



University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SPECIMEN EXAMINATION 2015/2016

MATH00040

Access to Science, Engineering and Agriculture: Mathematics 2

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Time Allowed: 2 hours

Instructions for Candidates

Candidates should attempt all questions.

The exam will be marked out of 75.

Not all questions are allocated the same number of marks.

Notes for Invigilators

Non programmable calculators are permitted.

The statistical tables provided are permitted.

The formula sheet provided is permitted.

1. (i) (a) Determine if the following matrix operations can be performed and if so perform them.

$$\begin{pmatrix} -3 & 1 \\ 2 & -1 \\ -1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & -1 & 1 \\ -2 & 2 & -2 \end{pmatrix}$$

$$\text{and } 3 \begin{pmatrix} 0 & -2 \\ -1 & -3 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ -3 & -1 \\ 0 & 0 \end{pmatrix}.$$

[4]

- (b) Find the length of the vector $(1, -2, 2)$ and hence find a unit vector in the same direction as $(1, -2, 2)$.

[2]

- (ii) (a) Determine if the following matrix operations can be performed and if so perform them.

$$\begin{pmatrix} -2 & -1 \\ -2 & 3 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} -1 & 4 \\ 2 & -3 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}^T \begin{pmatrix} 0 & 1 & -5 \end{pmatrix}.$$

[5]

- (b) Find the cross product $(3, 2, 1) \times (-1, -2, -3)$.

[4]

- (iii) (a) Using row reduction, determine if the matrix $\begin{pmatrix} 1 & -2 & 1 \\ -1 & 1 & -1 \\ 1 & -3 & 0 \end{pmatrix}$ has an inverse and find it if it exists.

[6]

- (b) Find the eigenvalues and corresponding eigenvectors of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$.

[6]

2. (i) For $z = 1 + 2i$ and $w = 3 - i$, calculate

$$|z|, \quad \bar{z}, \quad \operatorname{Re}(z), \quad \operatorname{Im}(z), \quad z + w, \quad z - w, \quad zw \quad \text{and} \quad \frac{z}{w}.$$

[5]

- (ii) Convert $\sqrt{3} - i$ into polar form and hence calculate $(\sqrt{3} - i)^4$, expressing your final answer both in exact polar form and in Cartesian form correct to three decimal places.

[4]

- (iii) Given that $1 + \sqrt{3}i = 2 \left(\cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right) \right)$, calculate all the third roots of $1 + \sqrt{3}i$, leaving your answers in polar form.

[3]

3. (i) (a) By finding the second derivative, classify all the critical points of the function $f(x) = -2x^3 + 3x^2 + 36x - 10$. [4]

- (b) Find the points where the global maximum and minimum of the function

$$f: [0, 5] \rightarrow \mathbb{R}$$

$$x \mapsto x^3 - 9x^2 + 24x - 15$$

occur. [4]

- (ii) Differentiate the functions

$$f(x) = \frac{e^{2x} \cos(3x)}{x^4 \ln(x)} \quad (\text{where } x > 0)$$

and $g(x) = \cos(x^3 + 2x^2 - x)$. [6]

4. (i) (a) Find the area lying between the graph of $f(x) = x^3 - 2x^2 - x + 2$ and the x -axis between the points $x = -2$ and $x = 0$. Hint: The graph of this function only crosses the x -axis at $x = -1$ in the interval $[-2, 0]$. [3]

- (b) Find the volume of revolution of the function $f(x) = \sqrt{1 - x^2}$ about the x -axis between $x = -1$ and $x = 1$. [2]

- (ii) (a) Find

$$\int -2x \cos(-3x) dx.$$

[4]

- (b) Evaluate

$$\int_{-1}^1 x^2(2x^3 + 3)^6 dx.$$

[4]

5. (i) In the UCD School of Mathematical Sciences, the teaching staff consists of twenty lecturers and thirty five tutors. Twelve of the lecturers are male and seventeen of the tutors are female. What is the probability of a staff member picked at random being either a tutor or a male? [3]

- (ii) Suppose that we toss a fair coin fifteen times. What is the probability of getting exactly nine heads? [3]

- (iii) It has been observed that the average rate of cars arriving at a petrol station between 5 pm and 6 pm on a Sunday evening is one per five minutes. What is the probability of at least five cars arriving between 5.30 pm and 6 pm on Sunday? [3]