Multiscaling nature of sonic velocities and lithology in the upper crystalline crust: evidence from the KTB Main Borehole

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Abstract. The observed scaling of sonic logs is generally described using a monofractal model (fractional Brownian motion with Hurst exponent typically between 0.1 and 0.2, or fractional Lévy motion); even though such a description recognizes the existence of the self-similarity of the logs, it offers only a limited account of the actual symmetry, as it misses the displayed heterogeneous fractality. Intermittency of the velocity increments is here observed, and calls for the use of a multifractal description. We quantify such a multifractality in the case of the P-wave sonic log recorded at the KTB Main Borehole. We also show that the associated lithology, inferred from the gamma log, has a multifractal distribution as well, though characterized by a stronger intermittency. It can be expected that the multifractality of the sonic log is partially induced by the lithology, though the observed discrepancy in the degree of intermittency between the two logs indicates that the scaling symmetry of the seismic velocity is probably due to many different contributions. Such statistics for both sonic and gamma logs have important implications on our understanding of wave propagation and rock strength and permeability in the crystalline crust, as they show the crust to be more heterogeneous than is usually modeled.

Introduction

It is commonly observed that sonic logs exhibit scaling properties, and are usually described and modeled as fractional Brownian motions with a Hurst parameter $H$ characteristic of the log. Typically, $H$ is found to be small, the power spectrum showing a log-log slope ($-\beta = -1 - 2H$) close to -1, i.e., the log is viewed as an instance of flicker noise. The 7 km deep KTB Main Borehole in Bavaria offers the opportunity to study scaling from a few meters to a few kilometers. Different studies have estimated $\beta$ for the P-wave velocity: $0.97 \pm 0.005$ [Wu et al., 1994], $1.11 \pm 0.04$ [Jones and Holliger, 1997], or $1.3 \pm 0.2$ [Dolan and Bean, 1997].

The origin of such a scaling symmetry is often attributed to the scaling of the fracture set along the borehole [Leary, 1991; Holliger, 1996], though Bean [1996] showed that scaling in the lithology distribution can also be taken as a contributing cause. In the case of a contribution from the fractures, the straightforward reasoning is that, as fractures and cracks are observed to be fractally distributed, and since they create seismic velocity fluctuations, they induce scaling properties for the latter. A very qualitative model can be used to describe such a link between fracture fractality and the scaling of the Vp log: consider that a fracture statistically generates a Gaussian fluctuation for the seismic velocity. If the fracture distribution is dense along the borehole, i.e. the fractal dimension $D$ is equal to 1, then the sonic log is equivalent to a Brownian motion: $v(z) = \int W(d^Dz)$, where $W$ is a Wiener measure. For $D \leq 1$, this 'diffusion' becomes anomalous, and the increments $v(z) - v(z')$ are identical to $W(\Delta z^D) \sim \Delta z^{D/2}$. The Hurst exponent $H$ of the sonic log is then related to $D$ by the relation $H = D/2$. For non-Gaussian (Lévy-stable) fluctuations, the same argument leads to $H = D/\alpha$ where $\alpha$ is the index of stability, and thus to fractional Lévy motion (as found for sedimentary rocks, Painter and Paterson [1994]).

The aim of this paper is to show that, while this usual scaling description is of primary importance, it offers only a limited view and modeling of the actual scaling symmetries observed for both sonic and gamma logs, as it neglects the intermittency exhibited by the logs.

Theory

The monofractal characterization corresponds to a simple scaling of both the probability density function (pdf) $P_t$ and the moments of the increment $\delta v = v(z + \ell) - v(z)$ of Vp over an interval $\ell$. If we rescale this interval by a factor $\lambda$, i.e., looking at the increments $\delta v_{\lambda t}$, we need to rescale the pdf following $P_{\lambda t}(\delta v_{\lambda t}) = \lambda^{-H} P_t(\delta v = \lambda^{-H} \delta v_{\lambda t})$ or more classically written $P_t(\delta v_{\lambda t} = \lambda^{-H} \delta v_{t}) \sim \lambda^{-H}$ as well as the moments $\langle \delta v_{\lambda t} \rangle = \lambda^{qH} \langle \delta v_{t} \rangle = \lambda^{qH} \Delta v$ where $H$ is the Hurst exponent, and $\zeta(q)$ is the so-called structure function [Monin and Yaglom, 1975]. The important point is that such a rescaling keeps the shape of the pdf invariant.

A much wider class of scaling processes corresponds to logs with a non-invariant shape for the pdf. Multifractal distributions [Frisch, 1995; Schertzer et al., 1997] indeed possess a generally infinite number of characteristic exponents expressed through the structure function $\zeta(q)$. Non-linearity of $\zeta(q)$ is obtained, and roughly speaking corresponds to a variation of the Hurst exponent for each moment of order $q$. While in the monofractal case, $\zeta(q) = qH$ (for a fractional Lévy motion, there exists two linear regimes with a transition at $q = \alpha$), the multifractal case leads to $\zeta(q) = qH(q)$, with $H(q)$ decreasing non-linearly as $q$ increases.
Multiscaling of the KTB Main Borehole sonic log

We analyze here the multiscaling properties of the sonic log for the KTB Main Borehole (Figure 1). Such an analysis has already been performed on other well-logs [Saucier and Muller, 1993, for sedimentary rocks; Herrmann, 1997]. In order to remove the smoothing effect of the logging tool (the tool measures a velocity on intervals of 1.52 m every 0.152 m), we sum the signal over every 10 consecutive bins. The existence of a scaling symmetry is nicely indicated by the rather straight log-log slope obtained for the power spectrum, computed and averaged on five non-overlapping sections of 900 datapoints (a section covering the range of scales from 1.52 m up to 1.37 km), and displayed in Figure 2. We estimate $\beta = 1.07 \pm 0.1$, which is in the range of estimates found in other studies.

In order to compute the structure function $\zeta(q)$, we proceed by averaging the $q$-powers of the increments of $v$ at different scales $\ell$, for a set of values of $q$. Note that we do not estimate $\zeta(q)$ for negative moments, as they are usually very sensitive to noise. Also, statistical convergence cannot be reached for too large moments $q$, and we thus limit ourselves to $0 < q \leq 4$, as estimates for $q \geq 4$ appear to be too affected by insufficient sampling. Figure 3 displays the obtained structure function $\zeta(q)$. A direct observation is that this function is far from being linear, and thus characterizes a multi- and not mono-scaling system. Indeed, Figure 4 emphasizes a major difference between the multiscaling $V_p$ and a fractional Brownian motion with Hurst exponent $H = 0.11$, and with the same mean and standard deviation as found for the KTB sonic log. The Hurst exponent is chosen equal to the estimate of the linear slope of $\zeta(q)$ at $q \to 0$ (see Figure 3). These two signals have very similar power spectra, yielding nearly identical spectral slopes. However, while a monoscaling analysis would not be able to distinguish between the two, it is apparent that the increments at the smallest scale $\ell = 1.52$ m effectively display intermittent clusters of spikes in the case of the sonic log, while they appear more homogeneously distributed in the monoscaling case of the fractional Brownian motion.

This intermittency explains the negative curvature of $\zeta(q)$: compared to its monoscaling counterpart, large moments are characterized by smaller values of $\zeta(q)$ (i.e., $H(q) - H \leq 0$ decreases as $q$ increases), thus by a smaller rate of convergence to zero of the large moments as $\ell \to 0$. As we go to smaller scales, the pdf of the increments, a proxy for the reflectivity coefficients, becomes relatively more and more distorted in favour of large absolute values. Also, note that the log-log slope $\beta$ of the power spectrum is theoretically associated with the value of the structure function for the order of moment $q = 2$: $\beta = 1 + \zeta(2)$. We see that the...
estimate $1 + \zeta(2) = 1.132 \pm 0.0075$ is indeed in the range of the estimate $\beta = 1.07 \pm 0.1$.

Multifractality of the lithology in the borehole

The gamma log recorded in the analyzed borehole (Figure 1) provides a proxy for the lithological composition. By performing the same multifractal analysis on this gamma log as the one performed in the preceding section, we actually analyze the scaling of the mixed composition in the borehole, since we merely average the gamma log on zones at various scales: this allows for the two modes encountered along the borehole (metabasite and paragneiss) to be both present in given proportions in large zones. The power spectrum (Figure 2) shows that the gamma log indeed possesses a scaling symmetry; the spectral exponent is estimated to $\beta = 1.46 \pm 0.13$. Figure 4 displays the increments of the gamma log (over an interval of 1.52 m); this signal appears to be even more intermittent than the one obtained with the increments of $V_p$ and naturally leads us to study the multiscaling properties of the gamma log.

Figure 3 shows the structure function obtained for the gamma log. The error bars were computed in the same way as was done for the sonic log. It is also clear in this case that the log indeed displays multifractal statistics. In comparison to the one obtained for $V_p$, it indicates a more intermittent signal as was expected from Figure 4.

Figure 4. From top to bottom: (top) increments $\delta V_p(z) = v(z + \ell) - v(z)$ at scale $\ell = 1.52 \text{m}$ for the KTB sonic log and (center) for a realization of a fractional Brownian motion of same mean and standard deviation, with $H = 0.11$, characterized by a linear structure function $\zeta(q) = 0.11q$ (dashed line in figure 3); (bottom) increments of the gamma log over 1.52 m intervals.

Conclusions

We show in this paper that both the sonic log (P-waves) and gamma log of the KTB Main Borehole possess multifractal statistics, and that the intermittency of their increments needs to be taken into account. The observation that the gamma log is more intermittent than the sonic log indicates that the multiscaling properties of the latter is likely to be due to many different contributions. As well as the lithology (Goff and Holliger, submitted to the J. of Geo-phys. Res., 1998, convincingly shows a correlation between the lithology and the sonic log) or for example the stress field (seismic velocities being tenso-sensitive), it can be expected (following other studies, e.g. Belfield, [1994]; Ouillon et al., [1995]) that fractures also possess a multiscaling symmetry and partly induce the multiscaling of the sonic log. The generalization of the ‘diffusion’ model presented in the introduction to multifractal distributions of fractures naturally leads to the multiscaling of the seismic velocity, in a way very similar to the one described by the ‘multifractal
model of asset returns [Mandelbrot et al., 1998]. This idea will be developed elsewhere. A study of the multiscale correlation between not only the sonic log and the lithology, but also with the fracture distribution when this information is available, should help to quantify the weights associated with the different contributions.

Finally, the numerical simulation of synthetic multifractal logs or even 2 or 3D velocity models can be performed, based on the results for the estimate of the structure function. Such synthetic models can then be used for numerical investigation of the effect of multiscale symmetry on, for example, the propagation of seismic waves, fluid flow in the crust, or the distribution of the stress field, conditioned by multiscale seismic velocities, rock permeabilities and rock strengths respectively.

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