Access to Science, Engineering and Agriculture: Mathematics 2
MATH00040
Chapter 4 Exercises

1. Find the following integrals.

   Note that these can be done just using Table 1 of Chapter 4 of the course notes.

   (a) $\int_0^1 5\,dx$

   (b) $\int -\pi\cos(e)\,dx$

   (c) $\int_{-1}^1 x^2\,dx$

   (d) $\int x^{\frac{9}{2}}\,dx$

   (e) $\int_1^2 x^{-5}\,dx$

   (f) $\int x^{\cos(2)}\,dx$

   (g) $\int_0^2 e^{4x}\,dx$

   (h) $\int e^{\frac{3}{2}x}\,dx$

   (i) $\int_{-1}^0 e^{-6x}\,dx$

   (j) $\int e^{\pi x}\,dx$

   (k) $\int_1^2 \frac{1}{x}\,dx$

   (l) $\int \sin(2x)\,dx$

   (m) $\int_0^{\frac{\pi}{3}} \sin(-3x)\,dx$

   (n) $\int \sin(ex)\,dx$

   (o) $\int_{-\pi}^{\frac{\pi}{3}} \cos(3x)\,dx$

(p) $\int \cos(-\pi x)\,dx$

2. Find the following integrals.

   Note that these can be done using Table 1 of Chapter 4 of the course notes together with the Sum and Multiple Rules.

   (a) $\int_{-1}^{1} 1 + 3x - 2x^2 + 3x^3 - 4x^4\,dx$

   (b) $\int -x^{-1} + 2\sin 4x\,dx$

   (c) $\int_{0}^{\pi} 3e^{-\frac{1}{2}x} - 2\cos\left(\frac{1}{2}x\right)\,dx$

   (d) $\int 4\cos(-3x) - e^{-\frac{3}{2}x}\,dx$

   (e) $\int_{-2}^{1} -2x^2 + e^{\cos(1)x}\,dx$

   (f) $\int 2\sin(3x) - 3\sin(2x) + 2\cos(3x) - 3\cos(2x)\,dx$

   (g) $\int_{1}^{3} e^2 + e^{2x} - 4\,dx$

   (h) $\int -3x^{-3} + 4x^4 + 5x^{-5} + 3x^0\,dx$

3. Find the following integrals.

   Note that these can be done using Table 1 of Chapter 4 of the course notes together with integration by substitution and the Sum and Multiple Rules.

   (a) $\int_{0}^{1} (x+5)^9\,dx$

   (b) $\int x^2(x^3-5)^5\,dx$

   (c) $\int_{0}^{\frac{\pi}{4}} \dfrac{\sin(x)}{\cos(x)}\,dx$

   (d) $\int \dfrac{\cos(x)}{\sin(x)}\,dx$

   (e) $\int_{1}^{2} x^{-2}e^{\frac{1}{x}}\,dx$

   (f) $\int \dfrac{12x^3 - 9x^2 + 6x - 3}{x^4 - x^3 + x^2 - x}\,dx$

   (g) $\int_{0}^{1} x\sqrt{1-x^2}\,dx$

(h) $\displaystyle\int (x^3 + x^2) \sin\left(x^4 + \frac{4}{3}x^3\right)\, dx$

(i) $\displaystyle\int_0^1 \frac{x}{\sqrt{x^2+4}}\, dx$

(j) $\displaystyle\int x^2 \left(x^3+1\right)^{\frac{3}{2}}\, dx$

(k) $\displaystyle\int_0^{\frac{\pi}{2}} \sin(x) e^{\cos(x)}\, dx$

(l) $\displaystyle\int x^3 \cos\left(x^4\right)\, dx$

(m) $\displaystyle\int_0^1 \sqrt{1-x^2}\, dx$

Hint: try $x = \sin(u)$ together with $dx = \dfrac{dx}{du}du$ and $\sin^2(u) + \cos^2(u) = 1$.
Note this question is a good bit harder than the other questions, and I won't give you anything like this in the assignments or the exam.

4. Find the following integrals.

Note that these can be done using Table 1 of Chapter 4 of the course notes together with integration by parts and the Sum and Multiple Rules.

(a) $\displaystyle\int 3xe^{2x}\, dx.$

(b) $\displaystyle\int_0^{\pi} 5x\cos(2x)\, dx.$

(c) $\displaystyle\int x\ln(x)\, dx.$

(d) $\displaystyle\int_1^2 \left(x^4+3\right)\ln(x)\, dx.$

(e) $\displaystyle\int x^2\sin(x)\, dx.$

(f) $\displaystyle\int_0^1 x^2 e^x\, dx.$

5. Find the following integrals.

Note that these can be done using Table 1 of Chapter 4 of the course notes together with partial fractions, integration by substitution and the Sum and Multiple Rules.

(a) $\displaystyle\int \frac{3x+2}{x^2-4}\, dx$ (where $x > 2$).

(b) $\displaystyle\int_0^1 \frac{-1}{x^2+9x+20}\, dx.$

(c) $\displaystyle\int \frac{-x-8}{x^2+4x}\, dx$ (where $x > 0$).

(d) $\displaystyle\int_2^3 \frac{3x^2+4x-1}{(x^2-1)(x+2)}\,dx.$

(e) $\displaystyle\int \frac{x-1}{(x-2)^2}\,dx$ (where $x>2$).

(f) $\displaystyle\int_0^1 \frac{3x^2+6x+4}{(x^2+2x+2)\,(x+1)}\,dx.$

6. (a) Find the area lying between the graph of $f(x)=x^7$ and the x-axis between the points $x=-1$ and $x=1$.

   (b) Find the area lying between the graph of $f(x)=\cos(3x)$ and the x-axis between the points $x=-\dfrac{\pi}{3}$ and $x=0$.

   (c) Find the area lying between the graph of $f(x)=e^{2x}$ and the x-axis between the points $x=-1$ and $x=1$.

   (d) Find the area lying between the graph of $f(x)=x^3-2x^2-x+2$ and the x-axis between the points $x=-2$ and $x=0$. Hint: The graph of this function only crosses the x-axis at $x=-1$ in the interval $[-2,0]$.

7. (a) Find the volume of revolution of the function $f(x)=x^2$ about the x-axis between $x=0$ and $x=1$.

   (b) Find the volume of revolution of the function $f(x)=\sqrt{\sin(x)}$ about the x-axis between $x=0$ and $x=\pi$.

   (c) Find the volume of revolution of the function $f(x)=-e^{-x}$ about the x-axis between $x=0$ and $x=1$.

   (d) Find the volume of revolution of the function $f(x)=\sin(x)$ about the x-axis between $x=0$ and $x=\pi$. Hint: Use the formula $\sin^2(x)=\dfrac{1+\cos(2x)}{2}$.