdot 1^4 - 1^3 - 5\cdot 1^2 + 24(1)\right)\right] \\ &\quad - \left[\left(\frac{1}{4}\cdot 3^4 - 3^3 - 5\cdot 3^2 + 24(3)\right) - \left(\frac{1}{4}\cdot 2^4 - 2^3 - 5\cdot 2^2 + 24(2)\right)\right] \\ &= \left[24 - \frac{73}{4}\right] - \left[\frac{81}{4} - 24\right] \\ &= \frac{19}{2}.\end{aligned}$$

(b) Using the formula $V = \pi \int_a^b f(x)^2 \, dx$, the volume is

$$\begin{aligned} V &= \pi \int_0^2 \left(\sqrt{x+2x^3}\right)^2 \, dx \\ &= \pi \int_0^2 x + 2x^3 \, dx \\ &= \pi \left[\frac{1}{2}x^2 + \frac{1}{2}x^4\right]_0^2 \\ &= \pi \left[\left(\frac{1}{2}2^2 + \frac{1}{2}2^4\right) - 0\right] \\ &= 10\pi. \end{aligned}$$

(iii) (a) Here we use integration by parts.
Let $f(x) = 3x$ and $g'(x) = e^{-2x}$, so that $f'(x) = 3$ and $g(x) = -\frac{1}{2}e^{-2x}$.
Hence, using the integration by parts formula,

$$\int 3xe^{-2x} \, dx = 3x\left(-\frac{1}{2}e^{-2x}\right) - \int -\frac{3}{2}e^{-2x} \, dx = -\frac{3}{2}xe^{-2x} - \frac{3}{4}e^{-2x} + c.$$

(b) Here we use integration by substitution.
Let $u = 3x^3 + 5$, so that so that $\frac{du}{dx} = 9x^2$.
Then $dx = \frac{du}{du/dx} = \frac{du}{9x^2}$.
Also, when $x = -1$, $u = 2$ and when $x = 1$, $u = 8$.
Hence

$$\begin{aligned} \int_{-1}^1 x^2(3x^3+5)^5 \, dx &= \int_2^8 x^2u^5 \cdot \frac{du}{9x^2} \\ &= \int_2^8 \frac{1}{9}u^5 \, du \\ &= \left[\frac{1}{54}u^6\right]_2^8 \\ &= \frac{1}{54}(8^6 - 2^6) \\ &= \frac{14560}{3} \end{aligned}$$

5. (i) (a) Let A be the event 'the staff member is a nurse' and let B be the event 'the staff member is male', so that $P(A \cup B)$ is the probability that the staff member is either a nurse or is male. Since there are 11 nurses out of a total of 16 staff members, $P(A) = \frac{11}{16}$. Also there are 2 male doctors and 4 male nurses, so $P(B) = \frac{6}{16}$. Again, since there are 4 male nurses, $P(A \cap B) = \frac{4}{16}$.

Hence we have that the probability that the staff member selected is either a nurse or is male is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{11}{16} + \frac{6}{16} - \frac{4}{16} = \frac{13}{16}.$$

(b) Let A be the event that a student fails the second test and let B be the event that a student fails the first test. Now, the probability we want is $P(A|B)$ and we are given in the question that $P(B) = 0.11$ and that $P(A \cap B) = 0.05$.
Hence if we pick a student who passed the first test at random, then the probability that they also passed the second test is

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.05}{0.11} = \frac{5}{11}.$$

(ii) Here we take $\lambda = 15$, where λ is the average number of cyclists arriving in the three minutes between 9am and 9.03am on Monday. We have to calculate $P(X \geqslant 11)$, where X is the number of cyclists arriving in the given three minutes.
Now $P(X \geqslant 11) = 1 - P(X \leqslant 10)$ and to find $P(X \leqslant 10)$, we look at the $x = 10$ row in the $\lambda = 15$ column in the tables. Hence

$$P(X \geqslant 11) = 1 - P(X \leqslant 10) = 1 - 0.1185 = 0.8815.$$

(iii) Suppose that X is a normally distributed random variable with mean 171 and standard deviation 10. Then we want to find $P(169 \leqslant X \leqslant 184)$. Now $\mu = 171$ and $\sigma = 10$, so that $\dfrac{169-\mu}{\sigma} = \dfrac{169-171}{10} = -0.2$ and $\dfrac{184-\mu}{\sigma} = \dfrac{184-171}{10} = 1.3$.
Hence, $P(169 \leqslant X \leqslant 184) = P(-0.2 \leqslant Z \leqslant 1.3)$.
However $P(-0.2 \leqslant Z \leqslant 1.3) = P(Z \leqslant 1.3) - P(Z \leqslant -0.2)$ and using the tables we have that $P(Z \leqslant -0.2) = 0.4207$ and $P(Z \leqslant 1.3) = 0.9032$. Thus the probability of a woman chosen at random in Ireland being both taller than 169cm and shorter than 184cm is

$$P(-0.2 \leqslant Z \leqslant 1.3) = P(Z \leqslant 1.3) - P(Z \leqslant -0.2) = 0.9032 - 0.4207 = 0.4825.$$