LIQUIDITY AND FEASIBLE DEBT RELIEF*

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March 2009

Abstract

This paper analyzes the determinants of secondary debt market liquidity, identifying conditions under which trading in competitive markets results in sufficient ownership concentration to induce ex post efficient debt relief. The feasibility of debt relief is path-dependent, hinging upon interim economic conditions. Secondary debt markets are likely to freeze during recessions, precisely when trading has high social value. This is due to three factors: severe free-riding reduces profits of large bondholders; uninformed small bondholders are reluctant to sell due to high informational sensitivity of debt; and large investors are more likely to face wealth constraints. However, there are potentially countervailing effects facilitating trade in bad states. Specifically, recessions may cause a broader set of uninformed bondholders to face intense liquidity shocks. The intensity of such shocks encourages them to sell, while the breadth of the shock facilitates concentrated ownership. Both effects ultimately promote voluntary debt relief by a large investor.

1 Introduction

As argued by Shleifer (2003), the sale of debt to a large number of dispersed lenders can deter borrowers from requesting debt relief despite their having the ability to pay. However, it has also been argued that widely-held debt interferes with the provision of debt relief for legitimately distressed borrowers. For example, Gilson, John and Lang (1990) find that broad ownership inhibits

*We thank Patrick Bolton, Francesca Cornelli, James Dow, Ron Giammarino, Neal Stoughton, Vikrant Vig and the seminar participants of HKUST, National University of Singapore, Singapore Management University, UNSW, and the University of Melbourne for helpful feedback. We are particularly grateful to Jin Yu and Natalia Ivanova for their research assistance.

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restructuring of corporate debt. Bolton and Jeanne (2007) argue that dispersed ownership makes it more difficult to achieve sovereign debt relief. Finally, a large number of commentators, e.g. Eggert (2007), have pointed to the diffuse ownership of mortgages arising from securitization as a principle factor preventing debt relief for distressed homeowners. Consistent with this view, Piskorski, Seru and Vig (2008) find that securitized loans have a higher foreclosure rate than loans held on a bank’s balance sheet, ceteris paribus. The economic costs of the failure to restructure debt can be significant. For example, Posner and Zingales (2009) estimate the deadweight cost due to failed mortgage renegotiations in the US to be $300 billion.

This raises a natural question: If debt restructuring is efficient ex post, will the market bring it about? That is, why doesn’t trading of debt claims in secondary markets bring about sufficiently concentrated ownership such that debt relief becomes feasible? This is not simply an academic question given that market-based TARP Investment Companies are the centerpiece of the Geithner Plan for resolving the credit crisis. In fact, some commentators contend the Geithner Plan does not go far enough in relying upon the market. For example, Puchala and Goldhill (2009) argue, “Unfortunately, early indications are that the scale and true risk of private capital will be very limited, putting private investors in a passive role in those elements where they can make the greatest contribution-valuation and work-out management.” Unfortunately, this debate falls into a theoretical void given that we lack a benchmark model for understanding the liquidity of secondary debt markets and the interplay between those markets and the debt relief process. The objective of our paper is to fill this void.

The model opens with an agent selling debt in a primary market. Following Hart and Moore (1994), the agent has the ability to withdraw human capital from the underlying project at an implementation stage, when his skills are essential. The prospect of strategic default induces dispersed debt ownership in the primary debt market, since atomistic lenders rebuff requests for debt relief. The focus of the analysis is on equilibrium in the secondary debt market where the atomistic bondholders have the option to trade with each other, as well as an anonymous large investor. We
are particularly interested in conditions under which the ownership structure of debt can efficiently shift from dispersed to concentrated in light of the potential for costly insolvency.

The model of the secondary debt market is in the spirit of the seminal work of Kyle (1985), making three critical departures. First, we analyze a concave debt claim. Second, there is a feedback effect from ownership structure to fundamental asset value since the former influences incentives for debt relief. Finally, we depart from the standard noise-trader assumption of Kyle and consider the trading incentives of uninformed atomistic bondholders facing preference shocks that bias them towards liquidating the bond positions they acquired in the primary market. That is, in our model atomistic bondholders trade-off liquidity preference against adverse selection costs in making their trading decisions. As shown below, each model feature plays an important part in explaining secondary market liquidity and the probability of achieving debt relief.

The model delivers a number of important insights regarding factors conducive to trade in secondary debt markets, concentrated ownership, and debt relief. The feasibility of debt relief ex post depends on interim economic conditions. In a weak interim economy, the probability of concentrated debt ownership is reduced by three factors: high free-riding costs lower the profits of the large bondholder; uninformed bondholders are reluctant to liquidate due to high informational sensitivity of debt in bad states; and large investors are more likely to face binding wealth constraints. In fact, we find that the wealth constraint channel has the potential to induce a large discontinuous reduction in the probability of debt relief. In this way, the model can explain the seizing up of secondary debt markets at precisely the points in time when trading of debt, from atomistic to large investors, has the highest social value.

However, the model also shows that secondary debt markets need not become illiquid during bad times. As recent events illustrate, recessions and panics are associated with strong liquidity demand. This liquidity demand can induce debt trading in cases where adverse selection might have otherwise caused atomistic investors to hold onto their debt claims. In addition to intensifying the demand for liquidity, bad economic times are also likely to broaden liquidity demand. We show that
broader liquidity shocks have the potential to promote concentrated debt ownership, and debt relief, since they provide better camouflage for large investors. More generally, these results show that liquidity shocks can have a beneficial effect in promoting trade. Intuitively, liquidity shocks induce trade that is not information-motivated. It follows that attempts to prop up distressed bondholders may have an unintended side-effect, inhibiting the formation of an ex post efficient concentration of debt ownership.

The model also delivers subtle comparative static insights. For example, the probability of debt relief can be non-monotone in the breadth of liquidity shocks hitting atomistic investors. In general, broader liquidity shocks increase the likelihood of debt relief. This is because broader liquidity shocks allow the large investor to acquire a larger stake. Since atomistic investors hold a lower percentage of debt in such cases, free-riding costs tend to fall. However, there is a competing effect that serves to increase free-riding costs. Specifically, atomistic bondholders may anticipate that a large debt stake will cause the large investor to grant debt relief in more states of nature ex post. When this second effect dominates, the probability of debt relief actually declines in the breadth of liquidity shocks.

Another subtle set of insights delivered by the model concerns correlation between liquidity shocks and the real economy. We find that the higher probability of liquidity shocks during recessions may encourage liquidation by uninformed bondholders. Such bondholders are exposed to adverse selection whenever order flow is not fully revealing. Intermediate levels of order flow confound market makers, and lead them to set pooling prices, since such order flows can arise from either: entry by the large investor cum liquidity shock or no entry and no shock. When the probability of a liquidity shock is high, market makers assign a higher probability to the former combination being responsible for intermediate order flow. The equilibrium price of debt in non-revealing states rises, encouraging liquidation by uninformed bondholders.

We turn now to related literature. A variant of the free-rider problem at the heart of our model was first analyzed by Grossman and Hart (1980) in the context of hostile takeovers in equities.
markets. Closer to our model is that developed by Shleifer and Vishny (1986), who analyze the interplay between free-ridership and the endogenous ownership structure of equity. They show that with public trading, a large investor will never find it profitable to acquire a stake consistent with takeover. Our model is also similar to those of Maug (1998) and Mello and Repullo (2004), who develop noise-trader models to analyze the incentive to implement value-enhancing takeovers with endogenous equity ownership structure. The key difference between the models is that we analyze debt, debt relief, and consider endogenous liquidation decisions by uninformed investors.

Dooley (2000), Shleifer (2003) and Bolton and Jeanne (2007) share our argument that dispersed debt may arise as an ex ante efficient response to borrower moral hazard. Gertner and Scharfstein (1991) analyze the use of exchange offers to overcome free-riding by atomistic lenders. They show a necessary condition for a successful exchange offer is that the new claim have higher priority. We do not explicitly analyze exchange offers, although one can view the default costs in our model as being a reduced-form representation of the costs of making an exchange offer. As argued by Gilson (1991), exchange offers are not costless, although they are less costly for corporations than the Chapter 11 process. Another reason for ruling out exchange offers is that it might be optimal ex ante to rule out such offers via bond covenants given that exchange offers can actually be used to expropriate bondholders.

There is a voluminous literature building upon the pioneering work of Kyle (1985) and Glosten and Milgrom (1985) which analyzes trading in markets occupied by informed and noise traders. However, most of this literature is concerned with linear equity claims and does not consider the role of adverse selection in deterring trade by uninformed investors. Further, there is no feedback from ownership structure to fundamental value in those models.

Our paper also contributes to a recent literature that attempts to explain bouts of extreme illiquidity. The liquidity preference in our model can be viewed as a reduced-form representation of the recent margin-constrained investor model of Brunnermeier and Pedersen (2008), for example. A key differentiating factor is that our model predicts that forced liquidations can actually be beneficial
in that the increase in non-informational liquidations serves to promote concentrated ownership and efficient debt relief.

2 The Economic Setting

There are four dates \( t \in \{t_0, t_1, t_2, t_3\} \) and two investor classes. There is a large investor \( I \) and a continuum of ex ante identical investors \( N \) with generic member \( n \). Each investor class enters the economy at \( t_0 \) endowed with one unit of the numeraire good. At \( t_0 \) there is a single financial asset that can be used to transfer wealth intertemporally, a debt claim with face value normalized at 1 sold in the primary market by agent \( A \). In addition, investors have access to a safe storage technology.

Consumption can take place at \( t_2 \) or \( t_3 \). The large investor is patient, having utility

\[
U^I = c_2 + c_3. \tag{1}
\]

Each atomistic investor has utility

\[
U^n = c_2 + [\chi^n(1 - \tau) + (1 - \chi^n)]c_3 \tag{2}
\]

where \( \tau \in (0, 1) \) and \( \chi^n \) is an indicator for the investor being impatient. Each investor \( n \) observes his own realized \( \chi^n \) just prior to secondary market trading at \( t_2 \).

The debt claim sold by \( A \) is backed by a risky project paying \( Y \in \{L, M, H\} \) at \( t_3 \). To capture default risk, assume

\[
A1 : 0 < L < M < 1 < H.
\]

As shown in Figure 1, the project payoffs result from a recombining binomial tree with jump probabilities equal to one-half at all nodes. In order for the project to generate a final payoff, it is essential that \( A \) implement the project at date \( t_1 \). If \( A \) quits at this point in time, the project is worthless. Once the implementation stage has been completed, \( A \) is no longer necessary and investors need only wait for \( Y \) to be realized at \( t_3 \). Although stylized, this feature of the model
Figure 1: Project Payoffs.

captures the idea that a founding manager-entrepreneur is likely to have skills and knowledge that are critical for early-stage success. After that stage, firms can and do often turn to the pool of professional managers to run the firm.

The ownership structure of debt is common knowledge and $F_j$ denotes the face value of debt outstanding at the start of period $t_j$. At the start of period $t_1$, $A$ has the ability to exploit the fact that his skills are essential for project implementation. In particular, he can make a take-it or leave-it offer to bondholders requesting debt relief: “If the total face value of debt is not voluntarily reduced to $F_1 < 1$, I will quit.” In this way, the model captures the strategic default problem stemming from inalienable human capital as described by Hart and Moore (1994).

Given the possibility of strategic default at the implementation stage, investor $I$ will not buy any debt in the primary market, regardless of price. To see this, suppose to the contrary that $I$ buys a fraction $s_0 \in (0, 1]$ of the debt at $t_0$. Then $A$ will propose $F_1 = 1 - (s_0 - \varepsilon)$ where $\varepsilon$ is arbitrarily small. For the atomistic investors, it is a weakly dominant strategy to reject requests for debt relief. However, investor $I$ is pivotal and will forgive $s_0 - \varepsilon$ of his debt. Anticipating such an outcome, $I$ will never buy any debt in the primary market. Thus, the free-rider problem in debt relief makes atomistic lenders tough, which is precisely what is necessary when confronting the threat of strategic default. However, the same free-rider problem also causes atomistic lenders to refuse to grant debt relief ex post when confronting the prospect of bona fide insolvency.

$^1$Consequently, the equilibrium in our model differs from that in the takeover model of Bagnoli and Lipman (1988) featuring any-and-all tender offers. In their model, it is optimal for an atom to accept if all other investors reject. In our model, an atom is indifferent between accept and reject if other investors reject, since he gets zero regardless.
The interim state, denoted \( \omega \in \{d, u\} \), is common knowledge at the start of \( t_2 \). We let \( \sigma^*_\omega \) denote the equilibrium probability of investor \( I \) buying debt in the secondary market when the interim state is \( \omega \). Since the large investor buys debt in order to restructure debt, \( \sigma^*_\omega \) is equivalent to the probability of debt restructuring.

At the start of \( t_2 \), a fraction \( \gamma_\omega \leq 1/2 \) of \( N \) (randomly selected) become vulnerable to preference shocks. Each \( n \in N \) knows whether or not he is vulnerable. Invulnerable investors in \( N \) are time-neutral with \( \chi^n = 0 \). Further, the invulnerable investors receive \( 1 - \gamma_\omega \) units of aggregate endowment at time \( t_2 \). For reasons that will become apparent, invulnerable investors in \( N \) are labeled \textit{market makers} (\textit{MM} below). For now it is sufficient to note that the large investor and market makers both apply a zero discount rate when assessing consumption at \( t_2 \) relative to \( t_3 \).

Vulnerable atomistic investors are labeled \textit{atoms} below, for brevity. The atoms are \textit{collectively} patient or impatient. If patient, atoms have \( \chi^n = 0 \) and are locked into their debt holdings due to transactions costs. Thus, patient atoms value debt at its fundamental long-term value. If impatient, atoms have \( \chi^n = 1 \), biasing them towards selling their debt at time \( t_2 \). Importantly, and in contrast to pure noise trading models, this simple setup with impatience allows us to treat the liquidation decision as an endogenous choice by uninformed bondholders. In our model, impatient atoms assess adverse selection costs before tendering their debt.

Recent events support the notion that investors place a premium on liquidity during contractions. To allow for this possibility, the liquidity preference parameter \( \tau \) is contingent upon \( \omega \), with

\[ A2 : \tau_d \geq \tau_u > 0. \]

The realized time preference of the atoms is private information to them. However, all agents know the state-contingent probability of atoms being impatient, \( \pi_\omega \). Thus, the parameter \( \pi_\omega \) captures the arrival intensity of liquidity shocks while \( \gamma_\omega \) captures the breadth of such shocks. To allow for the possibility that liquidity shocks hit a larger percentage of small investors during recessions, we assume

\[ A3 : \frac{1}{2} \geq \gamma_d \geq \gamma_u > 0. \]
For simplicity, we initially rule out the large investor facing any wealth constraints. However, our baseline analysis is readily extended to evaluate the effects of state-contingent wealth constraints.

We solve for the perfect Bayesian equilibrium (PBE) of the game commencing at time $t_2$. The game starts with nature publicly drawing the interim economic state $\omega$. Then nature draws the preference shock $\chi$. Given their respective information sets, the large investor and atoms submit orders to the $MM$. Their respective aggregate orders are denoted $x^I$ and $x^N$. There is no borrowing or short-selling. Following Kyle (1985), $MM$ observe total order flow $X$, and set prices competitively as is appropriate given that there is a measure $1 - \gamma_\omega$ continuum of market makers. After the secondary market clears, $I$ enters period $t_3$ with an endogenous stake $s$ in the debt claim. At this same point in time, the cash flow $Y$ is observed. If $Y = H$, lenders are paid 1 and the shareholder $A$ keeps $H - 1$. If $Y \in \{L, M\}$, lenders receive $Y(1 - \alpha)$ if there is no voluntary debt relief, where $\alpha \in (0, 1)$ captures deadweight costs of default. Alternatively, the pivotal investor $I$ can voluntarily forgive some debt. Iff $I$ forgives a sufficient amount of debt such that the resulting face value $F_3 = Y$, default costs are avoided.\(^2\) The measure zero bondholders would then be paid in full, receiving $1 - s$, leaving investor $I$ to collect $Y - (1 - s)$.

The time line in Figure 2 summarizes the model.

\(^2\)It is never optimal for the large investor to forgive more debt than needed to avoid default costs.
3 Baseline Model

To fix ideas, this section presents formal results derived under a set of baseline parameters.

\[ A2' : \tau_d = \tau_u = \tau \]
\[ A3' : \gamma_d = \gamma_u = \gamma. \]

Note that under these baseline assumptions, there is no difference between the up and down node in terms of the intensity or breadth of liquidity shocks. After the baseline analysis, we describe how incorporating state-contingent parameter values affects equilibrium.

As argued below, the correlation structure of liquidity shocks plays an important, yet more complex, role. This section deliberately turns off this causal mechanism to focus on other factors at work in determining the feasibility of debt relief. In particular, the baseline model assumes that the arrival intensity of liquidity shocks is equal across the interim nodes with

\[ A4 : \pi_d = \pi_u = \frac{1}{2}. \]

3.1 The final period

Suppose it is time \( t_3 \) and \( Y \in \{ L, M \} \), implying debt relief is necessary to avoid default costs. Given a debt stake \( s \), the large investor is willing to grant debt relief iff

\[ Y - (1 - s) \geq s(1 - \alpha)Y. \]  \( (3) \)

The following Lemma is a useful summary of the implications of the inequality above.

**Lemma 1.** A necessary condition for debt relief is that the large investor holds a stake at least as large as

\[ s(Y, \alpha) \equiv \frac{1-Y}{1-Y+\alpha Y}. \]  \( (4) \)

It is readily verified that the minimum stake \( s \) is decreasing in both \( Y \) and \( \alpha \), with

\[ \lim_{\alpha \downarrow 0} s(Y, \alpha) = 1 \]
\[ \lim_{\alpha \uparrow 1} s(Y, \alpha) = 1 - Y. \]
Lemma 1 is intuitive. If cash flow is high, debt relief is more likely since high cash flow reduces the transfer that debt relief provides to the hold-out atomistic bondholders. It is also apparent that the prospect of incurring high default costs encourages voluntary debt relief. For this reason, a more efficient bankruptcy forum can actually decrease social efficiency. This point is commonly overlooked in policy debates regarding bankruptcy reform.

3.2 Debt relief following expansions

Suppose it is the start of $t_2$ and the interim state is $u$. With probability one-half, debt relief will be necessary to avoid costly default. In particular, if the terminal node $M$ is reached, a necessary condition for successful debt restructuring is that the large investor hold a stake of at least $s(M,\alpha)$. Under what conditions will $I$ obtain such a stake via secondary market trading? To address this question consider the trading game taking place at time $t_2$.

We are interested in PBE such that debt relief occurs and compute prices consistent with that conjecture. In equilibrium, impatient atoms sell a block of endogenous size $\gamma^* \in (0,\gamma]$ in aggregate. For example, if impatient atoms strictly prefer to liquidate then $\gamma^* = \gamma$. In contrast, if impatient atoms are indifferent between liquidating and holding, then it is possible to support a PBE in which only a proper subset of them liquidates, inducing $\gamma^* < \gamma$.

In equilibrium the only way for $I$ to make weakly positive trading gains is to mask his trades by purchasing a block of size $\gamma^*$ whenever he buys debt in the secondary market. Further, he must play a mixed strategy. To this end, let $\sigma^*$ denote the equilibrium probability of $I$ placing a buy order in the PBE, which is then equal to the probability of debt relief.

Table 1 depicts outcomes in the trading game.
Table 1: Baseline order flow

<table>
<thead>
<tr>
<th>Buy</th>
<th>Shock</th>
<th>$x^I$</th>
<th>$x^N$</th>
<th>$X$</th>
<th>Price</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>N</td>
<td>$\gamma^*$</td>
<td>0</td>
<td>$\gamma^*$</td>
<td>$P^+_{\omega}$</td>
<td>$\frac{\sigma^*}{2}$</td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
<td>$\gamma^*$</td>
<td>$-\gamma^*$</td>
<td>0</td>
<td>$P^0_{\omega}$</td>
<td>$\frac{\sigma^*}{2}$</td>
</tr>
<tr>
<td>N</td>
<td>N</td>
<td>$\gamma^* - \gamma^* = 0$</td>
<td>0</td>
<td>0</td>
<td>$P^0_{\omega}$</td>
<td>$\frac{1-\sigma^*}{2}$</td>
</tr>
<tr>
<td>N</td>
<td>Y</td>
<td>$\gamma^* - \gamma^* = 0$</td>
<td>$-\gamma^*$</td>
<td>$-\gamma^*$</td>
<td>$P^-_{\omega}$</td>
<td>$\frac{1-\sigma^*}{2}$</td>
</tr>
</tbody>
</table>

Since $\gamma^* \leq 1/2$, the $MM$ have sufficient endowment and bondholdings to take the opposite side of any equilibrium order flow. If we step away from the standard assumption, e.g. Kyle (1985) that the $MM$ can only observe net order flow, the large investor can still confound the market makers by placing offsetting buy and sell orders as shown in the table.

Since $I$ obtains a debt stake of $\gamma^* \leq \gamma$ when he buys debt, a necessary condition for debt relief if $M$ is reached via $u$ is:

$$\gamma \geq g(M, \alpha).$$

(5)

If $\gamma < g(M, \alpha)$, debt relief is not feasible if $M$ is reached via $u$. That is, if liquidity shocks are too narrow, efficient debt relief is impossible to achieve. Intuitively, the large investor relies upon broad liquidity shocks to mask his buy orders. If the liquidity shocks only hit a low percentage of atomistic bondholders, the large investor cannot acquire a stake sufficiently large such that he is willing to grant debt relief ex post.

In all the analysis below, only three order flows occur on the equilibrium path, with

$$X \in \{-\gamma^*, 0, \gamma^*\}.$$  

(6)

When there is positive net order flow, the equilibrium is fully revealing and the $MM$ know the large investor is buying stake $\gamma^*$. At the opposite extreme, negative net order flow reveals that the large investor is not buying debt. At the up-node, the market makers set the following respective prices
upon observing positive and negative order flows:

\[ P_u^+ = 1 \]  
\[ P_u^- = \frac{1 + M(1 - \alpha)}{2}. \]

Zero net order flow is non-revealing, forcing the market makers to set a pooling price. In the baseline model, with \( \pi_\omega = 1/2 \), the MM respond to zero net order flow by setting the following price:

\[ P^0_\omega = \sigma P^+_\omega + (1 - \sigma)P^-_\omega. \]  

We first conjecture PBE in which all impatient atoms liquidate, i.e. \( \gamma^* = \gamma \). In order to confirm that the resulting trading outcomes are indeed a PBE we must verify that the large investor is playing an optimal strategy and then verify that impatient atoms prefer to liquidate.

At the up node, let \( G_u(\sigma, \hat{\gamma}) \) denote the expected trading gain perceived by the large investor in the event that with probability \( \sigma \) he places a buy order of size \( \hat{\gamma} \), where \( \hat{\gamma} \) is the measure of liquidating impatient atoms. The gain is equal to his expected payoff at time \( t_3 \) net of the expected price paid to acquire the stake \( \hat{\gamma} \):

\[ G_u(\sigma, \hat{\gamma}) = \frac{1}{2} \hat{\gamma} + \frac{1}{2}[M - (1 - \hat{\gamma})] - \frac{1}{2}[P^+_u + P^0_u]\hat{\gamma} \]

\[ = \frac{1}{2} \left[ \hat{\gamma}(1 - \sigma)(1 - M + \alpha M) \right] - \frac{1}{2}(1 - M) \]  

Since the function \( G_u \) is strictly decreasing in its first argument and increasing in its second argument, a necessary condition for the large investor to enter the secondary market is that \( G_u \) be strictly positive in the limit as \( \sigma \) converges to zero for \( \hat{\gamma} = \gamma \). From this one obtains the following necessary condition for the large investor to enter at the up node:

\[ \gamma > \gamma_u = 2g(M, \alpha). \]  

Once again, we see that broad liquidity shocks (high \( \gamma \)) are a necessary condition for debt relief. Further, since \( g \) is decreasing in \( \alpha \), condition (10) also reveals that high bankruptcy costs promote entry by the large investor. In fact, the necessary condition for debt relief specified in condition
(10) is more stringent than that specified in condition (5). The intuition is as follows. The latter condition simply ensures that the large investor would be willing to grant debt relief at time $t_3$, despite costs of free riding. The former condition ensures that the large investor can cover the costs of free-riding, a portion of which he will incur at time $t_3$, with additional free-riding being capitalized into debt prices at time $t_2$.

If $\gamma \leq \gamma_u$, the equilibrium entails $\sigma_u^* = 0$ and there is zero probability of debt relief. Since there is no informed trading in this case, the impatient atoms always liquidate and $\gamma_u^* = \gamma$. The market makers then take the opposite side of the trade.

The rest of this subsection considers the remaining case where $\gamma \geq \gamma_u$. In this case, there is a unique $\bar{\sigma}_u \in (0, 1)$ satisfying the large investor’s indifference condition $G_u(\bar{\sigma}_u, \gamma) = 0$, which must be satisfied given that he plays a mixed strategy. In particular,

$$\bar{\sigma}_u = 1 - \frac{2s(M, \alpha)}{\gamma}. \quad (11)$$

Throughout the paper, the variable $\bar{\sigma}_u$ captures endogenous investment incentives for the large investor. Differentiating equation (11) reveals that the buying intensity of the large investor should increase in the breadth of the liquidity shock and bankruptcy costs. In fact, the following signed comparative statics always hold, even when we move away from the baseline assumptions:

$$\frac{\partial \bar{\sigma}_u}{\partial \gamma} > 0, \quad \frac{\partial \bar{\sigma}_u}{\partial \alpha} > 0. \quad (12)$$

These comparative statics illustrate once again that broad liquidity shocks and higher bankruptcy costs have incentive effects which serve to promote debt market intervention by a large investor.

There exists a PBE with $\sigma_u^* = \bar{\sigma}_u$ iff each impatient atom prefers to sell his debt. This will hold if the expected value captured by selling is larger than the payoff to unilaterally deviating by holding onto the debt. If the large investor buys and forgives debt with probability $\sigma$, impatient atoms are willing to liquidate iff the expected payoff to selling exceeds the payoff to holding, or:

$$\sigma P_0^0 + (1 - \sigma)P_0^- \geq (1 - \tau)P_0^+ \iff \tau \geq \tau(\sigma, P_0^+, P_0^-) \equiv \frac{\sigma(1 - \sigma)(P_0^+ - P_0^-)}{P_0^- + \sigma(P_0^+ - P_0^-)}. \quad (13)$$
The function \( \tau \) captures the degree of adverse selection facing impatient atoms. This induces a simple trading rule: liquidate iff the liquidity preference, as captured by the parameter \( \tau \), is larger than the cost of adverse selection \( \tau \). Impatient atoms are always willing to trade for limiting values of \( \sigma \) since

\[
\lim_{\sigma \uparrow 1} \tau(\sigma, P_\omega^+, P_\omega^-) = \lim_{\sigma \downarrow 0} \tau(\sigma, P_\omega^+, P_\omega^-) = 0.
\]

Intuitively, impatient atoms are reluctant to trade due to fear of selling at too low a price given that the large investor has private information regarding his strategy. In particular, price is below fundamental value in the second row of Table 1 in the sense that a hold-out atom would capture a payoff of one if he deviated, which is greater than the pooling price \( P_\omega^0 \). If \( \sigma = 0 \), there is zero possibility of reaching that row. If \( \sigma = 1 \) that row can be reached, but the market makers set prices equal to fundamental value since there is no possibility of their confounding the second and third rows.

The function \( \tau(\cdot, P_\omega^+, P_\omega^-) \) reaches a unique maximum at an interior point denoted \( \sigma_{\omega}^{\text{max}} \). Further,

\[
\pi_\omega = \frac{1}{2} \Rightarrow \sigma_{\omega}^{\text{max}}(P_\omega^+, P_\omega^-) = \frac{-P_\omega^- + \sqrt{P_\omega^- P_\omega^+ + P_\omega^+ - P_\omega^-}}{P_\omega^+ - P_\omega^-} \in (0, 1). \quad (14)
\]

For sufficiently high values of \( \tau \), impatient atoms are willing to liquidate regardless of \( \sigma \), so that \( \tilde{\sigma}_u \) is an equilibrium outcome. Formally

\[
\tau \geq \tau(\sigma_{\omega}^{\text{max}}, P_\omega^+, P_\omega^-) \Rightarrow (\sigma^*_u, \gamma^*_u) = (\tilde{\sigma}_u, \gamma). \quad (15)
\]

Figure 3 depicts the remaining scenario where \( \tau < \tau(\sigma_{\omega}^{\text{max}}, P_\omega^+, P_\omega^-) \). The subscripts are omitted in the figure because the analysis is also relevant at the down node. There are two \( \sigma \) values satisfying \( \tau(\sigma, P^+, P^-) = \tau \), say \( \sigma_1^\omega \) and \( \sigma_2^\omega \). Checking the trading condition it follows that we can support the following PBE in which all impatient atoms liquidate:

\[
\tilde{\sigma}_u \leq \sigma_1^u \Rightarrow (\sigma_1^u, \gamma_u^u) = (\tilde{\sigma}_u, \gamma) \\
\tilde{\sigma}_u \geq \sigma_2^u \Rightarrow (\sigma_2^u, \gamma_u^u) = (\tilde{\sigma}_u, \gamma).
\]

(16)

The only remaining case is \( \tilde{\sigma}_u \in (\sigma_1^u, \sigma_2^u) \). In this case, \( \tilde{\sigma}_u \) cannot be sustained as a PBE since the impatient atoms are unwilling to liquidate. Here we can support a PBE in which the impatient
atoms play a “mixed strategy” in which only a proper subset of them liquidate. The reduction in the size of their block trade decreases trading gains for the large investor, which induces him to reduce $\sigma$ to $\sigma^u_1$, at which point the atoms are willing to mix. In this case, the low liquidity demand of the atoms reduces the probability of debt relief since $\sigma^*_u = \sigma^u_1 < \bar{\sigma}_u$. In this equilibrium, the atoms liquidate a block of size $\gamma^*_u = \gamma^{mix}_u < \gamma$ such that

$$G_u(\sigma^u_1, \gamma^{mix}_u) = 0.$$  \hspace{1cm} (17)

The following proposition summarizes the results derived in this subsection.

**Proposition 1 (Up Node).** If $\gamma \leq \gamma^*_u$, there is never any debt relief ($\sigma^* = 0$) and all impatient atoms liquidate ($\gamma^* = \gamma$). If $\gamma > \gamma^*_u$

$$\tau \geq \tau(\bar{\sigma}_u, P^+_u, P^-_u) \Rightarrow (\sigma^*_u, \gamma^*_u) = (\bar{\sigma}_u, \gamma)$$

$$\tau < \tau(\bar{\sigma}_u, P^+_u, P^-_u) \Rightarrow (\sigma^*_u, \gamma^*_u) = (\sigma^*_1, \gamma^{mix}_u).$$
Having evaluated the nature of equilibrium under the baseline parameters, it is now easy to analyze comparative static effects. As argued above, it may be reasonable to assume $\gamma_u$ is low, i.e. the breadth of liquidity shocks is low during good times. If the initial equilibrium features $\sigma_u^* = \sigma_1^u$, perturbing $\gamma_u$ has no effect on $\sigma_u^*$. If instead, $\sigma_u^* = \tilde{\sigma}_u$, a local reduction in $\gamma$ reduces $\sigma^*$. That is, low breadth of liquidity shocks during good times reduces the probability of debt relief. Intuitively, narrow liquidity shocks during good times reduces the equilibrium debt stake held by the large investor. At the up node, a reduction in large investor stakes has the unambiguous effect of increasing free-riding costs and deterring restructuring.

Consider next the effect of state-contingent liquidity preference intensity. During good times it is possible that $\tau_u$ is low, since atomistic bondholders face less urgency to come up with liquidity. In Figure 3, this effect would be captured by shifting down the horizontal $\tau$ line. If the initial equilibrium features $\sigma_u^* = \sigma_1^u$, reducing $\tau_u$ reduces $\sigma_u^*$. Putting this result together with the prior one, we arrive at the more general conclusion that low liquidity demand during good times serves to decrease the probability of debt relief, ceteris paribus.

Consider next comparative statics performed on the bankruptcy cost parameter $\alpha$. As shown above, higher bankruptcy costs increase the profitability of entry and $\tilde{\sigma}_u$. It follows that the equilibrium probability of debt relief is weakly increasing in $\alpha$. This again illustrates a beneficial incentive effect arising from the prospect of costly bankruptcy. Intuitively, higher bankruptcy costs reduce the equilibrium price of debt whenever there is a positive probability of costly default, increasing the return to acquiring a large debt stake.

In the interest of simplicity, we have assumed that the large investor is never wealth constrained, i.e. his initial wealth storage technology is riskless. However, wealth constraints are readily incorporated into the analysis. For example, one may suppose that the large investor’s initial storage technology (from $t_0$ to $t_2$) returns zero with probability $1 - \pi_\omega$. In this case, the large investor can buy debt on the secondary market with a maximum probability of $\pi_\omega$. To capture the notion that large investors are more likely to be wealth constrained during downturns, one may further assume
How does a wealth constraint for the large investor affect equilibrium? The effect is potentially dramatic. For example, if $\sigma^*$ described in Proposition 1 is less than $\sigma_u$, the wealth constraint has no effect on the equilibrium. If instead $\sigma^* > \sigma_u$, the equilibrium entry probability shifts to $\sigma_u$ provided $\tau \leq \tau(\sigma_u, P_u^+, P_u^-)$. However, if $\sigma^* > \sigma_u$ and $\tau > \tau(\sigma_u, P_u^+, P_u^-)$, the equilibrium entry probability exhibits a dramatic jump down to $\sigma_1^u$. That is, if large investors face wealth constraints, this can dramatically reduce the probability of debt relief. The absence of wealth constraints offer one mechanism, amongst others, supporting relatively liquid secondary debt markets during expansions.

### 3.3 Unconditional debt relief following contractions

Suppose next that node $d$ is reached. This subsection evaluates conditions under which a PBE can be supported such that restructuring will take place with positive probability regardless of which $Y \in \{L, M\}$ is realized, i.e. at both terminal nodes. In this case, the market makers would set the following prices upon observing positive and negative order flows, respectively:

\[
P_{db}^+ = 1 \quad \text{(18)}
\]
\[
P_{db}^- = \frac{(L+M)(1-\alpha)}{2}.
\]

Upon observing zero net order flow, the market makers set the pooling price in equation (8).

Following the same steps as in the preceding subsection, we compute the large investor’s expected gain to buying debt at the down node if debt relief would be subsequently granted at both nodes $L$ and $M$

\[
G_{db}(\sigma, \hat{\gamma}) = \frac{1}{2}[L - (1 - \hat{\gamma})] + \frac{1}{2}[M - (1 - \hat{\gamma})] - \frac{1}{2}[P_{d}^+ + P_{d}^0]\hat{\gamma} \quad \text{(19)}
\]

\[
= \frac{1}{2} \left[ \frac{\hat{\gamma}(1 - \sigma)(2 - (L + M)(1 - \alpha))}{2} - (2 - M - L) \right].
\]

Since $G_{db}$ is strictly decreasing in its first argument and increasing in its second, a necessary condition for debt relief is that this value is strictly positive as $\sigma$ converges to zero for $\hat{\gamma} = \gamma$. Thus, we arrive
at the following necessary condition for entry by the large investor under unconditional debt relief

$$\gamma > \gamma_{db} \equiv \frac{2(2 - L - M)}{2 - (L + M)(1 - \alpha)}.$$  \hfill (20)

If condition (20) is not satisfied, one cannot rule out entry at the down node. This is because the large investor may still find it profitable to enter the debt market in order to implement conditional debt relief whereby he forgives debt if node M is reached but not if node L is reached. Conditional debt relief is analyzed in the next subsection.

For the remainder of this subsection it is assumed that condition (20) is satisfied. In this case, there is a unique $$\tilde{\sigma}_{db} \in (0, 1)$$ satisfying the large investor’s mixing condition $$G_{db}(\tilde{\sigma}_{db}) = 0$$. In particular,

$$\tilde{\sigma}_{db} = 1 - \frac{2(2 - L - M)}{\gamma[2 - (L + M)(1 - \alpha)]}.$$ \hfill (21)

If $$\tau \geq \tau(\tilde{\sigma}_{db}^{\text{max}}, P_{db}^+, P_{db}^-)$$, the impatient atoms are always willing to liquidate and the PBE entails with $$\sigma_d^* = \tilde{\sigma}_{db}$$. If instead $$\tau < \tau(\tilde{\sigma}_{db}^{\text{max}}, P_{db}^+, P_{db}^-)$$ we return to Figure 3, with $$\sigma_{db}^1$$ and $$\sigma_{db}^2$$ denoting the two $$\sigma$$ values at which the atoms are just indifferent between liquidating and holding. If $$\tilde{\sigma}_{db} \notin (\sigma_{db}^1, \sigma_{db}^2)$$, the impatient atoms still prefer to liquidate and $$\sigma_d^* = \tilde{\sigma}_{db}$$. Finally, if $$\tilde{\sigma}_{db} \in (\sigma_{db}^1, \sigma_{db}^2)$$, $$\tilde{\sigma}_{db}$$ cannot be sustained as a PBE since the impatient atoms are unwilling to liquidate. Here we can support a PBE in which the impatient atoms liquidate a block of size $$\gamma_d^* = \gamma_{db}^{\text{mix}} < \gamma$$ such that

$$G_{db}(\sigma_{db}^1, \gamma_{db}^{\text{mix}}) = 0.$$ 

Again, unwillingness of the impatient atoms to trade at $$\tilde{\sigma}_{db}$$ induces a reduction in the equilibrium probability of debt relief to $$\sigma_{db}^1 < \tilde{\sigma}_{db}$$.

The following proposition summarizes the results derived in this subsection.

**Proposition 2 (Down Node).** If $$\gamma \leq \gamma_{db}$$, the large investor will never implement unconditional debt relief. If $$\gamma > \gamma_{db}$$, the large investor implements conditional debt relief as follows

$$\tau \geq \tau(\tilde{\sigma}_{db}, P_{db}^+, P_{db}^-) \Rightarrow (\sigma_d^*, \gamma_d^*) = (\tilde{\sigma}_{db}, \gamma)$$

$$\tau < \tau(\tilde{\sigma}_{db}, P_{db}^+, P_{db}^-) \Rightarrow (\sigma_d^*, \gamma_d^*) = (\sigma_{db}^1, \gamma_{db}^{\text{mix}}).$$
Comparison of Propositions 1 and 2 allows one to evaluate whether fully ex post efficient debt relief is more or less likely if the interim state is down rather than up. Under the baseline parameters, there are three factors making debt relief more likely if the interim state is up. First, the free-ridership capitalized into bond prices is less severe at the up node, increasing the gains to entry. To see this, note that the entry condition in (20) is more stringent than that stipulated in (10). For this same reason, conditional upon entry the large investor grants debt relief with higher probability if the interim node is up since

$$\tilde{\sigma}_{db} < \tilde{\sigma}_{u}. \quad (22)$$

In addition to the large investor’s incentives being more powerful at the up node, small uninformed bondholders are more willing to sell at that node. That is, the perceived adverse selection problem is less severe for the atoms at the up node. To see this formally, note that

$$\tau(\sigma, P^+_u, P^-_u) < \tau(\sigma, P^+_db, P^-_db) \quad \forall \quad \sigma \in (0, 1).$$

Moving away from the baseline parameters, the relative magnitudes of $\sigma^*_d$ and $\sigma^*_u$ is ambiguous. In particular, high values of $\tau$ at the down node would serve to encourage liquidation by the atoms, weakly increasing $\sigma^*_d$. In addition, high values of $\gamma$ at the down node would serve to increase the trading profits of the large investor, resulting in a strict increase in $\tilde{\sigma}_{db}$ and a weak increase in $\sigma^*_{db}$. Conversely, a high probability of the large investor becoming wealth constrained at the down node (low $\bar{\sigma}_d$) could induce a large fall in the equilibrium relief probability $\sigma^*_d$.

### 3.4 Conditional debt relief following contractions

Suppose again node $d$ is reached and suppose also that

$$s(L, \alpha) > \gamma \geq s(M, \alpha).$$

Only if $\gamma$ falls into that interval is it possible for investors to anticipate that debt relief will occur if $M$ is realized but not $L$. By way of contrast, it can never be the case that investors anticipate restructuring at $L$ but not at $M$ since any $\gamma$ inducing relief at $L$ necessarily induces relief at $M$. 


Continuing the analysis of the case in which debt relief is granted if \( M \) is reached via \( d \), the market makers set the following prices upon observing positive and negative order flows:

\[
P^+_{dm} = \frac{1 + L(1 - \alpha)}{2} \quad (23)
\]

\[
P^-_{dm} = \frac{(L + M)(1 - \alpha)}{2}.
\]

Following the same steps as in the preceding subsection, one can compute the large investor’s expected gain to buying debt at the down node conjecturing that relief would be granted at \( M \)

\[
G_{dm}(\sigma, \hat{\gamma}) = \frac{1}{2} \hat{\gamma}L(1 - \alpha) + \frac{1}{2}[M - (1 - \hat{\gamma})] - \frac{1}{2}[P^+_{dm} + P^0_{dm}]\hat{\gamma}
\]

\[
= \frac{1}{2} \left[ \frac{\hat{\gamma}(1 - \sigma)(1 - M + \alpha M)}{2} - (1 - M) \right] = G_u(\sigma, \hat{\gamma}).
\]

Since \( G_{dm} \) is strictly decreasing in its first argument and increasing in its second, a necessary condition for entry is that this value is strictly positive as \( \sigma \) converges to zero for \( \hat{\gamma} = \gamma \). Thus, we arrive at the following two necessary conditions for debt market entry with conditional debt relief

\[
(i) \quad \gamma > \gamma_{dm} = 2s(M, \alpha)
\]

\[
\text{and}
\]

\[
(ii) \quad \gamma < s(L, \alpha).
\]

It is also worth noting that

\[
\gamma_u = \gamma_{dm} < \gamma_{db}.
\]

Satisfaction of the two necessary conditions stated in (25) requires

\[
s(L, \alpha) > 2s(M, \alpha) \Leftrightarrow \alpha[(M - L) - L(1 - M)] < (1 - L)(1 - M).
\]

If condition (26) is violated, there is no possibility for conditional debt relief. When condition (26) is violated, any \( \gamma \) satisfying the first entry condition (i) would also induce the large bondholder to grant debt relief at node \( L \) as well as node \( M \). Therefore, when condition (26) is violated, the only possible equilibrium features unconditional debt relief, as described in Proposition 2. If instead
condition (26) is satisfied, but \( \gamma \leq \gamma_{dm} < \gamma_{db} \), debt market entry is never profitable for the large investor and equilibrium entails \((\sigma^*_d, \gamma^*_d) = (0, \gamma)\).

It is worth noting that the threshold \( \gamma \) values for entry into the secondary debt markets are the same at the up and down node, provided the latter entails a PBE with conditional debt relief. Intuitively, entry incentives are in this case the same at the up and down nodes because the extent of the free-rider problem is also the same.

For the remainder of this subsection it is assumed that condition (25) is satisfied. In this case, there is a unique \( \tilde{\sigma}_{dm} \in (0,1) \) such that \( G_{dm}(\tilde{\sigma}_{dm}) = 0 \). In particular,

\[
\tilde{\sigma}_{dm} = 1 - \frac{2g(M, \alpha)}{\gamma} = \tilde{\sigma}_u. \tag{27}
\]

Again, we see that the large investor’s incentives to trade on the secondary debt market are equal between the up and down node provided the latter entails only conditional debt relief.

The equivalence between the up and down nodes breaks down once one considers the trading incentives of the atomistic investors, however. To see this, note that

\[
\tau(\sigma, P^+_u, P^-_u) < \tau(\sigma, P^+_{dm}, P^-_{dm}) < \tau(\sigma, P^+_b, P^-_b) \quad \forall \quad \sigma \in (0,1). \tag{28}
\]

That is, the adverse selection problem perceived by the atoms is more severe at the down node than at the up node. Intuitively, the debt security is more informationally sensitive at the down node, inducing greater reluctance to trade for uninformed investors.

We turn finally to pinning down the PBE at the down node when (25) is satisfied. If \( \tau \geq \tau(\tilde{\sigma}_{dm}^{max}, P^+_{dm}, P^-_{dm}) \), the impatient atoms are always willing to liquidate and the PBE entails with \( \sigma^*_d = \tilde{\sigma}_{dm} \). If instead \( \tau < \tau(\tilde{\sigma}_{dm}^{max}, P^+_{dm}, P^-_{dm}) \) we return to Figure 3, with \( \sigma^1_{dm} \) and \( \sigma^2_{dm} \) denoting the two \( \sigma \) values at which the atoms are just indifferent between liquidating and holding. If \( \tilde{\sigma}_{dm} \notin (\sigma^1_{dm}, \sigma^2_{dm}) \), the impatient atoms still prefer to liquidate and \( \sigma^*_d = \tilde{\sigma}_{dm} \). Finally, if \( \tilde{\sigma}_{dm} \in (\sigma^1_{dm}, \sigma^2_{dm}) \), \( \tilde{\sigma}_{dm} \) cannot be sustained as a PBE since the impatient atoms are unwilling to liquidate. Here we can support a PBE in which the impatient atoms liquidate a block of size \( \gamma^*_d = \gamma_{mix}^{dm} < \gamma \) such that

\[
G_{dm}(\sigma^1_{dm}^{dm}, \gamma^m_{dm}) = 0.
\]
Again, unwillingness of the impatient atoms to trade at $\tilde{\sigma}_{dm}$ induces a reduction in the equilibrium probability of debt relief.

The following proposition summarizes the results derived in this subsection.

**Proposition 3 (Down Node).** If $\gamma \leq \gamma_{dm}$, neither conditional nor unconditional debt relief ($\sigma^* = 0$) will be granted and all impatient atoms liquidate ($\gamma^* = \gamma$). If condition (26) is violated or $\gamma \geq g(L,\alpha)$, only unconditional debt relief is credible, following Proposition 2. If $\gamma \in (\gamma_{dm}, g(L,\alpha))$, the large investor implements conditional debt relief as follows

$$\tau \geq \tau(\tilde{\sigma}_{dm}, P^+_{dm}, P^-_{dm}) \Rightarrow (\sigma^*_d, \gamma^*_d) = (\tilde{\sigma}_{dm}, \gamma)$$

$$\tau < \tau(\tilde{\sigma}_{dm}, P^+_{dm}, P^-_{dm}) \Rightarrow (\sigma^*_d, \gamma^*_d) = (\sigma^*_1, \gamma^*_{mix}).$$

Considering still the baseline parameters, it follows from condition (28) that even with conditional debt relief at the terminal node, $\sigma^*_d$ can fall below $\sigma^*_u$ due to the fact that severe adverse selection diminishes incentives for uninformed investors to trade at the down node. More generally, this result highlights that debt relief is *path-dependent*, with debt relief following contractions often being more difficult to implement due to the illiquidity of the debt market during contractions.

However, the secondary debt market need not be illiquid during downturns since high liquidity demand ($\tau$) during such periods encourages liquidation by uninformed investors. Further, broad liquidity shocks ($\gamma$) during downturns increase the return to acquiring large debt stakes.

### 4 Correlated Liquidity Shocks

This section allows the liquidity shock of atomistic investors to be correlated with production. This captures the fact that liquidity needs are likely to depend on the state of the economy, for example via endowment shocks. For simplicity of exposition, this section returns to the baseline assumptions regarding the other parameters, as stated in $A2'$ and $A3'$. Thus, the only state-contingency in underlying parameters is due to there being a higher likelihood of a liquidity shock at the down
node, with

\[ A4' : 1 > \pi_d > \pi_u > 0. \]

We first note that the analysis of the minimum stake of the large investor required for debt relief ex post is identical to the baseline analysis, since the correlation of liquidity shocks with the payoffs from the investment projects at time \( t_2 \) has no effect on the analysis of debt relief once the final period is reached.

### 4.1 Debt relief following expansions

We begin our analysis again by considering the start of \( t_2 \) when the state is \( u \). As in the previous section, debt relief will be necessary to avoid costly default if the terminal node \( M \) is reached. We therefore wish to determine conditions under which the large investor obtains a debt stake of at least \( s(M, \alpha) \), the minimum stake inducing debt relief if \( M \) occurs. To address that question, consider the trading game taking place at time \( t_2 \).

Again, we conjecture a PBE in which impatient atoms liquidate a block of size \( \gamma^* \leq \gamma \). In equilibrium, the large investor camouflages his trade by submitting a buy order of \( \gamma^* \) with probability \( \sigma^* \).

As in the previous section we consider a PBE such that debt relief occurs and therefore compute prices consistent with that conjecture. Table 2 depicts outcomes in the trading game and their probabilities.

<table>
<thead>
<tr>
<th>Buy</th>
<th>Shock</th>
<th>( x^I )</th>
<th>( x^N )</th>
<th>( X )</th>
<th>Price</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>N</td>
<td>( \gamma^* )</td>
<td>0</td>
<td>( \gamma^* )</td>
<td>( P_0^+ )</td>
<td>( \sigma^*(1 - \pi_\omega) )</td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
<td>( \gamma^* )</td>
<td>( -\gamma^* )</td>
<td>0</td>
<td>( P_0^0 )</td>
<td>( \sigma^*\pi_\omega )</td>
</tr>
<tr>
<td>N</td>
<td>N</td>
<td>( \gamma^* - \gamma^* = 0 )</td>
<td>0</td>
<td>0</td>
<td>( P_0^0 )</td>
<td>( (1 - \sigma^*)(1 - \pi_\omega) )</td>
</tr>
<tr>
<td>N</td>
<td>Y</td>
<td>( \gamma^* - \gamma^* = 0 )</td>
<td>( -\gamma^* )</td>
<td>( -\gamma^* )</td>
<td>( P_0^- )</td>
<td>( (1 - \sigma^*)\pi_\omega mega )</td>
</tr>
</tbody>
</table>
In contrast to Table 1, the probabilities of the various outcomes now take into account that liquidity shocks are correlated with real production.

Again, only three order flows occur on the equilibrium path. When there is positive or negative net order flow, the equilibrium is fully revealing. In all cases considered in this section, the prices at the revealing order flows are identical to those presented in the preceding section. However, correlation in liquidity shocks does change the nature of updating and hence price-setting by the MM. In particular:

\[
P_\omega^0 = \beta(\sigma, \pi) P_\omega^+ + [1 - \beta(\sigma, \pi)] P_\omega^-.
\]

(29)

\[
\beta(\sigma, \pi) \equiv \frac{\sigma \pi}{1 + 2\sigma \pi - \sigma - \pi}.
\]

Here \(\beta\) measures the market makers’ updated probability of the large investor buying given that observed order flow is zero. The updated probability has the following properties worth noting:

\[
\lim_{\sigma \downarrow 0} \beta(\sigma, \pi) = \lim_{\pi \downarrow 0} \beta(\sigma, \pi) = 0
\]

\[
\lim_{\sigma \uparrow 1} \beta(\sigma, \pi) = \lim_{\pi \uparrow 1} \beta(\sigma, \pi) = 1
\]

\[
\beta_\sigma(\sigma, \pi) = \frac{\pi(1 - \pi)}{(1 + 2\sigma \pi - \sigma - \pi)\gamma} > 0
\]

\[
\beta_\pi(\sigma, \pi) = \frac{\sigma(1 - \sigma)}{(1 + 2\sigma \pi - \sigma - \pi)\gamma} > 0.
\]

To verify that a PBE exists in which the large investor buys debt with positive probability, we redefine his expected trading gain function \(G_u\) as follows:

\[
G_u(\sigma, \pi, \hat{\gamma}) = \frac{1}{2}\hat{\gamma} + \frac{1}{2}[M - (1 - \hat{\gamma})] - [(1 - \pi_u)P_u^+ + \pi_u P_u^0]\hat{\gamma}
\]

(30)

\[
= \frac{\hat{\gamma}\pi(1 - \beta)(1 - M + \alpha M) - (1 - M)}{2}.
\]

Since \(G_u\) is again strictly decreasing in \(\sigma\) and increasing in \(\hat{\gamma}\), a necessary condition for debt relief is that it be strictly positive in the limit as \(\sigma\) converges to zero for \(\hat{\gamma} = \gamma\). Rearranging terms, one obtains the following entry condition at the up node:

\[
\gamma > \gamma_u = \bar{s}(M, \alpha)\pi_u^{-1}.
\]

(31)
It is interesting to observe that the $\gamma$ needed for entry is in fact inversely related to the probability of the liquidity shock. Thus, entry into the secondary debt market is promoted by broad liquidity shocks (high $\gamma$) and higher arrival intensity of those shocks (high $\pi$). As argued above, both $\gamma$ and $\pi$ are arguably lower during expansions, serving to deter entry by large investors, all else equal.

Assuming that (31) is indeed satisfied, there is a unique $\tilde{\sigma}_u \in (0, 1)$ satisfying the large investor’s mixing condition $G_u(\tilde{\sigma}, \pi_u, \gamma) = 0$. In particular,

$$\tilde{\sigma}_u = \frac{\gamma \pi_u (1 - M + \alpha M) - (1 - M)}{\gamma \pi_u (1 - M + \alpha M) - (1 - M) + \pi_u (1 - \pi_u)^{-1} (1 - M)}.$$ (32)

Since the focus of this section is the role of correlated liquidity shocks, it is worth noting that the large investor’s incentives, as captured by $\tilde{\sigma}_u$ are actually non-monotone in $\pi_u$. In particular:

$$\pi_u < \pi_u^{\text{max}} \Rightarrow \frac{\partial \tilde{\sigma}_u}{\partial \pi_u} > 0$$
$$\pi_u > \pi_u^{\text{max}} \Rightarrow \frac{\partial \tilde{\sigma}_u}{\partial \pi_u} < 0$$
$$\pi_u^{\text{max}} = \sqrt{\frac{1 - M}{\gamma(1 - M + \alpha M)}}.$$ (33)

Intuitively, the large investor dislikes extreme arrival intensities of the liquidity shock. To see this, note that if $\pi = 0$, the large investor has no camouflage for his trades. At the opposite extreme, when $\pi = 1$ the $MM$ set $\beta = 1$ and revise prices upward to such an extent that no profits can be earned by an informed trader.

We turn next to the trading incentives of the atoms. With correlated liquidity shocks, one must redefine the hurdle $\tau$ taking into account the new pricing rule used by the $MM$. Impatient atoms are willing to liquidate iff:

$$\sigma P^0_\omega + (1 - \sigma) P^-_\omega \geq (1 - \tau)[\sigma P^+_\omega + (1 - \sigma) P^-_\omega] \Leftrightarrow \tau \geq \tau(\sigma, \pi, P^+_\omega, P^-_\omega) \equiv \frac{\sigma(1 - \beta)(P^+_\omega - P^-_\omega)}{P^+_\omega + \sigma(P^+_\omega - P^-_\omega)}. \tag{33}$$

It is worth noting that, holding all else constant, an impatient atom’s incentive to liquidate is increasing in $\pi$ since

$$\frac{\partial \tau}{\partial \pi} = \frac{\partial \tau}{\partial \beta} \frac{\partial \beta}{\partial \pi} < 0.$$ (34)
Intuitively, high arrival intensity of the liquidity shock induces more favorable pricing from the perspective of the uninformed investors since $\beta_{\pi} > 0$. Since $\pi$ is likely to be lower during expansions, this factor would tend to reduce the probability of debt relief occurring after good interim states.

Having characterized the incentives of the large investor and impatient atoms, as summarized by the redefined $\tilde{\sigma}_u$ and $\tilde{\pi}$ respectively, the PBE for correlated shocks at the up node is identical in form to that specified in Proposition 1. Summarizing, this analysis of correlated shocks suggests that lower $\pi$ values during expansions serves to: discourage entry by the large investor; discourage liquidation by uninformed bondholders; and has an ambiguous effect on $\tilde{\sigma}_u$.

4.2 Unconditional debt relief following contractions

Consider next equilibrium at the down node. Following the same steps as in the preceding subsection, we begin by computing the large investor’s expected gain to buying debt at the down node under a conjectured PBE in which debt relief is granted at both possible terminal nodes. We redefine the trading gain function accounting for the new price setting equation with correlated shocks as given in equation (29):

$$G_{db}(\sigma, \pi_d, \hat{\gamma}) = \frac{1}{2}[L - (1 - \hat{\gamma})] + \frac{1}{2}[M - (1 - \hat{\gamma})] - [(1 - \pi_d)P_{db}^{+} + \pi_dP_{db}^{0}]\hat{\gamma}$$

(35)

Since $G_{db}$ is strictly decreasing in $\sigma$ and increasing in $\hat{\gamma}$, a necessary condition for unconditional debt relief via the down node is that $G_{db}$ is strictly positive as $\sigma$ converges to zero for $\hat{\gamma} = \gamma$. Thus, we arrive at the following necessary condition for entry in the conjectured PBE

$$\gamma > \gamma_{db} \equiv \left[\frac{(2 - L - M)}{2 - (L + M)(1 - \alpha)}\right] \pi_d^{-1}. \quad (36)$$

Retaining our focus on the role of correlated liquidity shocks, the above condition illustrates that the high probability of liquidity shocks during contractions promotes entry by large investors into the secondary debt market.
For the remainder of this subsection it is assumed that condition (36) is satisfied. In this case, there is a unique \( \tilde{\sigma}_{db} \in (0, 1) \) satisfying the large investor’s mixing condition which demands that 

\[ G_{db}(\tilde{\sigma}_{db}) = 0. \]

In particular,

\[
\tilde{\sigma}_{db} = \frac{\gamma \pi_d [2 - (L + M)(1 - \alpha)] - (2 - L - M)}{\gamma \pi_d [2 - (L + M)(1 - \alpha)] - (2 - L - M) + \pi_d (1 - \pi_d)^{-1}(2 - L - M)}.
\]  

(37)

Again, the large investor’s incentives, as captured by \( \tilde{\sigma}_{db} \) are non-monotone in \( \pi \):

\[
\pi_d < \pi_{db}^{max} \Rightarrow \frac{\partial \tilde{\sigma}_{db}}{\partial \pi_d} > 0 \quad (38)
\]

\[
\pi_d > \pi_{db}^{max} \Rightarrow \frac{\partial \tilde{\sigma}_{db}}{\partial \pi_d} < 0
\]

\[
\pi_{db}^{max} \equiv \sqrt{\frac{2 - L - M}{\gamma_d (2 - (L + M)(1 - \alpha))}}.
\]

Given the non-monotonicity of \( \tilde{\sigma}_{db} \) and \( \tilde{\sigma}_u \) it is impossible to rank these two restructuring probabilities if \( \pi_d > \pi_u \). However, for equal values of \( \pi \) the large investor has stronger incentives at the up node than at the down node, with

\[
\tilde{\sigma}_u(\pi) > \tilde{\sigma}_{db}(\pi) \quad \forall \quad \pi \in (0, 1).
\]  

(39)

Consider next the trading incentives of the impatient atoms, with the trading condition for correlated shocks given in (33). For equal \( \pi \) values, the perceived adverse selection problem is less severe for the atoms at the up node. To see this formally, we note that

\[
\tau(\sigma, \pi, P^+_u, P^-_u) < \tau(\sigma, \pi, P^+_u, P^-_u) \quad \forall \quad (\sigma, \pi) \in (0, 1) \times (0, 1).
\]  

(40)

However, it was shown in equation (34) that there is a countervailing effect encouraging trade at the down node since \( \tau \) is decreasing in \( \pi \), and \( \pi_d > \pi_u \).

Having characterized the incentives of the large investor and impatient atoms, as summarized by \( \tilde{\sigma}_{db} \) and \( \tau \) respectively, the PBE for correlated shocks at the down node, with unconditional debt relief, is identical in form to that specified in Proposition 2. Summarizing, this analysis of correlated shocks suggests that higher \( \pi \) values during contractions serve to: encourage entry by the large investor; encourages liquidation by uninformed bondholders; and has an ambiguous effect on \( \tilde{\sigma}_{db} \).
4.3 Conditional debt relief following contractions

Consider again the scenario in which node \( d \) is reached and suppose that

\[ g(L, \alpha) > \gamma \geq g(M, \alpha). \]

With correlated shocks we have the following trading gain for the large investor in a conjectured PBE with conditional debt relief:

\[
G_{dm}(\sigma, \pi, \hat{\gamma}) = \frac{1}{2}[M - (1 - \hat{\gamma})] + \frac{\hat{\gamma}}{2}L(1 - \alpha) - [(1 - \pi)P_{dm}^+ + \pi P_{dm}^0] \hat{\gamma} \\
= \frac{\hat{\gamma}\pi(1 - \beta)(1 - M + \alpha M) - (1 - M)}{2} = G_u(\sigma, \pi, \hat{\gamma}).
\]

Since the trading gain functions are equal at the up and down nodes in the present case, it follows that

\[
\gamma_{dm}(\pi) = \gamma_u(\pi) \\
\tilde{\sigma}_{dm}(\pi) = \tilde{\sigma}_u(\pi) \quad \forall \quad \pi \in (0, 1).
\]

However, the higher arrival intensity of liquidity shocks at the down node would here serve to encourage entry, with

\[ \pi_d > \pi_u \Rightarrow \gamma_{dm}(\pi_d) < \gamma_u(\pi_u). \]

Having characterized trading incentives for the large investor, consider next liquidation incentives for the atoms. We again consider the general trading condition for correlated shocks as given in equation (33). On one hand, there is greater intrinsic liquidity at the up node, due to lower adverse selection for uninformed bondholders, with

\[
\tilde{\tau}(\sigma, \pi, P_{up}^+, P_{up}^-) < \tilde{\tau}(\sigma, \pi, P_{dm}^+, P_{dm}^-) < \tilde{\tau}(\sigma, \pi, P_{db}^+, P_{db}^-) \quad \forall \quad (\sigma, \pi) \in (0, 1) \times (0, 1).
\]

However, it was shown in equation (34) that there is a countervailing effect encouraging trade at the down node since \( \tilde{\tau} \) is decreasing in \( \pi \), and \( \pi_d > \pi_u \).
Having characterized the incentives of the large investor and impatient atoms, as summarized by \( \tilde{\sigma}_{dm} \) and \( \tau \) respectively, the PBE for correlated shocks at the down node, with conditional debt relief, is identical in form to that specified in Proposition 3. Summarizing, this analysis of correlated shocks suggests that higher \( \pi \) values during contractions serve to: encourage entry by the large investor; encourages liquidation by uninformed investors; and has an ambiguous effect on \( \tilde{\sigma}_{dm} \).

5 Bond Valuation

This section derives the value of the bond when it is issued at time \( t_0 \) and explores how it is affected by the secondary debt market. As was shown in Section 2, the bond is sold to atomistic investors at time \( t_0 \). These investors will set the price so that they are indifferent between the storage technology and purchasing the bond. Thus, when pricing the bond, the investors rationally take into account that there is some probability that they will be subject to a liquidity preference shock at time \( t_2 \). As shown in Section 3, the bondholders who experience a liquidity shock either strictly prefer to sell the bond at time \( t_2 \) (in the pure strategy equilibrium) or are indifferent between selling and holding the bond (in the mixed strategy equilibrium). Since states \( u \) and \( d \) are equally likely and there is no discounting between time \( t_2 \) and \( t_0 \), the bond value at time \( t_0 \), \( V \), is therefore given by

\[
V = \frac{1}{2}v_u + \frac{1}{2}v_d,
\]

where

\[
v_{\omega} = (1 - \gamma\pi)[\sigma_{\omega}^*P_{\omega}^+ + (1 - \sigma_{\omega}^*)P_{\omega}^-] + \gamma\pi[\sigma_{\omega}^*P_{\omega}^0 + (1 - \sigma_{\omega}^*)P_{\omega}^-] \tag{44}
\]

\[
= \sigma_{\omega}^*P_{\omega}^+ + (1 - \sigma_{\omega}^*)P_{\omega}^- - \pi\gamma\sigma_{\omega}^*(1 - \beta_{\omega})(P_{\omega}^+ - P_{\omega}^-).
\]

The preceding equation reveals that bond price is equal to fundamental value less uninformed bondholders’ expected losses due to adverse selection.

To illustrate the effects of the model parameters \( \gamma \), \( \alpha \) and \( \pi \) on the initial bond value, \( V \), we provide numerical simulations. Most comparative static results can be derived analytically, based on
Figure 4: This figure shows the bond value $V$ and the probability of $I$ trading in states $u$ and $d$, $\sigma_u$ and $\sigma_d$ as a function of the broadness of the liquidity shock, $\gamma$.

the expressions from the preceding sections. The numerical analysis initially assumes a sufficiently high value of $\tau$ so that impatient bondholders always find it optimal to sell. This allows us to isolate the effect of changing parameters on the incentives of the large investor.

In the base case we set $\gamma = 0.35$, $\pi = 0.5$, $\alpha = 0.3$, $\bar{\sigma} = 1$, $M = 0.95$ and $L = 0.90$. We begin by exploring the effect of $\gamma$ on bond value. The top panel of Figure 4 plots the value of the bond at time $t_0$ as a function of the broadness of the liquidity shock, $\gamma$. Interestingly, for this numerical example, bond value is maximized at the interior value $\gamma = 0.43$. The bottom two panels of Figure 4 show the corresponding probabilities of debt relief. For low values of $\gamma$, the large investor does not enter the secondary debt market. Here, the value of the bond therefore corresponds to the case without debt relief ($\sigma^* = 0$). Starting at $\gamma$ approximately equal to 0.3, the liquidity in the secondary market becomes sufficiently high to make it worthwhile for $I$ to enter. The bottom panels reveal that trading starts in both states $u$ and $d$ at the same critical value of $\gamma$. This is because initially restructuring only occurs in state $M$, independent of whether this state is reached via the down or up node. Locally, the probability of debt relief increases in $\gamma$, a fact reflected in an increasing bond value. This reveals the importance of the incentive effect causing the large investor to buy debt more
aggressively when $\gamma$ increases. In particular, if one holds $\sigma$ constant, debt value necessarily falls with $\gamma$ since this increases adverse selection costs capitalized into the initial bond price. Apparently, however, the positive incentive effect dominates.

If $\gamma$ increases sufficiently, the large investor no longer trades at node $d$, as shown in the lower right panel. This results in a discrete reduction in the ex ante bond value. The intuition for this effect is simple. Once $\gamma$ increases sufficiently, atomistic bondholders anticipate a shift from conditional to unconditional debt relief. Free-ridership costs then increase discretely, inducing the large investor to drop out of the secondary market.

Next we analyze how the initial bond value depends on bankruptcy costs, $\alpha$. For low values of bankruptcy costs, the large investor does not grant debt relief. Debt relief becomes incentive compatible in state $M$ at $\alpha = 0.3$. Starting at this point, the large investor enters the secondary market in both states $u$ and $d$, as can be seen from the lower panels of Figure 5. There is only partial
restructuring following state $d$, i.e. for low default costs it is not optimal for the large investor to grant debt relief if state $L$ occurs. Locally, the buying intensity of the large investor increases in bankruptcy costs. However, for intermediate values of $\alpha$, atomistic investors anticipate that the large investor would be willing to grant debt relief ex post even in state $L$. This increases the cost of free-ridership, causing the large investor to drop out of the secondary debt market in state $d$.

Once $\alpha$ increases sufficiently, profit opportunities in the secondary market for $I$ improve sufficiently so that it again becomes optimal for him to enter the secondary market in state $d$, as shown in the lower right panel of Figure 5. In this range, unconditional debt relief occurs with positive probability.

The top panel of figure 5 reveals that bond value decreases monotonically in $\alpha$. This is a standard prediction. However, the model shows that the impact of an increase in bankruptcy costs is mitigated due to the incentives provided to the large investor. When $\alpha$ reaches 0.3, restructuring occurs in state $M$, no matter whether $M$ is reached following an expansion or a contraction. Locally, the debt pricing line flattens, reflecting an increasing probability of debt relief. As $\alpha$ continues to increase, the large investor stops trading in state $d$, which leads to a discrete drop in bond value. For sufficiently high values of $\alpha$, debt relief occurs both in states $M$ and $L$. In this range, a marginal increase in bankruptcy costs has only a small effect on debt value since restructuring occurs with high probability.

Next we consider the effect of changes in the probability of a liquidity shock, $\pi$. The initial value of the bond and the liquidity of the secondary market are illustrated in Figure 6. For low values of $\pi$, $I$ does not enter the secondary market. For these values, the secondary market is too illiquid and the bond value therefore reflects zero debt relief. Once $\pi$ exceeds a critical value, the secondary market becomes sufficiently liquid so that $I$ trades with positive probability. The resulting efficiency gain through debt relief is reflected in a rising bond value. However, the bond value reaches a maximum at an interior point. Intuitively, for very high values of $\pi$ it becomes increasingly difficult for the large trader to camouflage his presence. Whenever the market maker observes a net order flow of
Figure 6: This figure shows the bond value $V$ and the probability of $I$ trading in states $u$ and $d$, $\sigma_u$ and $\sigma_d$ as a function of the probability of a liquidity shock, $\pi$.

zero, he infers that there is a high probability that investor $I$ is in the market and he will price the bond accordingly. This reduces the profit opportunities for $I$ and as a result he reduces his trading probability. This leads to a decrease in the bond value.

We now turn to the impatient investors’ incentives to trade. The numerical analysis above assumed a sufficiently high value of $\tau$ such that impatient bondholders always found it optimal to sell. Recall that $\tau$ measures the degree of impatience, i.e. one unit of consumption at $t_3$ produces a utility of $1 - \tau$ whereas one unit of consumption at $t_2$ generates a utility value of 1. Recall also that $\tau$ measures the degree of adverse selection in the secondary market. For the following numerical simulations we assume that the impatience parameter in a liquidity shock is $\tau = 0.04$. All the simulations allow for endogenous variations in $\sigma$.

We here demonstrate how the probability of a liquidity shock, $\pi$ influences impatient bondholders’ incentives to trade, taking into account endogenous variations in $\sigma$. Figure 7 reveals that, for low values of $\pi$, investor $I$ is not in the market, and impatient bondholders sell without fearing adverse selection. For example, initially $\tau_u = \tau_d = 0$. As $\pi$ increases, the opportunities for $I$ to hide his trading improve and he starts trading with positive probability. This exposes uninformed bondholders to more severe adverse selection, raising both $\tau_u$ and $\tau_d$. Since the mispricing conditional
Figure 7: This figure shows the critical impatience parameters $\tau_u$ and $\tau_d$ that make the impatient bondholders indifferent between selling at $t_2$ and holding the bond until $t_3$. The actual impatience parameter in states $u$ and $d$ is assumed to be 0.04.

on $I$ being in the market is larger in state $d$ than in state $u$, the rise in $\tau_d$ is more pronounced than in $\tau_u$. As shown in the figure, impatient atoms would be unwilling to trade at intermediate values of $\pi$ due to high perceived adverse selection costs at such points. Consequently, equilibrium would there entail mixing by the impatient atoms in that only a proper subset of them liquidates.

6 Concluding Remarks

The toughness of dispersed lenders can serve to deter strategic default. However, when the likelihood of bona fide insolvency increases, a more concentrated ownership structure becomes optimal, since concentration facilitates efficient debt relief. This paper has identified conditions under which trading of debt in secondary markets can bring about an efficient shift from dispersed to concentrated ownership. Specifically, we have developed a theoretical framework to analyze a secondary market where endogenously determined ownership structure affects the value of a traded debt security.

The main obstacle inhibiting ex post efficient ownership concentration is free-ridership by small bondholders. While the cost of debt relief is only borne by the large bondholder, all bondholders enjoy the resulting efficiency gain from the avoidance of costly bankruptcy proceedings. Thus,
an investor will only acquire a large stake in the secondary market if the benefit from efficient restructuring is not fully incorporated in the purchasing price. In our framework this is possible since some investors may become subject to a liquidity shock, and thus may have a preference to sell.

We identify several reasons why the free-rider problem is likely to be most severe in economic downturns. First, both the likelihood of financial distress and the magnitude of the required debt relief in the financial distress states are higher in economic downturns. Thus, the cost of free-ridership borne by a large bondholder is more severe. Second, in an economic contraction, the bond price becomes more information sensitive. Thus, even when investors experience a liquidity preference, they may still not wish to sell in a market in which adverse selection is high. Finally, even a potentially large investor may face wealth constraints in an economic downturn, thus preventing efficient concentration of debt ownership. Therefore, precisely in the states when concentrated debt ownership is crucial, it is particularly difficult for the market to bring such ownership changes about.

We also identify several channels which can prevent the freezing of secondary debt markets during economic downturns. First, in a downturn small investors may be hit with more intense liquidity preference shocks, an effect which encourages them to sell their debtholdings. Second, in a downturn such shocks can be expected to hit more investors. Both effects tend to facilitate the acquisition of large stakes by large investors. Finally, the arrival intensity of liquidity shocks may be higher during bad times. This too encourages liquidation by small investors.

There are several directions for future research in this area. First, we believe that empirical work on the dynamics of debt ownership would be fruitful. Our knowledge of the evolution of debt ownership structure is limited. It would be interesting to see whether debt ownership concentration does, in fact, increase as the possibility of default increases. Second, we believe that several of our insights also apply to the market for securitized debt instruments. Recently, numerous SIVs have entered financial distress. In several cases large players, such as Goldman Sachs, have acquired concentrated ownership stakes in such legal entities. Although we are confident that the basic intu-
ition of our analysis carries over to these markets, a model which carefully reflects the institutional realities of this market would be an important contribution.
References


