Access to Science, Engineering and Agriculture: Mathematics 2 MATH00040 Chapter 1 Exercises

1. Evaluate each of the following.

   (a) $5\begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix}$

   (b) $\frac{1}{5}\begin{pmatrix} 2 & -5 \\ 10 & 3 \end{pmatrix}$

   (c) $-\frac{2}{3}\begin{pmatrix} 3 & -5 \\ 4 & -9 \end{pmatrix}$

2. Let

$$A = \begin{pmatrix} 2 & -1 & 5 \\ 0 & 3 & 1 \end{pmatrix} \quad B = \begin{pmatrix} -2 & 4 \\ 5 & -3 \end{pmatrix} \quad C = \begin{pmatrix} 2 & -4 & 7 \end{pmatrix}$$

$$D = \begin{pmatrix} 4 & 2 \\ -6 & 0 \\ 3 & 1 \end{pmatrix} \quad E = \begin{pmatrix} 6 & 0 \\ -2 & 1 \end{pmatrix} \quad F = \begin{pmatrix} -6 & -5 & 3 \\ -1 & 5 & -3 \end{pmatrix}$$

   (a) Write down the following elements of these matrices:

      (i) a_{23}

      (ii) d_{21}

      (iii) e_{12}

      (iv) f_{33}

   (b) Write down the sizes of each of these matrices.

   (c) (i) Which of the following combinations of matrices can be formed ?

$$A+B \quad A+F \quad B+B \quad B+D \quad B+E \quad F-A$$

      (ii) Evaluate those combinations in Part (i) which can be formed.

   (d) (i) Which of the following matrix products are defined ?

$$AB \quad BF \quad BE \quad EB \quad FD \quad DA \quad B^3 \quad (CD)E \quad C(DE)$$

      (ii) Write down the size of each product in Part (i) that is defined.

      (iii) Evaluate those matrix products in Part (i) which are defined.

3. Let

$$A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 6 & 5 \\ 2 & 1 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 4 & 2 \\ 2 & 3 & 5 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 2 \\ 4 & 0 \\ 0 & 3 \end{pmatrix}$$

(a) Write down the sizes of each of these matrices.

(b) (i) Which of the following combinations of matrices can be formed ?

$$A^T + B \quad 2B^T + 3C^T \quad 3C^T + A \quad 2C^T - B^T \quad 2D + 3D \quad A - 2A^T + A$$

(ii) Evaluate those combinations in Part (i) which can be formed.

(c) (i) Which of the following matrix products are defined ?

$$AB \quad CA^T \quad C^TD^T \quad AD^T \quad DB \quad (A^2)^T \quad (A^T)^2 \quad (D^2)^T \quad (D^T)^2$$

(ii) Write down the size of each product in Part (i) that is defined.

(iii) Evaluate those matrix products in Part (i) which are defined.

4. Let

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad C = \begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix} \quad D = \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 5 \\ -2 & 10 \end{pmatrix} \quad F = \begin{pmatrix} 2 & 4 \\ -3 & 0 \end{pmatrix} \quad G = \begin{pmatrix} 0 & 0 \\ 2 & -5 \end{pmatrix} \quad H = \begin{pmatrix} -2 & -3 \\ -1 & -6 \end{pmatrix}$$

(a) Determine whether or not these matrices have inverses.

(b) For those that do find the inverse.

5. Using row reduction, determine if each of the following systems of linear equations has a solution. For those that do give the solution.

(a)
$$\begin{aligned} x + y + 2z &= 3 \\ 2x + 2y + 3z &= 5 \\ x - y &= 5 \end{aligned}$$

(b)
$$\begin{aligned} x + 2y &= 0 \\ y - z &= 2 \\ x + y + z &= -2 \end{aligned}$$

(c)
$$\begin{aligned} x + 2y + 4z &= 6 \\ y + z &= 1 \\ x + 3y + 5z &= 10 \end{aligned}$$

(d) $$\begin{array}{rcrcrcr} 2x & - & y & + & z & = & 1 \\ x & + & y & - & z & = & 8 \\ -x & + & 3y & + & 2z & = & -8 \end{array}$$

(e) $$\begin{array}{rcrcrcr} x & + & y & + & z & = & 6 \\ -x & + & y & - & 3z & = & -2 \\ 2x & + & y & + & 3z & = & 6 \end{array}$$

(f) $$\begin{array}{rcrcrcr} -x & + & 2y & - & 4z & = & 6 \\ x & + & 3y & - & z & = & 4 \\ x & + & 2y & & & = & 2 \end{array}$$

(g) $$\begin{array}{rcrcr} x & + & 4y & = & -7 \\ 2x & - & y & = & 4 \\ -x & + & 2y & = & -5 \end{array}$$

(h) $$\begin{array}{rcrcrcr} x & + & y & - & z & = & 0 \\ & & y & - & 2z & = & 0 \\ 3x & - & y & + & 5z & = & 0 \end{array}$$

(i) $$\begin{array}{rcrcrcr} 3x & - & 11y & - & 3z & = & 3 \\ 2x & - & 6y & - & 2z & = & 1 \\ 5x & - & 17y & - & 6z & = & 2 \\ 4x & - & 8y & & & = & 7 \end{array}$$

(j) $$\begin{array}{rcrcrcr} 4x & + & 2y & + & z & = & 0 \\ -2x & - & y & + & 5z & = & 11 \\ 6x & + & 3y & - & z & = & \text{-5} \end{array}$$

6. Using row reduction, determine if each of the following matrices has an inverse and find it if it exists. Check your answers by multiplying the inverse with the original matrix in each case.

(a) $A = \begin{pmatrix} 1 & 3 & -1 \\ -2 & -5 & 1 \\ 4 & 11 & -2 \end{pmatrix}$

(b) $B = \begin{pmatrix} 1 & 3 & 5 \\ 3 & 1 & 7 \\ 2 & 4 & 8 \end{pmatrix}$

(c) $C = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & -4 \\ 3 & 2 & 10 \end{pmatrix}$

(d) $D = \begin{pmatrix} 2 & 4 & 6 \\ 1 & 2 & 4 \\ 5 & 10 & 5 \end{pmatrix}$

(e) $E = \begin{pmatrix} 1 & 2 & 4 \\ -2 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(f) $F = \begin{pmatrix} 1 & 1 & -4 \\ 2 & 1 & -6 \\ -3 & -1 & 9 \end{pmatrix}$

(g) $G = \begin{pmatrix} 1 & 4 & 1 \\ 1 & 6 & 3 \\ 2 & 3 & 0 \end{pmatrix}$

(h) $H = \begin{pmatrix} 1 & -4 & 1 \\ 1 & 6 & -3 \\ -2 & 3 & 0 \end{pmatrix}$

7. Calculate the determinant of each matrix in Question 6 and hence confirm your answers there about which matrices have inverses.

8. Find the lengths of the following vectors and hence find unit vectors in the directions of the vectors.

   (a) $(1, 2)$
   (b) $(-1, 3)$
   (c) $(1, 0, -2, 3)$
   (d) $(0, 0, 0)$

9. Find the angles (in radians to 2 decimal places) between the following pairs of vectors.

   (a) $(1, 2)$ and $(2, 1)$.
   (b) $(1, 0, 1)$ and $(-1, 2, 0)$.
   (c) $(1, 2, 1)$ and $(-1, 2, 2)$.
   (d) $(1, 1, 2)$ and $(-1, -1, 1)$.
   (e) $(0, 0)$ and $(1, 1)$.

10. Find the following cross products.

(a) $(1, 0, 0) \times (0, 1, 0)$

(b) $(0, 1, 0) \times (1, 0, 0)$

(c) $(1, 2, -4) \times (-2, -4, 8)$

(d) $(1, 2, -1) \times (-1, 2, 3)$

(e) $(1, 2) \times (2, 1)$

11. Find the eigenvalues and corresponding eigenvectors of each of the following matrices.

(a) $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} -2 & 3 \\ -2 & 5 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

(e) $\begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$

(f) $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$