



University College Dublin  
An Coláiste Ollscoile, Baile Átha Cliath

**SEMESTER 2 EXAMINATION 2017/2018**

**MATH00040**

**Access to Science, Engineering and Agriculture: Mathematics 2**

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**Time Allowed: 2 hours**

**Instructions for Candidates**

Candidates should attempt all questions.

The exam will be marked out of 75 marks.

Not all questions are allocated the same number of marks.

**Notes for Invigilators**

Non programmable calculators are permitted.

The statistical tables provided are permitted.

The formula sheet provided is permitted.

1. (a) Determine if the following matrix operations can be performed and if so perform them.

$$2 \begin{pmatrix} 1 & -2 \\ 3 & -1 \\ -4 & 0 \end{pmatrix} - \begin{pmatrix} 2 & -1 & -3 \\ -1 & -2 & 1 \end{pmatrix}$$

$$\text{and} \quad \begin{pmatrix} 0 & -1 & -1 \\ 1 & 3 & 2 \end{pmatrix} - 2 \begin{pmatrix} 1 & 3 & -2 \\ -3 & -1 & 2 \end{pmatrix}.$$

[4]

- (b) Determine if the following matrix operations can be performed and if so perform them.

$$\begin{pmatrix} -2 & 3 & 2 \\ -2 & 3 & -2 \end{pmatrix} \begin{pmatrix} -1 & -2 \\ -3 & 2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 & 2 & -3 \end{pmatrix}^T \begin{pmatrix} 1 & -2 & 3 \end{pmatrix}.$$

[5]

- (c) Find the angle (in radians to 2 decimal places) between the vectors  $(1, 2, 1)$  and  $(-1, 2, 2)$ .

[2]

- (d) Find the determinant of the matrix  $\begin{pmatrix} 1 & -4 & 1 \\ 1 & 6 & -3 \\ -2 & 3 & 0 \end{pmatrix}$ .

[4]

- (e) Using row reduction, determine if the following system of linear equations has a solution and give the solution if it has.

$$3x - 11y - 3z = 3$$

$$2x - 6y - 2z = 1$$

$$5x - 17y - 6z = 2$$

$$4x - 8y = 7$$

[6]

- (f) Find the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}.$$

[6]

2. (a) For  $z = 2 - i$  and  $w = -3 + 2i$ , calculate  $|z|$ ,  $\bar{z}$ ,  $\operatorname{Re}(z)$ ,  $\operatorname{Im}(z)$ ,  $z + w$ ,  $z - w$ ,  $zw$  and  $\frac{z}{w}$ .  
 Note that the first six calculations are worth a half mark each, and the last two are worth one mark each. [5]
- (b) Convert  $-2 + 2i$  into polar form and hence calculate  $(-2 + 2i)^4$ , expressing your final answer both in polar form and in Cartesian form. [4]
- (c) Given that  $-1 - \sqrt{3}i = 2 \left( \cos \left( -\frac{2\pi}{3} \right) + i \sin \left( -\frac{2\pi}{3} \right) \right)$ , calculate all the third roots of  $-1 - \sqrt{3}i$ , leaving your answers in polar form. [3]
3. (a) (i) Classify all the critical points of the function  $f(x) = x^3 - 3x^2 - 9x - 7$ . [4]

Find the points where the global maximum and minimum of the function

$$f: [-1, 0] \rightarrow \mathbb{R}$$

$$x \mapsto e^{-3x} + 7x$$

occur. [4]

- (b) Differentiate the functions

$$f(x) = \frac{e^{-2x} \cos(4x)}{\ln(3x)} \quad (\text{where } x > 0)$$

$$\text{and } g(x) = \sin(-2x^3 + 3x^2 + x - 5)$$

[6]

4. (a) (i) Find the area lying between the graph of  $f(x) = \cos(2x)$  and the  $x$ -axis between the points  $x = 0$  and  $x = \frac{3\pi}{4}$ . [3]
- (ii) Find the volume of revolution of the function  $f(x) = -e^{-x}$  about the  $x$ -axis between  $x = 0$  and  $x = 1$ . [2]

- (b) (i) Find

$$\int -\frac{1}{x^2 + 9x + 20} dx.$$

[4]

- (ii) Evaluate

$$\int_0^{\frac{\pi}{2}} \sin(x)e^{\cos(x)} dx.$$

[4]

5. (a) In a supermarket survey, it was found that the probability that a customer likes chocolate ice cream is 0.55 and the probability that a customer likes both chocolate ice cream and strawberry ice cream is 0.45. What is the probability that someone who likes chocolate ice cream also likes strawberry ice cream? [3]
- (b) Suppose that we are given a bag containing four white balls and six black balls and suppose that we draw eleven balls from the bag, replacing each ball before drawing the next. What is the probability of drawing at least eight black balls? [3]
- (c) Suppose that the heights of adult females in Ireland are normally distributed with mean 170cm and standard deviation 8cm. What is the probability of a woman chosen at random in Ireland being taller than 176cm? [3]

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