Access to Science, Engineering and Agriculture: Mathematics 2 MATH00040 Assignment 3 Due Date: By 7.30pm on Wednesday 27/3/19

Show all your workings - part of overall mark

1. Find the derivatives of each of the following functions.
   (a) $f(x) = 4x^3e^{-x} + \cos(-2x)\ln(-5x)$ (where $x < 0$).
   (b) $f(x) = x^{-\frac{1}{2}}\cos(2x) - x^3\sin(-4x)$
   (c) $f(x) = \dfrac{\sin(2x)}{\cos(3x)}$ (where $\cos(3x) \neq 0$).
   (d) $f(x) = \dfrac{x^3}{\ln(3x)}$ (where $x > \frac{1}{3}$).
   (e) $f(x) = \dfrac{x^2\cos(3x)}{e^{2x}\sin(x)}$ (where $\sin(x) \neq 0$).
   (f) $f(x) = \dfrac{e^x(x^3 - 2x^2 + 5)}{\cos(-4x)}$ (where $\cos(-4x) \neq 0$).
   (g) $f(x) = \cos(x^3 - 2x + 5)$
   (h) $f(x) = \sin(\sin(\sin(x)))$
   (i) $f(x) = e^{xe^x}$

2. Find all the critical points of each of the following functions.
   (a) $f(x) = x^3 + 3x^2 - 24x - 7$
   (b) $f(x) = -x^3 + 9x^2 - 24x + 10$
   (c) $f(x) = e^{-4x} + 5x$
   (d) $f(x) = \sin(2x)$

3. Find the points where the global maxima and minima of each of the following functions occur.
   (a)
   $$\begin{aligned} f\colon [0,6] &\to \mathbb{R} \\ x &\mapsto -x^3 + 9x^2 - 24x + 10 \end{aligned}$$
   (b)
   $$\begin{aligned} f\colon [1,6] &\to \mathbb{R} \\ x &\mapsto -x^3 + 9x^2 - 24x + 10 \end{aligned}$$

(c)

$$\begin{aligned} f\colon [0,5] &\to \mathbb{R} \\ x &\mapsto -x^3 + 9x^2 - 24x + 10 \end{aligned}$$

(d)

$$\begin{aligned} f\colon [2,4] &\to \mathbb{R} \\ x &\mapsto -x^3 + 9x^2 - 24x + 10 \end{aligned}$$

(e)

$$\begin{aligned} f\colon [2,3] &\to \mathbb{R} \\ x &\mapsto -x^3 + 9x^2 - 24x + 10 \end{aligned}$$

(f)

$$\begin{aligned} f\colon [3,4] &\to \mathbb{R} \\ x &\mapsto -x^3 + 9x^2 - 24x + 10 \end{aligned}$$

(g)

$$\begin{aligned} f\colon [-2,-1] &\to \mathbb{R} \\ x &\mapsto e^{-4x} + 5x \end{aligned}$$

(h)

$$\begin{aligned} f\colon [-1,0] &\to \mathbb{R} \\ x &\mapsto e^{-4x} + 5x \end{aligned}$$

(i)

$$\begin{aligned} f\colon [0,1] &\to \mathbb{R} \\ x &\mapsto e^{-4x} + 5x \end{aligned}$$

4. By finding the second derivative, classify all the critical points of each of the following functions.

   (a) $f(x) = x^3 + 3x^2 - 24x - 7$

   (b) $f(x) = -x^3 + 9x^2 - 24x + 10$

   (c) $f(x) = e^{-4x} + 5x$

   (d) $f(x) = \sin(2x)$

5. (a) Starting with the initial guess $x_0 = 2$, apply two iterations of the Newton-Raphson method to obtain an approximate solution of the equation $x^3 - 9x^2 + 12x + 10 = 0$.

   (b) Starting with the initial guess $x_0 = 1$, apply two iterations of the Newton-Raphson method to obtain an approximate solution of the equation $\cos(2x) - x = 0$.