Conundrum or Complication: A Study of Yield Curve Dynamics under Unusual Economic Conditions and Monetary Policies.

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Abstract

The definition of the decline of long term yields in the light of increasing short term yields as a conundrum by Chairman Greenspan in February 2005 has generated a significant amount of research. This paper presents a study of yield curve dynamics over this period using economic surprise data as the diagnostic tool. Results are presented for both US and Japanese data which indicate a non-linear response of the yield curve to economic data and monetary policy over the period in question. Further, a limited model is presented that is consistent with the observations. This can lead to an explanation of the conundrum in terms of a non-linear yield response to expected long term inflation and a variable expected long term real rate.

Key Words: federal reserve, term structure of interest rates, inflation
JEL Codes: E43, E44, E52, E58
1. Introduction

The exercise of monetary policy is fraught with the perils of unintended consequences to deliberate movements in the limited number of policy instruments available to central bankers. As such considerable effort has been made in developing models, both heuristic and fundamental, to aid monetary policy setting institutions in making decisions. The heuristic models derive primarily from the work of Taylor (Taylor 1993) where relationships were posited relating to the appropriate level of short rates in a changing measured inflation and output environment. On the more fundamental economic front a considerable theoretical advance was made with the introduction of the New Keynesian Phillips Curve (Clarida, Gali et al. 1999). This has led to the development of a class of yield curve models where the evolution of the yield curves may be related to the evolution of a number of economic variables in an arbitrage free manner (Ang and Piazzesi 2003). The use of these approaches, either separately or in combination have given policy setters are much greater insight into the mechanics of the evolution of the yield curve, and as such allowed them greater confidence in their predictions of the consequences of their actions (Gallmeyer, Hollifield et al. 2005).

During the latter stages of Chairman Greenspan’s tenure as head of the Federal Open Market Committee however a situation arose where the response of the yield curve to a sharp reduction, and later increase, of the short rate was completely at variance to the predictions of most classes of models. In this case in the period 2002 – 2004 as the short rate was lowered to a historical level of 1%, the long end of the term structure actually rose in yield, Figure 1. In general, yield curve models would predict that falling short rates would lead to a lowering of the long end although there was rarely agreement between models on the actual degree of yield reductions. This abnormal behaviour of the long end of the yield curve was memorably described by Chairman Greenspan as a “conundrum” (Greenspan 2005). However the relevance of such unanticipated movement is greater than that of a mere puzzle. The principal purpose of central bank easing is to increase liquidity in the financial system and, by taking real rates effectively negative, to prompt an economic recovery in the overall economy. If at the same time borrowing costs over the long term are effectively rising, as was observed in this time, the impact of a key weapon of a central bank in
avoiding recessionary trends is to some extent being mitigated. Similarly, short rates are raised in order to reduce liquidity in the greater economy. If at the same time, long maturity yields are dropping, making it cheaper to borrow for term, then the total impact of the tightening is to some extent compromised. Thus, it is of interest to understand the reaction of the yield curve in this environment so as to greater determine the impact and effectiveness of monetary policy going forward (McGough, Rudebusch et al. 2005).

In addition there was the heuristic observation that the yield curve tended to predict, with varying degrees of accuracy, a number of economic variables (Estrella 2005). These observations were not consistent with the relevant market data over the period in question.

Figure 1: Greenspan’s Conundrum - After mid 2002 reducing short term rates were accompanied by an increase in long term yields. The increase in short term rates from June 2004 was then accompanied by a decrease in long term yields.

Since the naming of Greenspan’s conundrum, work has been carried out to attempt to understand the dynamics that were relevant at the time. Rudebusch (Rudebusch 2006) carried out a detailed study to investigate the conundrum in terms of a number of joint yield curve / macroeconomic models. There were two main conclusions from this study. Firstly, it was shown that bond purchases/sales at the long end of the curve had no discernable effect on abnormal movements of long yields. This is an important result as previously there had been a general assumption that the impact of foreign investors was a significant determinant of long end yields over the relevant period. The second, more disquieting result was that the current generation of yield curve models could not adequately explain the observed
behaviour. A more recent paper has attempted to explain the conundrum in terms of Goodhart’s Law (Thornton 2007) - a financial version of the Heisenberg uncertainty principle, however if this were true in practice, it would leave the science of monetary policy as described in Clarida et al in very poor shape (Rudebusch 1995).

In this paper it is proposed to study Greenspan’s conundrum using the economic surprise data approach first used to study the behaviour of fixed income instruments directly after the release of economic data (Ederington and Lee 1993) (Green 2004). In these and later studies, the degree of surprise was identified by the difference between the realised economic data and that predicted by a panel of economists. This original studies showed that the bond market reacted in a statistically significant manner consistent with the level of surprise across a wide range of economic data. The application of surprise data was greatly enhanced by the work of Gürkaynak et al (Gürkaynak 2005) where a macroeconomic approach to the evolution of the yield curve was considered in the light of the surprise data. This compared the observed behaviour of the yield curve to the predictions of a number of NKPC models. Whilst agreement was found in some cases, the response of the yield curve to inflation data such as the consumer price index (CPI) and the consumer price index less food and energy (CPI-x), was of less statistical significance, and to a lower magnitude that that expected by theory. This inadequate response to inflation data led the authors to posit the existence of a variable long term market expectation of inflation which went to some extent to explain the observed results. However with reference to the FOMC’s stated policy of basing policy decisions based broadly, but not exclusively, on realised and expectations of CPI-x and non-farm payroll data remains an enigma within a conundrum. (The pronouncements of a number of FOMC members outlining policy may be on the Federal Reserve website).

Thus even without the naming the observed phenomenon as a conundrum it is clear that there is some inadequacy in current understanding of the evolution of yield curves to realised inflation within a macroeconomic framework. In this paper the evolution of the yield curve in the period 2000 – 2007 will be studied. As noted earlier, over this period short term rates experienced an unprecedented range of values starting at over 6% in 2000, dropping to 1% in 2002 and then rising to almost 5% by the end of 2006. In such circumstances, it may well be questioned whether results and deductions obtained from extreme data may be applicable to general monetary policy.
However, in the current environment (February 2008) where monetary policy has been loosened considerably due to adverse economic conditions, at the same time as inflationary conditions not necessarily being benign, the study may still have relevance.

The paper is structured as follows. Section 2 gives a description of the data and the relatively simple analysis methods used in its study. Section 3 gives the results as derived primarily to US data. A limited amount of data from the Japanese market is also presented. Whilst not comparable in quality to the US data, it was the only major market, ex the US, that exhibited significant inflationary issues over the period in question. Section 4 presents a discussion of the results and outlines a macroeconomic treatment that may be used to explain the observed results. Section 5 concludes the paper.

2. Data and Analysis

The economic surprise data is collected by Bloomberg and MMS. For most developed economies, which include for the purpose of this study the United States and Japan, the announcement date for economic data is well known in advance. As such, the data collectors assemble the predictions of a large number of economists, from investment banks and research institutes. These predictions are available on the day of the announcement and the magnitude of the surprise is defined as the difference between the median prediction and the realised data. There is an assumption here that the market makers and other market participants responsible for setting prices effectively agree with the predictions of the “median economist”. Previously reported results indicate that this is a reasonable assumption. In the literature the surprise factor is usually normalised by the volatility of the surprise, determined over the whole dataset, however as this study will concentrate almost exclusively on CPI-x data, this is not carried out here. Within this study only two types of economic data will be considered: US capacity utilisation and US and Japanese inflation. The capacity utilisation data is collected by the US Federal Reserve and is released in the middle of each calendar month. From a macroeconomic point of view capacity utilisation is closely related to the concept of output gap (Lars Ljungqvist 2004). The inflation indices CPI-x (USD) and TCPI-x (JPY) are
measures of inflation where the highly variable food and energy components have been removed from the basket used in the computation of the index. The actual data used are the monthly returns on the index, as the index for the US date generally increases across the sample time window. The data is collected by the US Bureau of Labour Statistics and, along with the GDP deflator is the principal measure of short term price inflation in the United States. The Japanese inflation data is collected by the Statistics Bureau of the Ministry of Internal Affairs and Communications. The “T” prefix indicated that the data used is for the Tokyo region. This data is calculated and released before that for the rest of the country and as such is more relevant as surprise data.

Figure 2: The monthly index of inflation and the change on a month to month basis.

The yield curve data used in this study are short term interbank offered interest rates (Libor) provided by the British Bankers Association (BBA). This data is freely available on the web. The long dated yield curve information is interest swap data collected by the International Swap Dealers Association (ISDA). This data was freely available on the web until February 2007 and is now available on Reuters. The data is collected on a daily basis. The reason this data has been used is that it depends on polling data. That is, on any day in question, the data collectors poll a range of actual trading firms at a specified time, to get a range of quotes on the interest rates in question. The highest and lowest quotes are discarded and the recorded quote is the average of the residual quotes. On days that an insufficient number of quotes are
received by the data collector, normally 5, no value is recorded. This data has a significant disadvantage that data is not therefore available on every trading day. It has the significant advantage however that for every that there is a data point, this is a data point that is consistent with the majority view of the major market participants, at that time, on that day. As confirmation of this hypothesis, both the BBA data and the ISDA data are used to cash settle caps/floors and swap options respectively on the appropriate days. Thus it is in the financial interest of all market participants to ensure that the rates that are recorded are the right ones. Swap data has been usefully applied in previous studies of the evolution of the yield curve (Piazzesi 2005).

For the long dated interest rate data, it is not clear from previous studies over whether it is appropriate to use the basic par swap yield curve data or forward rate data based on zero coupon bond rates derived from the par interest rate data. Either price data or market yield data has been used in a number of studies. However in Gürkaynak 2005, 1yr forward rates were used. The use of forward rate data is predicated on the expectations hypothesis (James 2004) which deems that the forward rate is effectively isolated from short rate concerns (Carriero, Favero et al. 2006). Irrespective of beliefs or otherwise on the expectation hypothesis, within this study no preference is taken. Results are presented for both the responses of par swap rates of maturity: 1yr, 5yr, 10yr and 30yr and for forward swap rate maturities of: 1yr, 1yr forward, 3yr, 2yr forward, 5yr, 5yr forward and 20yr, 10yrs forward. For the Japanese data only 10yr par swap rate data is presented.

In order to analyse the response of the different parts of the yield curve to surprises in the economic data, robust regression techniques are used (Sheather 1990). Whilst the use of robust regression is implicit in the use of Huber-White statistics in Green 2004, its application does not appear to be widespread in the analysis of surprise data. Given the volatility of financial markets and the number of influences to which individual instruments are subject, it would be surprising if the only influence on a given day was that of a given economic data release. Further, it would be very surprising if the release of the economic data dominated all other information on that day for all the datapoints under consideration. An example is the activity of the financial markets after the tragic events of 9/11, where daily changes bore little impact of released economic data.
The use of robust analysis techniques allows for the possibility that not all data within the sample acts according to a given relationship, and allows the user to negatively weight, in a statistically consistent manner, those datapoints that are not consistent with the overall relationship that may exist in the data. This does of course necessitate the use of Huber-White statistics to increase the relevant standard errors to compensate for the fact of the negative weighting of some of the datapoints. In addition to the standard regression estimation statistics such as standard error and $r^2$, Bayesian Information Criteria (Schwarz 1978) will be used to assess the validity of the regression models.

3. Results

The results presented in this section illustrate what may be considered to be an unusual period in the evolution of the US yield curve however in a number of ways it reacted exactly in accordance with theoretical expectations and previously reported data. The long term drift (month to month) of the slope of the yield curve displays a high degree of correlation with the realised level of Capacity Utilisation, figure 3. If the capacity utilisation is used as a proxy for output gap, this result is completely in agreement with the predictions of NPKC theory.

Figure 3: The evolution of the spread between 1yr and 10yr swap yields and Capacity Utilisation. The $r^2$ for the two time series is 0.91.

However using the same sample, the daily response of the yield curve to surprises in Capacity Utilisations show no statistically significant relationship. This is not in
agreement with results present in Gürkaynak et al using forward curves derived from a fitted treasury curve.

When the CPI-X data is considered however, the response of the yield curve data to surprise information of inflation is somewhat consistent with that reported in previously mentioned studies, table 1. Whilst there is a statistically significant response, it appears to be inconsistent with the weight that the FOMC placed on the statistic.

Table 1: Linear regression responses of swap rates and derived forward rates to surprise CPI-x news

<table>
<thead>
<tr>
<th></th>
<th>Datapoints</th>
<th>Co-Efficient</th>
<th>Correlation r²</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1yr</td>
<td>70</td>
<td>0.074*</td>
<td>5.91%</td>
<td>-199.15 / -194.92</td>
</tr>
<tr>
<td>5yr</td>
<td>70</td>
<td>0.119*</td>
<td>5.28%</td>
<td>-145.31 / -141.22</td>
</tr>
<tr>
<td>10yr</td>
<td>70</td>
<td>0.103</td>
<td>3.24%</td>
<td>-114.96 / -108.77</td>
</tr>
<tr>
<td>30yr</td>
<td>69</td>
<td>0.083</td>
<td>2.35%</td>
<td>-118.67 / -111.84</td>
</tr>
<tr>
<td>1yr, 1yr Fwd</td>
<td>70</td>
<td>0.223**</td>
<td>11.8%</td>
<td>-90.62 / -90.92*</td>
</tr>
<tr>
<td>3yr, 2yr Fwd</td>
<td>71</td>
<td>0.142*</td>
<td>5.58%</td>
<td>-104.54 / -100.10</td>
</tr>
<tr>
<td>5yr, 5yr Fwd</td>
<td>73</td>
<td>0.193**</td>
<td>6.84%</td>
<td>-70.44 / -67.03</td>
</tr>
<tr>
<td>20yr, 10yr Fwd</td>
<td>71</td>
<td>0.0852</td>
<td>2.28%</td>
<td>-109.56 / -102.67</td>
</tr>
</tbody>
</table>

Note: For the Huber-White standard errors, *** indicates significance at the 1% level, ** at the 5% level and * at the 10% level. The Bayesian information criterion (BIC) indicates significance with a function that attempts to minimise the number of parameters. * indicates that the BIC of the regression is less than that of the data on its own. The regressions include constant parameters that are not shown.

From figure 1, it is clear that there is a definite response to declining inflation as the FOMC was observed to aggressively cut short term rates in order to loosen monetary policy. However this heuristic argument is not directly observed in the data. The solution is to attempt to understand the response of the long end of the yield curve, to changes in realised inflation, in terms of a currently “unobserved variable”. The further part of the analysis is an attempt to determine that “unobserved variable” and further to determine if the variable is in fact observable.
There are a number of ways in which to attempt to determine the nature of the unobserved variable. Firstly there is the large vector autoregression approach advocated in the pioneering work of Campbell and Shiller (Campbell and Shiller 1987). However in this study, significant weight has been placed on ensuring that the data used in the analysis are synchronous. As this will not be the case if multiple economic data streams are used, this approach will not be used here. However it is possible to investigate multiple linear regressions, using the economic data as lagged regressors. However for a number of economic data time series (CPI, CPI-x, PPI, PPI-x, Non farm payrolls, Capacity Utilisation, Retail Sales, and the ISM survey data), no significant impact on the response of the long end of the yield curve to inflation surprise data was found.

Similarly, the impact of the evolution of different maturity yields was studied, however for these cases there was no significant impact on the inflation response.

As a result of the effective failure of linear methodologies to accurately explain the data, the use of a non-linear response was investigated. This is motivated by the observation, that over the period in question, the yield curve went through unprecedented changes and as such, the level of interest rates may have a significant impact on the response of the yield curve. In this case an externally excited threshold regression model (Tong 1990) was used where the response of the yield to surprise data was predicated on the level of a different interest rate. Whilst statistically significant results were found for a number of different external rates, the highest level of confidence is found when using the 1yr rate as the parameter that controls the threshold response, table 2. As can be seen from table 2, over the period in question the level of the 1yr interest rate had a significant impact on the response of long term interest rates to surprise inflation data. This is at the expense of the number of data points in each sample. However the results are quite clear. For 1yr interest rates above 3% there is a very significant positive response of the long term data to surprises in CPI-X. This is what would be expected. However when a study of the response of logterm yields to inflation surprises was made with 1 year rates below 2% the data indicates that there is again a statistically significant response to the inflation data, however it has reversed sign, table 3.
Table 2: Responses of swap rates and derived forward rates to surprise CPI-X news when the 1yr swap rate > 3%

<table>
<thead>
<tr>
<th></th>
<th>Datapoints</th>
<th>Co-Efficient</th>
<th>Correlation r²</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1yr</td>
<td>39</td>
<td>0.126*</td>
<td>9.19%</td>
<td>-133.85 / -129.64</td>
</tr>
<tr>
<td>5yr</td>
<td>40</td>
<td>0.155**</td>
<td>12.3%</td>
<td>-107.24 / -105.09</td>
</tr>
<tr>
<td>10yr</td>
<td>40</td>
<td>0.152**</td>
<td>9.25%</td>
<td>-97.61 / -94.12</td>
</tr>
<tr>
<td>30yr</td>
<td>35</td>
<td>0.223***</td>
<td>18.4%</td>
<td>-88.02 / -88.02*</td>
</tr>
<tr>
<td>1yr, 1yr Fwd</td>
<td>40</td>
<td>0.176*</td>
<td>11.4%</td>
<td>-93.89 / -91.34</td>
</tr>
<tr>
<td>3yr, 2yr Fwd</td>
<td>41</td>
<td>0.211**</td>
<td>14.6%</td>
<td>-90.68 / -89.75</td>
</tr>
<tr>
<td>5yr, 5yr Fwd</td>
<td>38</td>
<td>0.177</td>
<td>9.69%</td>
<td>-83.04 / -79.63</td>
</tr>
<tr>
<td>20yr, 10yr Fwd</td>
<td>36</td>
<td>0.187</td>
<td>11.2%</td>
<td>-84.31 / -81.42</td>
</tr>
</tbody>
</table>

In other words the +0.223 bp move in 30 year interest rates for every 1bp surprise in inflation, has change to a -0.237bp move in 30 year interest rates. This is, for low short term interest rates, the response to a surprise increase in measured inflation, is that long term interest rates decline. This is completely at odds with current models and indeed any intuitive thought about the evolution of interest rates.

Table 3: Responses of swap rates and derived forward rates to surprise CPI-X news when the 1yr swap rate < 2%

<table>
<thead>
<tr>
<th></th>
<th>Datapoints</th>
<th>Co-Efficient</th>
<th>Correlation r²</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1yr</td>
<td>17</td>
<td>0.016</td>
<td>0.7%</td>
<td>-80.64 / -75.1</td>
</tr>
<tr>
<td>5yr</td>
<td>16</td>
<td>-0.158</td>
<td>5.2%</td>
<td>-40.63 / -35.93</td>
</tr>
<tr>
<td>10yr</td>
<td>17</td>
<td>-0.266*</td>
<td>12.6%</td>
<td>-38.66 / -35.29</td>
</tr>
<tr>
<td>30yr</td>
<td>17</td>
<td>-0.237**</td>
<td>14.8%</td>
<td>-45.38 / -42.44</td>
</tr>
<tr>
<td>1yr, 1yr Fwd</td>
<td>17</td>
<td>-0.051</td>
<td>0.0046</td>
<td>-37.71 / -32.12</td>
</tr>
<tr>
<td>3yr, 2yr Fwd</td>
<td>17</td>
<td>-0.115</td>
<td>0.01871</td>
<td>-33.86 / -28.51</td>
</tr>
<tr>
<td>5yr, 5yr Fwd</td>
<td>18</td>
<td>-0.172</td>
<td>0.03802</td>
<td>-32.21 / -27.13</td>
</tr>
<tr>
<td>20yr, 10yr Fwd</td>
<td>18</td>
<td>-0.138</td>
<td>0.06386</td>
<td>-46.98 / -42.39</td>
</tr>
</tbody>
</table>

There is a further observation that the response of par rates displays a far greater level of statistical significance to that observed using forward rates. Studies were carried out to investigate if there were instrument sensitivity relationships such as duration or DV01 (Dollar Value of 1 basis point) that may explain the difference however no statistically significant relationship was found. Further work needs to be
carried out to investigate the obviously related evolution of par and forward rates around the release of surprise data.

A similar study may be carried out with Japanese data. Over the period in question, the Bank of Japan was generally in a Zero Interest Rate Period (ZIRP) where short term rates were kept at effectively zero percent. As such short term interest rates showed almost no variability across the whole sample window, Figure 4.

Figure 4: The evolution of the JPY 1mo Libor short term rate and the 10yr swap rate over the Zero Interest Rate Policy (ZIRP) period. It should be noted that over this period the exclusively Japanese bank interbank bid rate (Tibid) often went negative.

As a result using a 1yr rate, as was the case for the US data gave no significant results. For the JPY data the highest level of confidence was found using the 5yr swap rate as the external threshold parameter.

Table 4: Responses of JPY 10yr swap rates to surprise TCPI-x data using the 5yr swap rate as a threshold parameter

<table>
<thead>
<tr>
<th>Datapoints</th>
<th>Co-Efficient</th>
<th>Correlation r²</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>10yr, no threshold</td>
<td>45</td>
<td>0.022</td>
<td>1.32%</td>
</tr>
<tr>
<td>10yr, 5yr &gt; 0.70%</td>
<td>18</td>
<td>0.304**</td>
<td>40.17%</td>
</tr>
<tr>
<td>10yr, 5yr &lt; 0.7%</td>
<td>24</td>
<td>-0.007</td>
<td>0.39%</td>
</tr>
</tbody>
</table>

The results of the study are presented in Table 4. As can be seen there is statistically significant evidence of threshold behaviour for 5yr rates in excess of 0.7%, however the below threshold behaviour, observed in the US data is not
replicated. In addition, the JPY data suffers from a paucity of data points which may compromise the value of the statistical relationships observed.

As well as looking for the threshold relationship in the surprise data, direct relationships may also exist in the response of the yield curve to changes in expectations of economic variable on a month to month basis. Results based on the monthly change of interest rate versus the difference between the realised data and the economists’ expectation of that data, one month later are presented in Tables 5 and 6. In this case the threshold value of the 1yr rate was 2.45%. As can be seen from the data, the results for the longer term changes are both statistically significant and are consistent with those found for the one day surprise data. That is, for short term interest rates below a threshold level, the sign of the response of long term yields to the inflation data, changes sign.

Whilst the confidence in the response function in terms of the monthly data is not as strong as that for the daily surprise data, what is significant here is that the direction and magnitude of the results are consistent.

Table 5: Responses of swap rates and derived forward rates to month to month evolution of CPI-x with 1 yr rates > 2.45%

<table>
<thead>
<tr>
<th>Rate</th>
<th>Datapoints</th>
<th>Co-Efficient</th>
<th>Correlation r²</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1yr</td>
<td>44</td>
<td>0.568</td>
<td>4.88%</td>
<td>34.53 / 39.90</td>
</tr>
<tr>
<td>5yr</td>
<td>44</td>
<td>0.660*</td>
<td>6.38%</td>
<td>37.06 / 41.73</td>
</tr>
<tr>
<td>10yr</td>
<td>43</td>
<td>0.651**</td>
<td>7.17%</td>
<td>27.17 / 31.50</td>
</tr>
<tr>
<td>30yr</td>
<td>42</td>
<td>0.699***</td>
<td>15.96%</td>
<td>10.57 / 10.74</td>
</tr>
<tr>
<td>1yr, 1yr Fwd</td>
<td>45</td>
<td>1.006**</td>
<td>8.60%</td>
<td>56.90 / 60.47</td>
</tr>
<tr>
<td>3yr, 2yr Fwd</td>
<td>45</td>
<td>0.546*</td>
<td>3.92%</td>
<td>45.5 / 51.31</td>
</tr>
<tr>
<td>5yr, 5yr Fwd</td>
<td>45</td>
<td>0.624**</td>
<td>6.59%</td>
<td>35.13 / 39.68</td>
</tr>
<tr>
<td>20yr, 10yr Fwd</td>
<td>42</td>
<td>0.549**</td>
<td>7.89%</td>
<td>14.24 / 18.27</td>
</tr>
</tbody>
</table>

In addition the external threshold rates are consistent. As such it is clear from the data that the yield curve, at the time identified by Chairman Greenspan, was evolving over both short term (daily) and the longer term (monthly) in a fashion that was not in agreement with current yield curve models.
Table 6: Responses of swap rates and derived forward rates to month to month evolution of CPI-x with 1 yr rates < 2.45%

<table>
<thead>
<tr>
<th></th>
<th>Datapoints</th>
<th>Co-Efficient</th>
<th>Correlation r²</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1yr</td>
<td>24</td>
<td>-0.221</td>
<td>2.58%</td>
<td>-19.55/-13.82</td>
</tr>
<tr>
<td>5yr</td>
<td>23</td>
<td>-0.884*</td>
<td>11.53%</td>
<td>4.26 / 7.72</td>
</tr>
<tr>
<td>10yr</td>
<td>23</td>
<td>-0.940*</td>
<td>12.90%</td>
<td>4.47 / 7.56</td>
</tr>
<tr>
<td>30yr</td>
<td>24</td>
<td>-1.186**</td>
<td>28.29%</td>
<td>4.18 / 2.55*</td>
</tr>
<tr>
<td>1yr, 1yr Fwd</td>
<td>26</td>
<td>-0.858*</td>
<td>10.72%</td>
<td>15.27 / 18.84</td>
</tr>
<tr>
<td>3yr, 2yr Fwd</td>
<td>24</td>
<td>-1.125*</td>
<td>11.55%</td>
<td>16.41 / 19.83</td>
</tr>
<tr>
<td>5yr, 5yr Fwd</td>
<td>26</td>
<td>-1.51**</td>
<td>20.84%</td>
<td>25.72 / 26.16</td>
</tr>
<tr>
<td>20yr, 10yr Fwd</td>
<td>25</td>
<td>-0.994**</td>
<td>19.44%</td>
<td>4.91 / 5.94</td>
</tr>
</tbody>
</table>

4. Discussion

The results indicate that at low levels of short term rates, the response of the long end of the yield to changes in inflation reverses sign. As a matter of historical record, it was during the period of low short term rates that Chairman Greenspan experienced his conundrum in the evolution of long term interest rates. Thus whilst it is not clear whether the observed behaviour was purely responsible, explaining the non-linear response is undoubtedly part of the solution to the problem. Similar to the approach taken in Gürkaynak et al, an attempt here is made to sketch a possible explanation whose main qualification is that it fits the observations and is, to some extend, consistent with current macroeconomic yield curve models.

The observation that the response of the long end of the yield curve seems to depend on a critical level in the short term rate is difficult to reconcile with any reasonable model of the evolution of the yield curve. There have been a number of models where non-linear effects were built into the yield curve, (Pfann, Schotman et al. 1996) however little work has been carried out that would predict threshold effects in such a macro manner. In addition, the “special” threshold level of ~ 2.50% for US data seems also difficult to reconcile with current models given that little attention is given in those models to specific interest rate levels.
A point that was made earlier however is that it there may be an unobserved variable that is causing the threshold effects. The observation that the highest statistical significance is seen using the 1yr rate, does not of itself mean that the 1yr rate is the unobserved variable, merely that its dynamics must closely match that of the true unobserved variable. With this in mind it is worth looking at the Fisher equation (Fisher 1907), where the level of nominal interest rates of maturity $t - y_t$ are defined in terms of inflation - $\pi_t$ and real rates - $r_t$.

$$y_t = \pi_t + r_t$$

Using this equation it is possible to define an expected 1yr real rate as the difference between the 1yr interest rate and the annualised inflation rate as measured by CPI-x. In figure 5 a quarterly average of the expected 1yr real rate and the 1yr interest rate is shown.

Figure 5: The evolution of the one year interest rate, the quarterly averaged expected 1yr real rate and the critical 1yr threshold rate (2.5%).

As can be seen from figure 5, the level at which the 1yr rate crosses the threshold both decreasing and increasing, is quite synchronous with the movement of real rates from positive to negative. In other words the time period where the inflation response of the yield curve reverses may be associated with a period where there was an expectation of negative real rates over a considerable time horizon, 1 year. The use of the 1yr rate, as opposed to a shorter maturity interest rate, is important here because of the role of
the expectations of short rates in its construction. A reason why no real statistically
significant threshold was found using the directly determined real rates is that it uses
the realised inflation as the expectation of inflation. In the threshold period under
study there are a number of times when the measured real rates were positive,
however the expectation of real rates would have been negative. As such, doing a
regression using such volatile and possibly misrepresenting data, even a robust one
would not be able to identify a statistically significant relationship.

Even if it were accepted that the expected real rate is the appropriate missing
variable, some work still has to be carried out to explain the reversal of the sign of the
response to inflation news. To quote from Gürkaynak et al, “It is hard to see why
financial markets would modify their estimate of r* (long term expectation of real
rates) in response to monetary policy surprises”. However it is the purpose of this
section to propose that that is exactly what happened in practice under the particular
environment that persisted at the time under study. For positive real and expected real
rates, the results that are shown here are in agreement with previously reported data
and it is plausible to say that the sensitivity to measured inflation is primarily the
result of changes in the markets perception of the steady state level of inflation – π*.

However for the situation when the real rates and, more importantly, their
expectations are negative it is clear that a different effect is taking place. It is
proposed that the response of the yield curve to inflation news is constrained by the
expectation of real rates. Firstly it is proposed that the there is a lower critical level,
πc, of inflation built into the yield curve below which the market will not take long
term yields. In other words even if short term inflation is low, even negative, and long
term expectations are less than πc, the long term inflation expectation that would be
extracted from the yield curve would still be πc.

It is accepted that this assumption is not consistent with a large number of
studies carried out essentially using the Campbell and Shiller methodology (Campbell
and Shiller 1991) etc. However for the extended periods of time that these studies
covered, the inflation rate was either quasi-normal (which I will define as being in a
range acceptable to the precepts of the Federal Reserve) or excessive. For only very
short periods of time, if at all, were inflationary conditions considered to be heading
in the direction of disinflation. Thus the impact of the dynamics of a possible
disinflationary period will have a negligible impact on the results of these studies. It
must be stressed that it is an a priori assumption that part of the reason that the market
dynamics were unusual over this period was because the economic and policy
environment was itself unusual.

It is further proposed that the yield curve response critical inflation level ($\pi_c$) is reached when expectations of real yields turn from positive to negative. One further assumption is needed to explain the observed results and that is that over the period in question the market did not expect any significant changes of short term rates due to short term movements in the measured rate of inflation. It is clear that over most of the period covered by the threshold level Fed Fund rates had been cut to 1% as a precautionary defence against possible deflation. In this environment however the Fisher equation still held. Thus a change in inflation, could not lead to a change in nominal rates and therefore had to be balanced by a change in real rates, but with the opposite sign. Given that the long term inflation expectation built into the yield curve, would not change long term yields as it was below the critical level, the change in the expectation of real rates would lead to a change in the level of long term nominal rates but in a direction opposite to that expected from a simple examination of the Fisher equation.

Over the long term the Fisher Equation is altered to be

$$y^* = r^* + (\pi^* | E(r_t > 0), \pi^c)$$

however in the shorter term it still holds as

$$y_t = \pi_t + r_t$$

and changes will be:

$$\delta(y_t) = \delta(\pi_t) + \delta(r_t) = 0$$

$$\delta(\pi_t) = - \delta(r_t)$$
And the changes in long term rates will be:

\[ y^* + \delta(y^*) = r^* - \alpha \delta(\pi_t) + \pi_c \]

where \( \alpha \) is the change in long term expectations of real rates as a result of a change in short term expectations. It is accepted that the current explanation is somewhat convoluted however it provides a ready explanation as to why long term expectations of the real rates move. It is because they are the only variable left in what is, in effect an accounting relationship. The direction of the move, whilst unexpected, is consistent with the conditions that were necessary to create it.

In the light of the results presented and the discussion, the conundrum can be partially understood in terms of a non-linear response of the yield curve to changing monetary and economic conditions. Once economic and monetary conditions were thought to be a quasi-emergency condition, the response of long term yields no longer acted as would be expected from a linear perspective. Declining inflation, and negative inflation surprises may have led to a decline in long term inflation expectations, these were not reflected in long term rates. Rather the relative increase in (already negative) short term real rates resulted in an increase in long term real rate expectations, that manifested itself in an effective increase in long term yields. Once monetary policy was “normalised” and short term interest rates started to rise, the long term rates declined slowly as the excess long term expectations of real rates were effectively removed from the determination of long term yields. This was observed as a decline in long term yields even as both short term yields and short term inflation expectations were rising.

5. Conclusions

Results have been presented that indicate that there is a degree of non-linearity in the response of yield curves to changing monetary and/or economic conditions. A working hypothesis of the behaviour of the yield curve has been outlined which seeks to describe the market behaviour known as Greenspan’s Conundrum. A key
component of this hypothesis is the conclusion that the dynamics of the yield curve are, under certain circumstances, a function of the market's confidence in the actions of the policy makers as well as expectations in the future values of relevant economic data. Future work, to be presented in a later paper, will be to look at the response of different currency yield curves to economic surprise data in the light of the utility function of the relevant central bank.
References:


