



University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 2 EXAMINATION 2015/2016

MATH00040

Access to Science, Engineering and Agriculture: Mathematics 2

Professor G. McGuire

Dr. Anthony Cronin

Dr. Anthony Brown*

Time Allowed: 2 hours

Instructions for Candidates

Candidates should attempt all questions.

The exam will be marked out of 75 marks.

Not all questions are allocated the same number of marks.

Notes for Invigilators

Non programmable calculators are permitted.

The statistical tables provided are permitted.

The formula sheet provided is permitted.

1. (a) Determine if the following matrix operations can be performed and if so perform them.

$$\begin{pmatrix} -1 & 2 \\ 3 & -1 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 0 \\ 2 & -1 & -2 \end{pmatrix}$$

and $\begin{pmatrix} 0 & -1 & 2 \\ -1 & 0 & 3 \end{pmatrix} - 3 \begin{pmatrix} 1 & 3 & -1 \\ -3 & -1 & 0 \end{pmatrix}.$

[4]

- (b) Determine if the matrix $\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$ has an inverse and find it if it has. [2]

- (c) Determine if the following matrix operations can be performed and if so perform them.

$$\begin{pmatrix} -1 & -2 \\ 3 & -2 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} -1 & 4 \\ 2 & -3 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}^T.$$

[5]

- (d) Find the determinant of $\begin{pmatrix} 1 & -1 & -3 \\ -2 & -3 & 4 \\ 1 & -2 & -5 \end{pmatrix}.$ [4]

- (e) Using row reduction, determine if the following system of linear equations has a solution and give the solution if it has.

$$\begin{aligned} 2x - y + z &= 1 \\ x + y - z &= 8 \\ -x + 3y + 2z &= -8 \end{aligned}$$

[6]

- (f) Find the eigenvalues and corresponding eigenvectors of the matrix $\begin{pmatrix} -2 & 3 \\ -2 & 5 \end{pmatrix}.$ [6]

2. (a) For $z = 2 - i$ and $w = 4 + i$, calculate $|z|$, \bar{z} , $\operatorname{Re}(z)$, $\operatorname{Im}(z)$, $z + w$, $z - w$, zw and $\frac{z}{w}$. [5]
- (b) Convert $-\sqrt{3} + i$ into polar form and hence calculate $(-\sqrt{3} + i)^3$, expressing your final answer both in polar form and in Cartesian form. [4]
- (c) Given that $-1 + i = \sqrt{2} \left(\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \right)$, calculate all the third roots of $-1 + i$, leaving your answers in polar form. [3]
3. (a) (i) Classify all the critical points of the function $f(x) = -2x^3 + 12x^2 + 30x - 10$. [4]
- (ii) Find the points where the global maximum and minimum of the function

$$f: [0, 4] \rightarrow \mathbb{R}$$

$$x \mapsto x^3 - 9x^2 + 15x + 3$$

occur. [4]

- (b) Differentiate the functions

$$f(x) = \frac{e^{-2x} \sin(3x)}{\ln(x)} \quad (\text{where } x > 1)$$

and $g(x) = \cos(2x^3 - 2x^2 - x + 3)$.

[6]

4. (a) (i) Find the area lying between the graph of $f(x) = \cos(3x)$ and the x -axis between the points $x = 0$ and $x = \frac{\pi}{3}$. [3]
- (ii) Find the volume of revolution of the function $f(x) = \sqrt{1 - x^3}$ about the x -axis between $x = -1$ and $x = 1$. [2]
- (b) (i) Find

$$\int 2xe^{-3x} dx.$$

[4]

- (ii) Evaluate

$$\int_3^4 \frac{3x + 2}{x^2 - 4} dx.$$

[4]

5. (a) In a supermarket survey, it was found that the probability that a customer likes chocolate ice cream is 0.75 and the probability that a customer likes both chocolate ice cream and strawberry ice cream is 0.45. What is the probability that a customer who likes chocolate ice cream also likes strawberry ice cream? [3]
- (b) Suppose that the heights of adult females in Ireland are normally distributed with mean 170cm and standard deviation 8cm. What is the probability of a woman chosen at random in Ireland being shorter than 174cm? [3]
- (c) A particular type of bacterium is randomly distributed in a certain river at an average concentration of one per 50 cm^3 of water. If we draw from the river a test tube containing 200 cm^3 of water, what is the chance that the sample contains at least four of these bacteria? [3]

—o0o—