



University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 2 EXAMINATION 2016/2017

MATH00040

Access to Science, Engineering and Agriculture: Mathematics 2

Professor G. McGuire

Dr. Anthony Cronin

Dr. Anthony Brown*

Time Allowed: 2 hours

Instructions for Candidates

Candidates should attempt all questions.

The exam will be marked out of 75 marks.

Not all questions are allocated the same number of marks.

Notes for Invigilators

Non programmable calculators are permitted.

The statistical tables provided are permitted.

The formula sheet provided is permitted.

1. (a) Determine if the following matrix operations can be performed and if so perform them.

$$2 \begin{pmatrix} 1 & -2 \\ 3 & -1 \\ -4 & 0 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ -3 & -1 \\ -2 & 1 \end{pmatrix}$$

$$\text{and} \begin{pmatrix} 0 & -1 \\ -1 & 0 \\ 3 & 2 \end{pmatrix} - 2 \begin{pmatrix} 1 & 3 \\ -3 & -1 \end{pmatrix}.$$

[4]

- (b) Determine if the matrix $\begin{pmatrix} -1 & 1 \\ -2 & 3 \end{pmatrix}$ has an inverse and find it if it has. [2]

- (c) Determine if the following matrix operations can be performed and if so perform them.

$$\begin{pmatrix} -1 & -2 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} -1 & 4 & 2 \\ 2 & -3 & -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 2 & -3 \end{pmatrix}^T.$$

[5]

- (d) Find the cross product $(1, 2, -1) \times (-1, 2, 3)$. [4]

- (e) Using row reduction, determine if the following matrix has an inverse and give the inverse if it has.

$$\begin{pmatrix} 1 & 1 & -4 \\ 2 & 1 & -6 \\ -3 & -1 & 9 \end{pmatrix}$$

[6]

- (f) Find the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}.$$

[6]

2. (a) For $z = 1 - 2i$ and $w = -2 + 3i$, calculate $|z|$, \bar{z} , $\operatorname{Re}(z)$, $\operatorname{Im}(z)$, $z + w$, $z - w$, zw and $\frac{z}{w}$.
 Note that the first six calculations are worth a half mark each, and the last two are worth one mark each. [5]
- (b) Convert $-\sqrt{3} - i$ into polar form and hence calculate $(-\sqrt{3} - i)^3$, expressing your final answer both in polar form and in Cartesian form. [4]
- (c) Given that $-1 + \sqrt{3}i = 2 \left(\cos \left(\frac{2\pi}{3} \right) + i \sin \left(\frac{2\pi}{3} \right) \right)$, calculate all the third roots of $-1 + \sqrt{3}i$, leaving your answers in polar form. [3]
3. (a) (i) Classify all the critical points of the function $f(x) = -2x^3 - 9x^2 + 24x - 1$. [4]
- (ii) Find the points where the global maximum and minimum of the function

$$f: [0, 4] \rightarrow \mathbb{R}$$

$$x \mapsto x^3 - 3x^2 - 9x + 12$$

occur. [4]

- (b) Differentiate the functions

$$f(x) = \frac{\ln(2x) \cos(3x)}{e^{4x}} \quad (\text{where } x > 0)$$

$$\text{and } g(x) = e^{2x^3 - x^2 + 2x - 4}$$

[6]

4. (a) (i) Find the area lying between the graph of $f(x) = \sin(3x)$ and the x -axis between the points $x = -\frac{\pi}{6}$ and $x = \frac{\pi}{3}$. [3]
- (ii) Find the volume of revolution of the function $f(x) = \sqrt{\cos(x)}$ about the x -axis between $x = 0$ and $x = \frac{\pi}{2}$. [2]

- (b) (i) Find

$$\int 2x \sin(3x) dx.$$

[4]

- (ii) Evaluate

$$\int_0^3 \frac{x}{\sqrt{x^2 + 16}} dx.$$

[4]

5. (a) In a group of 95 undergraduate students at UCD, there are 58 first years, 44 females and 26 female first years. If one of these students is selected at random, what is the probability that they will either be a first year or male? [3]
- (b) It has been observed that the average rate of cars arriving at a petrol station between 5pm and 6pm on a Sunday evening is one per five minutes. What is the probability of less than seven cars arriving between 5pm and 5.30pm on Sunday? [3]
- (c) A particular brand of light bulb is known to have a life normally distributed with mean 1425 hours and standard deviation 100 hours. What is the probability of a randomly selected light bulb of this brand lasting between 1300 hours and 1500 hours? [3]

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