Access to Science, Engineering and Agriculture: Mathematics 2
MATH00040
Chapter 3 Exercises

1. Find the derivatives of the following functions.

   Note that these can be done just using Table 1 of Chapter 3 of the course notes.

   (a) $f(x) = 5$
   (b) $f(x) = -\pi\cos(e)$
   (c) $f(x) = x^2$
   (d) $f(x) = x^{\frac{9}{2}}$
   (e) $f(x) = x^{-5}$
   (f) $f(x) = x^{\cos(2)}$
   (g) $f(x) = e^{4x}$
   (h) $f(x) = e^{\frac{3}{2}x}$
   (i) $f(x) = e^{-6x}$
   (j) $f(x) = e^{\pi x}$
   (k) $f(x) = \ln(4x)$ (where $x > 0$).
   (l) $f(x) = \ln(-\pi x)$ (where $x < 0$).
   (m) $f(x) = \ln\left(\frac{1}{2}x\right)$ (where $x > 0$).
   (n) $f(x) = \sin(2x)$
   (o) $f(x) = \sin(-2x)$
   (p) $f(x) = \sin(ex)$
   (q) $f(x) = \cos(3x)$
   (r) $f(x) = \cos(-3x)$
   (s) $f(x) = \cos(-\pi x)$

2. Find the derivatives of the following functions.

   Note that these can be done using Table 1 of Chapter 3 of the course notes together with the Sum and Multiple Rules.

   (a) $f(x) = 1 + 3x - 2x^2 + 3x^3 - 4x^4$
   (b) $f(x) = -x^{-1} + 2\sin 4x$
   (c) $f(x) = 3e^{-\frac{1}{2}x} - 2\cos(\frac{1}{2}x)$

(d) $f(x) = 2\ln(-x) + 4\cos(-3x) - e^{-\frac{3}{2}x}$ (where $x < 0$).

(e) $f(x) = -2x^2 + 3\ln(3x) + e^{\cos(1)x}$ (where $x > 0$).

(f) $f(x) = 2\sin(3x) - 3\sin(2x) + 2\cos(3x) - 3\cos(2x)$

(g) $f(x) = e^2 + e^{2x} - 4$

(h) $f(x) = -3x^{-3} + 4x^4 + 5x^{-5} + 3x^0$

3. Find the derivatives of the following functions.

   Note that these can be done using Table 1 of Chapter 3 of the course notes together with the Sum, Multiple and Product Rules.

   (a) $f(x) = 3x^2\sin(2x) - e^{2x}\cos(x)$

   (b) $f(x) = (2x^2 - 3x^{-3} + 5x^4)\sin(-5x) + e^{-3x}(\sin(2x) - \cos(x))$

   (c) $f(x) = \sin(x)\sin(x) + \cos(x)\cos(x)$

   (d) $f(x) = \sin(x)\sin(x) - \cos(x)\cos(x)$

   (e) $f(x) = e^{2x}\ln(2x) - x^2 + x^3 + 1$ (where $x > 0$).

   (f) $f(x) = e^0\cos(2x) - \ln(1)\sin(-x)$

   (g) $f(x) = x^2e^{3x}\cos(4x)$

4. Find the derivatives of the following functions.

   Note that these can be done using Table 1 of Chapter 3 of the course notes together with the Sum, Multiple, Product and Quotient Rules.

   (a) $f(x) = \dfrac{x}{\sin(x)}$ (where $\sin(x) \neq 0$).

   (b) $f(x) = \dfrac{2e^{3x}}{x^2 + x + 1}$

   (c) $f(x) = \dfrac{x\cos(2x)}{\ln(2x)}$ (where $x > \frac{1}{2}$).

   (d) $f(x) = \dfrac{(x^3 - 2x)\cos(-2x)}{\sin(x)\cos(x)}$ (where $\sin(x)\cos(x) \neq 0$).

   (e) $f(x) = \dfrac{\cos(x)}{\sin(x)}$ (where $\sin(x) \neq 0$).

   (f) $f(x) = \dfrac{2\cos(3x)\ln(3x) + x^2}{2e^{-x} + x^4}$ (where $x > 0$).

5. Find the derivatives of the following functions.

   Note that these can be done using Table 1 of Chapter 3 of the course notes together with the Sum, Multiple, Product, Quotient and Chain Rules.

   (a) $f(x) = e^{x^2}$

   (b) $f(x) = \sin(2x^2 + 3x)$

   (c) $f(x) = \ln(\sin(x))$ (where $\sin(x) > 0$)

(d) $f(x) = \cos(xe^x)$

(e) $f(x) = \sin(\sin(x))$

(f) $f(x) = \cos(\sin(\cos(x)))$

(g) $f(x) = e^{\sin(x^2)}$

6. Find all the critical points of each of the following functions.

(a) $f(x) = x^3 - 3x^2 - 9x + 12$

(b) $f(x) = x^3 + 3x^2 - 5$

(c) $f(x) = -2x^3 - 9x^2 + 24x - 1$

(d) $f(x) = 2x^3 + 3x^2 + 6x + 5$

(e) $f(x) = e^{-3x} + 7x$

(f) $f(x) = e^{4x} + 5x$

(g) $f(x) = \sin(x)$

(h) $f(x) = \cos(2x)$

7. Find the points where the global maxima and minima of each of the following functions occur.

(a) (i)

$$\begin{aligned} f\colon [-4, 6] &\to \mathbb{R} \\ x &\mapsto x^3 - 3x^2 - 9x + 12 \end{aligned}$$

(ii)

$$\begin{aligned} f\colon [-3, 5] &\to \mathbb{R} \\ x &\mapsto x^3 - 3x^2 - 9x + 12 \end{aligned}$$

(iii)

$$\begin{aligned} f\colon [-2, 4] &\to \mathbb{R} \\ x &\mapsto x^3 - 3x^2 - 9x + 12 \end{aligned}$$

(iv)

$$\begin{aligned} f\colon [0, 3] &\to \mathbb{R} \\ x &\mapsto x^3 - 3x^2 - 9x + 12 \end{aligned}$$

(b) (i)

$$\begin{aligned} f\colon [-3, 1] &\to \mathbb{R} \\ x &\mapsto x^3 + 3x^2 - 5 \end{aligned}$$

(ii)

$$\begin{aligned} f\colon [-3,0] &\to \mathbb{R} \\ x &\mapsto x^3 + 3x^2 - 5 \end{aligned}$$

(iii)

$$\begin{aligned} f\colon [-2,1] &\to \mathbb{R} \\ x &\mapsto x^3 + 3x^2 - 5 \end{aligned}$$

(iv)

$$\begin{aligned} f\colon [-1,0] &\to \mathbb{R} \\ x &\mapsto x^3 + 3x^2 - 5 \end{aligned}$$

(c) (i)

$$\begin{aligned} f\colon [-7,3] &\to \mathbb{R} \\ x &\mapsto -2x^3 - 9x^2 + 24x - 1 \end{aligned}$$

(ii)

$$\begin{aligned} f\colon [-6,4] &\to \mathbb{R} \\ x &\mapsto -2x^3 - 9x^2 + 24x - 1 \end{aligned}$$

(iii)

$$\begin{aligned} f\colon [-4,1] &\to \mathbb{R} \\ x &\mapsto -2x^3 - 9x^2 + 24x - 1 \end{aligned}$$

(iv)

$$\begin{aligned} f\colon [2,3] &\to \mathbb{R} \\ x &\mapsto -2x^3 - 9x^2 + 24x - 1 \end{aligned}$$

(d) (i)

$$\begin{aligned} f\colon [-1,1] &\to \mathbb{R} \\ x &\mapsto 2x^3 + 3x^2 + 6x + 5 \end{aligned}$$

(ii)

$$\begin{aligned} f\colon [-1,3] &\to \mathbb{R} \\ x &\mapsto 2x^3 + 3x^2 + 6x + 5 \end{aligned}$$

(iii)

$$\begin{aligned} f\colon [-4,1] &\to \mathbb{R} \\ x &\mapsto 2x^3 + 3x^2 + 6x + 5 \end{aligned}$$

(iv)

$$\begin{aligned} f\colon [-4,3] &\to \mathbb{R} \\ x &\mapsto 2x^3+3x^2+6x+5 \end{aligned}$$

(e) (i)

$$\begin{aligned} f\colon [-2-,1] &\to \mathbb{R} \\ x &\mapsto e^{-3x}+7x \end{aligned}$$

(ii)

$$\begin{aligned} f\colon [-1,0] &\to \mathbb{R} \\ x &\mapsto e^{-3x}+7x \end{aligned}$$

(iii)

$$\begin{aligned} f\colon [0,1] &\to \mathbb{R} \\ x &\mapsto e^{-3x}+7x \end{aligned}$$

(f) (i)

$$\begin{aligned} f\colon [-2,-1] &\to \mathbb{R} \\ x &\mapsto e^{4x}+5x \end{aligned}$$

(ii)

$$\begin{aligned} f\colon [-1,0] &\to \mathbb{R} \\ x &\mapsto e^{4x}+5x \end{aligned}$$

(iii)

$$\begin{aligned} f\colon [0,1] &\to \mathbb{R} \\ x &\mapsto e^{4x}+5x \end{aligned}$$

(g) (i)

$$\begin{aligned} f\colon \mathbb{R} &\to \mathbb{R} \\ x &\mapsto \sin(x) \end{aligned}$$

(ii)

$$\begin{aligned} f\colon \left[-\frac{\pi}{4},\frac{\pi}{4}\right] &\to \mathbb{R} \\ x &\mapsto \sin(x) \end{aligned}$$

(iii)

$$\begin{aligned} f\colon [0,2\pi] &\to \mathbb{R} \\ x &\mapsto \sin(x) \end{aligned}$$

(h) (i)

$$\begin{aligned} f\colon \mathbb{R} &\to \mathbb{R} \\ x &\mapsto \cos(2x) \end{aligned}$$

(ii)

$$\begin{aligned} f\colon \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] &\to \mathbb{R} \\ x &\mapsto \cos(2x) \end{aligned}$$

(iii)

$$\begin{aligned} f\colon [0, 2\pi] &\to \mathbb{R} \\ x &\mapsto \cos(2x) \end{aligned}$$

8. By finding the second derivative, classify all the critical points of each of the following functions.

   (a) $f(x) = x^3 - 3x^2 - 9x + 12$
   (b) $f(x) = x^3 + 3x^2 - 5$
   (c) $f(x) = -2x^3 - 9x^2 + 24x - 1$
   (d) $f(x) = 2x^3 + 3x^2 + 6x + 5$
   (e) $f(x) = e^{-3x} + 7x$
   (f) $f(x) = e^{4x} + 5x$
   (g) $f(x) = \sin(x)$
   (h) $f(x) = \cos(2x)$

9. (a) Starting with the initial guess $x_0 = 4$, apply two iterations of the Newton-Raphson method to obtain an approximate solution of the equation $x^3 - 3x^2 - 9x + 12 = 0$.

   (b) Starting with the initial guess $x_0 = 1$, apply two iterations of the Newton-Raphson method to obtain an approximate solution of the equation $x^3 + 3x^2 - 5 = 0$.

   (c) Starting with the initial guess $x_0 = 2$, apply two iterations of the Newton-Raphson method to obtain an approximate solution of the equation $-2x^3 - 9x^2 + 24x - 12 = 0$.

   (d) Starting with the initial guess $x_0 = 0$, apply two iterations of the Newton-Raphson method to obtain an approximate solution of the equation $\cos(x) - x = 0$.