Access to Science, Engineering and Agriculture: Mathematics 2 MATH00040 Chapter 2 Solutions

1. (a)

$$|1+i| = \sqrt{1^2+1^2} = \sqrt{1+1} = \sqrt{2},$$
$$\overline{1+i} = 1-i, \quad \mathrm{Re}(1+i) = 1, \quad \mathrm{Im}(1+i) = 1.$$

$$|2+2i| = \sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2},$$
$$\overline{2+2i} = 2-2i, \quad \mathrm{Re}(2+2i) = 2, \quad \mathrm{Im}(2+2i) = 2.$$

$$(1+i)+(2+2i) = (1+2)+(1+2)i = 3+3i,$$
$$(1+i)-(2+2i) = (1-2)+(1-2)i = -1-i,$$
$$(1+i)(2+2i) = ((1)(2)-(1)(2)) + ((1)(2)+(1)(2))i = 0+4i = 4i,$$
$$\frac{1+i}{2+2i} = \frac{1+i}{2+2i}\cdot\frac{2-2i}{2-2i} = \frac{4+0i}{8} = \frac{1}{2} \text{ or } \frac{1+i}{2+2i} = \frac{1+i}{2(1+i)} = \frac{1}{2},$$
$$\frac{2+2i}{1+i} = \frac{2+2i}{1+i}\cdot\frac{1-i}{1-i} = \frac{4+0i}{2} = 2 \text{ or } \frac{2+2i}{1+i} = \frac{2(1+i)}{1+i} = 2.$$

(b)

$$|1-2i| = \sqrt{1^2+(-2)^2} = \sqrt{1+4} = \sqrt{5},$$
$$\overline{1-2i} = 1+2i, \quad \mathrm{Re}(1-2i) = 1, \quad \mathrm{Im}(1-2i) = -2.$$

$$|-2+3i| = \sqrt{(-2)^2+3^2} = \sqrt{4+9} = \sqrt{13},$$
$$\overline{-2+3i} = -2-3i, \quad \mathrm{Re}(-2+3i) = -2, \quad \mathrm{Im}(-2+3i) = 3.$$

$$(1-2i)+(-2+3i) = (1-2)+(-2+3)i = -1+i,$$
$$(1-2i)-(-2+3i) = (1+2)+(-2-3)i = 3-5i,$$
$$(1-2i)(-2+3i) = ((1)(-2)-(-2)(3)) + ((1)(3)+(-2)(-2))i = 4+7i$$
$$\frac{1-2i}{-2+3i} = \frac{1-2i}{-2+3i}\cdot\frac{-2-3i}{-2-3i} = \frac{-8+i}{13} = -\frac{8}{13}+\frac{1}{13}i,$$
$$\frac{-2+3i}{1-2i} = \frac{-2+3i}{1-2i}\cdot\frac{1+2i}{1+2i} = \frac{-8-i}{5} = -\frac{8}{5}-\frac{1}{5}i.$$

(c)

$$|-3i| = \sqrt{0^2+(-3)^2} = \sqrt{0+9} = \sqrt{9} = 3,$$
$$\overline{-3i} = 3i, \quad \mathrm{Re}(-3i) = 0, \quad \mathrm{Im}(-3i) = -3.$$

$$\begin{aligned}|4| &= \sqrt{4^2+0^2} = \sqrt{16} = 4,\\ \overline{4} &= 4, \quad \mathrm{Re}(4) = 4, \quad \mathrm{Im}(4) = 0.\end{aligned}$$

$$\begin{aligned}(-3i)+(4) &= 4-3i,\\ (-3i)-(4) &= -4-3i,\\ (-3i)(4) &= -12i\\ \frac{-3i}{4} &= -\frac{3}{4}i,\\ \frac{4}{-3i} &= \frac{4}{-3i}\cdot\frac{3i}{3i} = \frac{12i}{9} = \frac{4}{3}i.\end{aligned}$$

(d)

$$\begin{aligned}|-2-4i| &= \sqrt{(-2)^2+(-4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5},\\ \overline{-2-4i} &= -2+4i, \quad \mathrm{Re}(-2-4i) = -2, \quad \mathrm{Im}(-2-4i) = -4.\end{aligned}$$

$$\begin{aligned}|2-i| &= \sqrt{2^2+(-1)^2} = \sqrt{4+1} = \sqrt{5},\\ \overline{2-i} &= 2+i, \quad \mathrm{Re}(2-i) = 2, \quad \mathrm{Im}(2-i) = -1.\end{aligned}$$

$$\begin{aligned}(-2-4i)+(2-i) &= (-2+2)+(-4-1)i = 0-5i = -5i,\\ (-2-4i)-(2-i) &= (-2-2)+(-4+1)i = -4-3i,\\ (-2-4i)(2-i) &= ((-2)(2)-(-4)(-1))+((-2)(-1)+(-4)(2))i = -8-6i\\ \frac{-2-4i}{2-i} &= \frac{-2-4i}{2-i}\cdot\frac{2+i}{2+i} = \frac{0-10i}{5} = -2i,\\ \frac{2-i}{-2-4i} &= \frac{2-i}{-2-4i}\cdot\frac{-2+4i}{-2+4i} = \frac{0+10i}{20} = \frac{1}{2}i.\end{aligned}$$

2. (a) Since 1 lies on the positive real axis, we can immediately say that its argument is $\theta = 0$. Since its magnitude (i.e., its distance from the origin) is 1, its polar form is

$$1 = 1\left(\cos(0) + i\sin(0)\right) = \cos(0) + i\sin(0).$$

(b) Since the real and the imaginary parts of $2+2i$ are both positive, the argument, θ, of $2+2i$ is given by

$$\theta = \tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = \frac{\pi}{4}.$$

Also, the magnitude, r, of $2+2i$ is $r = \sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$. Hence, $2+2i$ in polar form is

$$2+2i = 2\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right).$$

(c) Since $3i$ lies on the positive imaginary axis, we can immediately say that its argument is $\theta = \frac{\pi}{2}$. Since its magnitude (i.e., its distance from the origin) is 3, its polar form is

$$3i = 3\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right).$$

(d) The real part of $-2 + \frac{2}{\sqrt{3}}i$ is negative and its imaginary part is positive, so we are in the situation of Figure 6 in the Complex Numbers notes. Hence the argument of $-2 + \frac{2}{\sqrt{3}}i$ is

$$\theta = \pi - \phi = \pi - \tan^{-1}\left(\left|\frac{2/\sqrt{3}}{-2}\right|\right) = \pi - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}.$$

Also, the magnitude, r, of $-2 + \frac{2}{\sqrt{3}}i$ is

$$r = \sqrt{(-2)^2 + \left(\frac{2}{\sqrt{3}}\right)^2} = \sqrt{4 + \frac{4}{3}} = \sqrt{\frac{16}{3}} = \frac{4}{\sqrt{3}}.$$

Hence, $-2 + \frac{2}{\sqrt{3}}i$ in polar form is

$$\begin{aligned} -2 + \frac{2}{\sqrt{3}}i &= \frac{4}{\sqrt{3}}\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right) \\ &= \frac{4\sqrt{3}}{3}\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right). \end{aligned}$$

(e) Since -4 lies on the negative real axis, we can immediately say that its argument is $\theta = \pi$. Since its magnitude (i.e., its distance from the origin) is 4, its polar form is

$$-4 = 4\left(\cos(\pi) + i\sin(\pi)\right).$$

(f) The real and imaginary parts of $-\sqrt{3} - i$ are both negative, so we are in the situation of Figure 7 in the Complex Numbers notes. Hence the argument of $-\sqrt{3} - i$ is

$$\theta = \phi - \pi = \tan^{-1}\left(\left|\frac{-1}{-\sqrt{3}}\right|\right) - \pi = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \pi = \frac{\pi}{6} - \pi = -\frac{5\pi}{6}.$$

Also, the magnitude of $-\sqrt{3} - i$ is

$$r = \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2} = \sqrt{3+1} = \sqrt{4} = 2.$$

Hence, $-\sqrt{3} - i$ in polar form is

$$-\sqrt{3} - i = 2\left(\cos\left(-\frac{5\pi}{6}\right) + i\sin\left(-\frac{5\pi}{6}\right)\right) \text{ or } 2\left(\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right)\right).$$

(g) Since $-4i$ lies on the negative imaginary axis, we can immediately say that its argument is $\theta = -\frac{\pi}{2}$. Since its magnitude (i.e., its distance from the origin) is 4, its polar form is

$$-4i = 4\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right) \text{ or } 4\left(\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right).$$

(h) The real part of $1-i$ is positive and its imaginary part is negative, so we are in the situation of Figure 5 in the Complex Numbers notes. Hence the argument of $1-i$ is

$$\theta = -\phi = -\tan^{-1}\left(\left|\frac{-1}{1}\right|\right) = -\tan^{-1}(1) = -\frac{\pi}{4}.$$

Also, the magnitude of $1-i$ is

$$r = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}.$$

Hence, $1-i$ in polar form is

$$1-i = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right) \text{ or } \sqrt{2}\left(\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right).$$

3. In all these problems we will use Corollary 2.3.9 from the Complex Numbers notes. That is we will use

$$(r(\cos(\theta) + i\sin(\theta)))^n = r^n(\cos(n\theta) + i\sin(n\theta)).$$

(a)

$$\begin{aligned} 1^2 &= (1(\cos(0) + i\sin(0)))^2 \\ &= 1^2(\cos(2(0)) + i\sin(2(0))) \\ &= 1(\cos(0) + i\sin(0)) \\ &= 1 + 0i \\ &= 1. \end{aligned}$$

(b)

$$\begin{aligned} (2+2i)^3 &= \left(2\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)\right)^3 \\ &= (2\sqrt{2})^3\left(\cos\left(3\cdot\frac{\pi}{4}\right) + i\sin\left(3\cdot\frac{\pi}{4}\right)\right) \\ &= 16\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right) \\ &= -16 + 16i. \end{aligned}$$

(c)

$$\begin{aligned} (3i))^4 &= \left(3\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)\right)^4 \\ &= 3^4\left(\cos\left(4\cdot\frac{\pi}{2}\right) + i\sin\left(4\cdot\frac{\pi}{2}\right)\right) \\ &= 81(\cos(2\pi) + i\sin(2\pi)) \\ &= 81. \end{aligned}$$

(d)

$$\begin{aligned}\left(\frac{4\sqrt{3}}{3}\left(\cos\left(\frac{5\pi}{6}\right)+i\sin\left(\frac{5\pi}{6}\right)\right)\right)^5 &= \left(\frac{4\sqrt{3}}{3}\right)^5\left(\cos\left(5\cdot\frac{5\pi}{6}\right)+i\sin\left(5\cdot\frac{5\pi}{6}\right)\right)\\ &= \frac{4^5\cdot 3^2\sqrt{3}}{3^5}\left(\cos\left(\frac{25\pi}{6}\right)+i\sin\left(\frac{25\pi}{6}\right)\right)\\ &= \frac{1024\sqrt{3}}{27}\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)\\ &= \frac{1024}{18}+\frac{1024\sqrt{3}}{54}i\\ &\simeq 56.889+32.845i.\end{aligned}$$

(e)

$$\begin{aligned}\left(4\left(\cos\left(\pi\right)+i\sin\left(\pi\right)\right)\right)^6 &= 4^6\left(\cos\left(6\pi\right)+i\sin\left(6\pi\right)\right)\\ &= 4096\left(\cos\left(0\right)+i\sin\left(0\right)\right)\\ &= 4096.\end{aligned}$$

(f)

$$\begin{aligned}\left(2\left(\cos\left(\frac{7\pi}{6}\right)+i\sin\left(\frac{7\pi}{6}\right)\right)\right)^7 &= 2^7\left(\cos\left(7\cdot\frac{7\pi}{6}\right)+i\sin\left(7\cdot\frac{7\pi}{6}\right)\right)\\ &= 128\left(\cos\left(\frac{49\pi}{6}\right)+i\sin\left(\frac{49\pi}{6}\right)\right)\\ &= 128\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)\\ &= 64\sqrt{3}+64i\\ &\simeq 110.851+64i.\end{aligned}$$

(g)

$$\begin{aligned}\left(4\left(\cos\left(\frac{3\pi}{2}\right)+i\sin\left(\frac{3\pi}{2}\right)\right)\right)^8 &= 4^8\left(\cos\left(8\cdot\frac{3\pi}{2}\right)+i\sin\left(8\cdot\frac{3\pi}{2}\right)\right)\\ &= 65536\left(\cos\left(12\pi\right)+i\sin\left(12\pi\right)\right)\\ &= 65536\left(\cos\left(0\right)+i\sin\left(0\right)\right)\\ &= 65536.\end{aligned}$$

(h)

$$\begin{aligned}\left(\sqrt{2}\left(\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)\right)\right)^9 &= \left(\sqrt{2}\right)^9\left(\cos\left(9\cdot\frac{7\pi}{4}\right)+i\sin\left(9\cdot\frac{7\pi}{4}\right)\right)\\ &= 16\sqrt{2}\left(\cos\left(\frac{63\pi}{4}\right)+i\sin\left(\frac{63\pi}{4}\right)\right)\\ &= 16\sqrt{2}\left(\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)\right)\\ &= 16-16i.\end{aligned}$$

4. In all of these solutions we will use the formula

$$z_k = r^{\frac{1}{n}} \left(\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i \sin\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)\right) \qquad k = 0, 1, \ldots, n-1,$$

where the z_k are the n'th roots.

(a) In this case $1 = 1\,(\cos(0) + i\sin(0))$ and we are looking for the third (i.e., cube) roots, so we take $n = 3$.
Since $1^{\frac{1}{3}} = 1$, the roots are

$$z_k = \cos\left(\frac{0}{3} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{0}{3} + \frac{2k\pi}{3}\right) \qquad k = 0, 1, 2.$$

That is

$$\begin{aligned} z_0 &= \cos(0) + i\sin(0) = 1, \\ z_1 &= \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) \simeq -0.5 + 0.866i, \\ z_2 &= \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right) \simeq -0.5 - 0.866i. \end{aligned}$$

(b) In this case

$$2 + 2i = 2\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right).$$

and we are looking for the fourth roots, so we take $n = 4$.
Since $\left(2\sqrt{2}\right)^{\frac{1}{4}} = \left(2^{\frac{3}{2}}\right)^{\frac{1}{4}} = 2^{\frac{3}{8}}$, the roots are

$$z_k = 2^{\frac{3}{8}}\left(\cos\left(\frac{\pi}{16} + \frac{2k\pi}{4}\right) + i\sin\left(\frac{\pi}{16} + \frac{2k\pi}{4}\right)\right) \qquad k = 0, 1, 2, 3.$$

That is

$$\begin{aligned} z_0 &= 2^{\frac{3}{8}}\left(\cos\left(\frac{\pi}{16}\right) + i\sin\left(\frac{\pi}{16}\right)\right) \simeq 1.272 + 0.253i, \\ z_1 &= 2^{\frac{3}{8}}\left(\cos\left(\frac{9\pi}{16}\right) + i\sin\left(\frac{9\pi}{16}\right)\right) \simeq -0.253 + 1.272i, \\ z_2 &= 2^{\frac{3}{8}}\left(\cos\left(\frac{17\pi}{16}\right) + i\sin\left(\frac{17\pi}{16}\right)\right) \simeq -1.272 - 0.253i, \\ z_3 &= 2^{\frac{3}{8}}\left(\cos\left(\frac{25\pi}{16}\right) + i\sin\left(\frac{25\pi}{16}\right)\right) \simeq 0.253 - 1.272i. \end{aligned}$$

(c) In this case

$$3i = 3\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right),$$

and we are looking for the fifth roots, so we take $n = 5$.
Thus the roots are

$$z_k = 3^{\frac{1}{5}}\left(\cos\left(\frac{\pi}{10} + \frac{2k\pi}{5}\right) + i\sin\left(\frac{\pi}{10} + \frac{2k\pi}{5}\right)\right) \qquad k = 0, 1, 2, 3, 4.$$

That is

$$z_0 = 3^{\frac{1}{5}}\left(\cos\left(\frac{\pi}{10}\right) + i\sin\left(\frac{\pi}{10}\right)\right) \simeq 1.185 + 0.385i,$$
$$z_1 = 3^{\frac{1}{5}}\left(\cos\left(\frac{5\pi}{10}\right) + i\sin\left(\frac{5\pi}{10}\right)\right) = 1.246i,$$
$$z_2 = 3^{\frac{1}{5}}\left(\cos\left(\frac{9\pi}{10}\right) + i\sin\left(\frac{9\pi}{10}\right)\right) \simeq -1.185 + 0.385i,$$
$$z_3 = 3^{\frac{1}{5}}\left(\cos\left(\frac{13\pi}{10}\right) + i\sin\left(\frac{13\pi}{10}\right)\right) \simeq -0.732 - 1.008i,$$
$$z_4 = 3^{\frac{1}{5}}\left(\cos\left(\frac{17\pi}{10}\right) + i\sin\left(\frac{17\pi}{10}\right)\right) \simeq 0.732 - 1.008i.$$

(d) In this case

$$-2 + \frac{2}{\sqrt{3}}i = \frac{4}{\sqrt{3}}\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right),$$

and we are looking for the sixth roots, so we take $n = 6$.

Since $\left(\frac{4}{\sqrt{3}}\right)^{\frac{1}{6}} = \frac{\left(16^{\frac{1}{2}}\right)^{\frac{1}{6}}}{\left(3^{\frac{1}{2}}\right)^{\frac{1}{6}}} = \frac{16^{\frac{1}{12}}}{3^{\frac{1}{12}}} = \left(\frac{16}{3}\right)^{\frac{1}{12}}$, the roots are

$$z_k = \left(\frac{16}{3}\right)^{\frac{1}{12}}\left(\cos\left(\frac{5\pi}{36} + \frac{2k\pi}{6}\right) + i\sin\left(\frac{5\pi}{36} + \frac{2k\pi}{6}\right)\right) \quad k = 0, 1, 2, 3, 4, 5.$$

That is

$$z_0 = \left(\frac{16}{3}\right)^{\frac{1}{12}}\left(\cos\left(\frac{5\pi}{36}\right) + i\sin\left(\frac{5\pi}{36}\right)\right) \simeq 1.042 + 0.486i$$
$$z_1 = \left(\frac{16}{3}\right)^{\frac{1}{12}}\left(\cos\left(\frac{17\pi}{36}\right) + i\sin\left(\frac{17\pi}{36}\right)\right) \simeq 0.100 + 1.145i$$
$$z_2 = \left(\frac{16}{3}\right)^{\frac{1}{12}}\left(\cos\left(\frac{29\pi}{36}\right) + i\sin\left(\frac{29\pi}{36}\right)\right) \simeq -0.942 + 0.659i$$
$$z_3 = \left(\frac{16}{3}\right)^{\frac{1}{12}}\left(\cos\left(\frac{41\pi}{36}\right) + i\sin\left(\frac{41\pi}{36}\right)\right) \simeq -1.042 - 0.486i$$
$$z_4 = \left(\frac{16}{3}\right)^{\frac{1}{12}}\left(\cos\left(\frac{53\pi}{36}\right) + i\sin\left(\frac{53\pi}{36}\right)\right) \simeq -0.100 - 1.145i$$
$$z_5 = \left(\frac{16}{3}\right)^{\frac{1}{12}}\left(\cos\left(\frac{65\pi}{36}\right) + i\sin\left(\frac{65\pi}{36}\right)\right) \simeq 0.942 - 0.659i$$

(e) In this case

$$-4 = 4\left(\cos(\pi) + i\sin(\pi)\right),$$

and we are looking for the seventh roots, so we take $n = 7$.
Thus the roots are

$$z_k = 4^{\frac{1}{7}}\left(\cos\left(\frac{\pi}{7} + \frac{2k\pi}{7}\right) + i\sin\left(\frac{\pi}{7} + \frac{2k\pi}{7}\right)\right) \quad k = 0, 1, 2, 3, 4, 5, 6.$$

That is

$$
\begin{aligned}
z_0 &= 4^{\frac{1}{7}}\left(\cos\left(\frac{\pi}{7}\right)+i\sin\left(\frac{\pi}{7}\right)\right)\simeq 1.098+0.529i,\\
z_1 &= 4^{\frac{1}{7}}\left(\cos\left(\frac{3\pi}{7}\right)+i\sin\left(\frac{3\pi}{7}\right)\right)\simeq 0.271+1.188i,\\
z_2 &= 4^{\frac{1}{7}}\left(\cos\left(\frac{5\pi}{7}\right)+i\sin\left(\frac{5\pi}{7}\right)\right)\simeq -0.760+0.953i,\\
z_3 &= 4^{\frac{1}{7}}\left(\cos\left(\frac{7\pi}{7}\right)+i\sin\left(\frac{7\pi}{7}\right)\right)\simeq -1.219,\\
z_4 &= 4^{\frac{1}{7}}\left(\cos\left(\frac{9\pi}{7}\right)+i\sin\left(\frac{9\pi}{7}\right)\right)\simeq -0.760-0.953i,\\
z_5 &= 4^{\frac{1}{7}}\left(\cos\left(\frac{11\pi}{7}\right)+i\sin\left(\frac{11\pi}{7}\right)\right)\simeq 0.271-1.188i,\\
z_6 &= 4^{\frac{1}{7}}\left(\cos\left(\frac{13\pi}{7}\right)+i\sin\left(\frac{13\pi}{7}\right)\right)\simeq 1.098-0.529i.
\end{aligned}
$$

(f) In this case

$$-\sqrt{3}-i = 2\left(\cos\left(\frac{7\pi}{6}\right)+i\sin\left(\frac{7\pi}{6}\right)\right),$$

and we are looking for the eighth roots, so we take $n = 8$.
Thus, the roots are

$$z_k = 2^{\frac{1}{8}}\left(\cos\left(\frac{7\pi}{48}+\frac{2k\pi}{8}\right)+i\sin\left(\frac{7\pi}{48}+\frac{2k\pi}{8}\right)\right)\qquad k=0,1,2,3,4,5,6,7.$$

That is

$$
\begin{aligned}
z_0 &= 2^{\frac{1}{8}}\left(\cos\left(\frac{7\pi}{48}\right)+i\sin\left(\frac{7\pi}{48}\right)\right)\simeq 0.978+0.482i,\\
z_1 &= 2^{\frac{1}{8}}\left(\cos\left(\frac{19\pi}{48}\right)+i\sin\left(\frac{19\pi}{48}\right)\right)\simeq 0.351+1.033i,\\
z_2 &= 2^{\frac{1}{8}}\left(\cos\left(\frac{31\pi}{48}\right)+i\sin\left(\frac{31\pi}{48}\right)\right)\simeq -0.482+0.978i,\\
z_3 &= 2^{\frac{1}{8}}\left(\cos\left(\frac{43\pi}{48}\right)+i\sin\left(\frac{43\pi}{48}\right)\right)\simeq -1.033+0.351i,\\
z_4 &= 2^{\frac{1}{8}}\left(\cos\left(\frac{55\pi}{48}\right)+i\sin\left(\frac{55\pi}{48}\right)\right)\simeq -0.978-0.482i,\\
z_5 &= 2^{\frac{1}{8}}\left(\cos\left(\frac{67\pi}{48}\right)+i\sin\left(\frac{67\pi}{48}\right)\right)\simeq -0.351-1.033i,\\
z_6 &= 2^{\frac{1}{8}}\left(\cos\left(\frac{79\pi}{48}\right)+i\sin\left(\frac{79\pi}{48}\right)\right)\simeq 0.482-0.978i,\\
z_7 &= 2^{\frac{1}{8}}\left(\cos\left(\frac{91\pi}{48}\right)+i\sin\left(\frac{91\pi}{48}\right)\right)\simeq 1.032-0.351i.
\end{aligned}
$$

(g) In this case

$$-4i = 4\left(\cos\left(\frac{3\pi}{2}\right)+i\sin\left(\frac{3\pi}{2}\right)\right),$$

and we are looking for the ninth roots, so we take $n = 9$.
Thus, the roots are

$$z_k = 4^{\frac{1}{9}} \left(\cos\left(\frac{3\pi}{18} + \frac{2k\pi}{9} \right) + i \sin\left(\frac{3\pi}{18} + \frac{2k\pi}{9} \right) \right) \qquad k = 0, 1, 2, 3, 4, 5, 6, 7, 8.$$

That is

$$\begin{aligned}
z_0 &= 4^{\frac{1}{9}} \left(\cos\left(\frac{3\pi}{18} \right) + i \sin\left(\frac{3\pi}{18} \right) \right) \simeq 1.010 + 0.583i, \\
z_1 &= 4^{\frac{1}{9}} \left(\cos\left(\frac{7\pi}{18} \right) + i \sin\left(\frac{7\pi}{18} \right) \right) \simeq 0.399 + 1.096i, \\
z_2 &= 4^{\frac{1}{9}} \left(\cos\left(\frac{11\pi}{18} \right) + i \sin\left(\frac{11\pi}{18} \right) \right) \simeq -0.399 + 1.096i, \\
z_3 &= 4^{\frac{1}{9}} \left(\cos\left(\frac{15\pi}{18} \right) + i \sin\left(\frac{15\pi}{18} \right) \right) \simeq -1.010 + 0.583i, \\
z_4 &= 4^{\frac{1}{9}} \left(\cos\left(\frac{19\pi}{18} \right) + i \sin\left(\frac{19\pi}{18} \right) \right) \simeq -1.149 - 0.203i, \\
z_5 &= 4^{\frac{1}{9}} \left(\cos\left(\frac{23\pi}{18} \right) + i \sin\left(\frac{23\pi}{18} \right) \right) \simeq -0.750 - 0.894i, \\
z_6 &= 4^{\frac{1}{9}} \left(\cos\left(\frac{27\pi}{18} \right) + i \sin\left(\frac{27\pi}{18} \right) \right) \simeq -1.167i, \\
z_7 &= 4^{\frac{1}{9}} \left(\cos\left(\frac{31\pi}{18} \right) + i \sin\left(\frac{31\pi}{18} \right) \right) \simeq 0.750 - 0.894i, \\
z_8 &= 4^{\frac{1}{9}} \left(\cos\left(\frac{35\pi}{18} \right) + i \sin\left(\frac{35\pi}{18} \right) \right) \simeq 1.149 - 0.203i.
\end{aligned}$$

(h) In this case

$$1 - i = \sqrt{2} \left(\cos\left(\frac{7\pi}{4} \right) + i \sin\left(\frac{7\pi}{4} \right) \right),$$

and we are looking for the tenth roots, so we take $n = 10$.
Since $\left(\sqrt{2}\right)^{\frac{1}{10}} = \left(2^{\frac{1}{2}}\right)^{\frac{1}{10}} = 2^{\frac{1}{20}}$, the roots are

$$z_k = 2^{\frac{1}{20}} \left(\cos\left(\frac{7\pi}{40} + \frac{2k\pi}{10} \right) + i \sin\left(\frac{3\pi}{40} + \frac{2k\pi}{10} \right) \right) \qquad k = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.$$

That is

$$\begin{aligned}
z_0 &= 2^{\frac{1}{20}} \left(\cos\left(\frac{7\pi}{40} \right) + i \sin\left(\frac{7\pi}{40} \right) \right) \simeq 0.883 + 0.541i, \\
z_1 &= 2^{\frac{1}{20}} \left(\cos\left(\frac{15\pi}{40} \right) + i \sin\left(\frac{15\pi}{40} \right) \right) \simeq 0.396 + 0.956i, \\
z_2 &= 2^{\frac{1}{20}} \left(\cos\left(\frac{23\pi}{40} \right) + i \sin\left(\frac{23\pi}{40} \right) \right) \simeq -0.242 + 1.007i, \\
z_3 &= 2^{\frac{1}{20}} \left(\cos\left(\frac{31\pi}{40} \right) + i \sin\left(\frac{31\pi}{40} \right) \right) \simeq -0.787 + 0.672i,
\end{aligned}$$

$$z_4 = 2^{\frac{1}{20}} \left(\cos\left(\frac{39\pi}{40}\right) + i \sin\left(\frac{39\pi}{40}\right)\right) \simeq -1.032 = 0.081i,$$
$$z_5 = 2^{\frac{1}{20}} \left(\cos\left(\frac{47\pi}{40}\right) + i \sin\left(\frac{47\pi}{40}\right)\right) \simeq -0.883 - 0.541i,$$
$$z_6 = 2^{\frac{1}{20}} \left(\cos\left(\frac{55\pi}{40}\right) + i \sin\left(\frac{55\pi}{40}\right)\right) \simeq -0.396 - 0.956i,$$
$$z_7 = 2^{\frac{1}{20}} \left(\cos\left(\frac{63\pi}{40}\right) + i \sin\left(\frac{63\pi}{40}\right)\right) \simeq 0.242 - 1.007i,$$
$$z_8 = 2^{\frac{1}{20}} \left(\cos\left(\frac{71\pi}{40}\right) + i \sin\left(\frac{71\pi}{40}\right)\right) \simeq 0.787 - 0.672i,$$
$$z_9 = 2^{\frac{1}{20}} \left(\cos\left(\frac{79\pi}{40}\right) + i \sin\left(\frac{79\pi}{40}\right)\right) \simeq 1.032 - 0.081i.$$