Distorted Trade Barriers
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Abstract

Since firm heterogeneity has been introduced into international trade models, the importance of firm entry and exit (the extensive margin) has been highlighted. In fact, Chaney (2008) illustrates how accounting for this extensive margin and heterogenous firms alters the standard gravity equation; thereby reversing the previously predicted effect the elasticity of substitution has on the elasticity of trade flows. Furthermore, Cole (forthcoming) points out that ad valorem tariffs affect the extensive margin quite differently than the commonly used iceberg transport cost. In this paper, I show that the elasticity of trade flows with respect to tariffs is more elastic than that of iceberg transport costs. Thus, elasticity estimates derived from variables such as distance may underestimate the effect caused by a change in tariffs.

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1 Introduction

It is quite common in the literature to use iceberg transport costs (shipping more than one unit of output for one unit to arrive as a portion “melts” away) to represent variable trade barriers; both tariffs and shipping costs alike. For many models, this equivalence is a completely reasonable assumption. For instance, in models of perfect competition, iceberg transport costs and *ad valorem* tariffs are completely equivalent. However, despite the market price being identical under both types of trade barrier in models with monopolistic competition, the level of firm profit is not; and the level of profit determines firm entry and exit— the extensive margin. Since monopolistic competition has been the workhorse model for international trade over the past 30 years, it is important to investigate the potential discrepancies between iceberg transport costs and *ad valorem* tariffs.

Since firm heterogeneity has been introduced into international trade models, the importance of the extensive margin has been highlighted. In fact, Chaney (2008) illustrates how accounting for this extensive margin and heterogenous firms alters the standard gravity equation; thereby reversing the previously predicted effect the elasticity of substitution has on the elasticity of trade flows. Specifically, Krugman (1980) predicts that a higher elasticity of substitution between goods magnifies the impact of trade barriers on trade flows, where Chaney (2008) shows that the effect on trade flows is actually dampened by higher levels of elasticity of substitution. The reasoning behind this reversal is driven by how the elasticity of substitution affects both the intensive and extensive margins. That is, the intensive margin is more sensitive but the extensive margin is less sensitive with a higher level of elasticity of substitution. Furthermore, with respect to iceberg transport costs, Chaney finds that the effect on the intensive and extensive margins exactly cancel out. It is this point in which I provide clarification. I utilize the Chaney framework and show that elasticity of trade flows with respect to an *ad valorem* tariff is not a constant but is decreasing in the elasticity of substitution.

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1 The different effect on firm profit is shown explicitly in Cole (forthcoming), which has fixed cost heterogeneity with quasi-linear utility and analyzes how iceberg transport costs and *ad valorem* tariffs affect the mass of varieties and welfare differently.
substitution, and that it is always greater than the elasticity of trade flows with respect to 
icberg transport costs. Thus, elasticity estimates derived from variables such as distance 
may underestimate the effect caused by a change in tariffs. So, if a policy maker were int-
tending to maximize tariff revenue or to follow the welfare by adhering to the longstanding 
rule of setting the tariff equal to the inverse of the elasticity of export supply, she would set 
the tariff too high.

The rest of the paper proceeds as follows. Section 2 quickly sets up the model. Section 3 
introduces trade and finds the elasticities of trade flows with respect to both trade barriers. 
Section 4 concludes.

2 Setup

I follow Chaney (2008) very closely, maintaining notation and setup, with two main ex-
ceptions. First, I additionally allow for an ad valorem tariff, \( s^h_{ij} \), on goods shipped from country 
\( i \) to country \( j \) in sector \( h \) where \( t^h_{ij} = 1 + s^h_{ij} > 1 \). Secondly, I allow for the government 
to contribute a portion of its tariff revenue to a global mutual fund that consists of firm 
profits and government bonds, all other tariff revenue is thrown away. This last assumption 
is helpful for two reasons: One, it provides a reasonable point of comparison between the two 
trade barriers by restricting consumer income to be identical; and two, it keeps the model 
tractable.

There are \( N \) potentially asymmetric countries that produce goods using only labor. Coun-
try \( n \) has a population of \( L_n \). Consumers in each country maximize utility derived from the 
consumption of goods from \( H + 1 \) sectors. Sector 0 provides a single freely traded homoge-
neous good that pins down the wage in country \( n \), \( w_n \). The other \( H \) sectors are made of a

\footnote{It will be shown that the elasticity of trade flows with respect to an ad valorem tariff is more elastic 
as long the elasticity of substitution is strictly greater than one, which is the standard assumption in the literature.}

\footnote{It is inherent to the iceberg transport cost assumption that output (and income) is lost to the economy 
whereas tariffs create revenue for the government. Of course, implicit in this bond assumption is that 
governments are keeping a budget in line with this policy.}

\footnote{I assume that every country produces a positive amount of \( q_0 \).}
continuum of differentiated goods. If a consumer consumes \( q_0 \) units of good 0, and \( q_h(\omega) \) units of each variety \( \omega \) of good \( h \), for all varieties in the set \( \Omega_h \) (determined in equilibrium), she gets a utility \( U \),

\[
U \equiv q_0^{\mu_0} \prod_{h=1}^{H} \left( \int_{\Omega_h} q_h(\omega)^{(\sigma_h-1)/\sigma_h} d\omega \right)^{[\sigma_h/(\sigma_h-1)]},
\]

where \( \mu_0 + \sum_{h=1}^{H} \mu_h = 1 \), and where \( \sigma_h > 1 \) is the elasticity of substitution between two varieties of good \( h \).

### 2.1 Trade Barriers and Technology

There are three types of trade barriers, two variable (tariffs, \( t_{ij}^h \), and iceberg transport costs, \( \tau_{ij}^h \)) and a fixed cost (\( f_{ij}^h \)). Each firm in sector \( h \) draws a random unit labor productivity \( \phi \) from a Pareto distribution with shape parameter \( \gamma_h \).

Following Chaney (2008), I assume the total mass of potential entrants in each sector is proportional to \( w_j L_j \). The cost of producing \( q \) units of a good and selling them in country \( j \) for a firm with productivity \( \phi \) is

\[
c_{ij}^h(p, q) = (t_{ij}^h - 1)pq + \tau_{ij}^h q + \frac{w_i}{\phi} q + f_{ij}^h.
\]

The price is the usual constant markup,

\[
p_{ij}^h(\phi) = \frac{\sigma_h}{(\sigma_h - 1)} \frac{t_{ij}^h \tau_{ij}^h w_i}{\phi}
\]

Note that the tariff and transport cost have the same affect on the price paid by consumers, however the actual profit level will be lower under an identical tariff which will have an effect on both the intensive and extensive margin.

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\(^5\)Productivity is distributed over \([1, +\infty)\) according to \( P(\hat{\phi}_h < \phi) = G_h(\phi) = 1 - \phi^{-\gamma_h} \), with \( \gamma_h > \sigma_h - 1 \).
2.2 Demand for Differentiated Goods

The total income spent by workers in country \( j \), \( Y_j \), is the sum of their labor income \( (w_j L_j) \) and of the dividends they get from their portfolio \( (w_j L_j \pi) \), where \( \pi \) is the dividend per share of the global mutual fund which consists of aggregated firm profits and government bonds.

Fresh off the boat (pre-tariff) exports from country \( i \) to country \( j \) in sector \( h \), by a firm with a labor productivity \( \varphi \), are

\[
x_{ij}^h(\varphi) = \frac{p_{ij}^h(\varphi)q_{ij}^h(\varphi)}{t_{ij}^h} = \frac{\mu_h Y_j}{t_{ij}^h} \left( \frac{p_{ij}^h(\varphi)}{P_j^h} \right)^{1-\sigma_h}
\]

where \( P_j^h \) is the ideal price index for good \( h \) in country \( j \). If only those firms above the productivity threshold \( \varphi_{kj}^h \) in country \( k \) and sector \( h \) export to country \( j \), the ideal price index for good \( h \) in country \( j \), \( P_j^h \), and dividends per share, \( \pi \), are defined as

\[
P_j^h = \left( \sum_{k=1}^{N} w_k L_k \int_{\varphi_{kj}^h}^{\infty} \left( \frac{\sigma_h}{(\sigma_h - 1) \varphi} \right)^{1-\sigma_h} dG_h(\varphi) \right)^{1/(1-\sigma_h)}
\]

\[
\pi = \frac{\sum_{h=1}^{H} \sum_{k,l=1}^{N} w_k L_k \left( \int_{\varphi_{kl}^h}^{\infty} \pi_{kl}^h(\varphi) + b_{kl}^h(\varphi)dG(\varphi) \right)}{\sum_{n=1}^{N} w_n L_n}
\]

where

\[
\pi_{kl}^h(\varphi) = \left( \frac{1}{(\sigma_h - 1)} \right)^{t_{kl}^h w_k^h(\varphi) - f_{kl}^h}
\]

are the net profits that firm with productivity \( \varphi \) in country \( k \) and sector \( h \) earns from exporting to country \( l \), and

\[
b_{kl}^h = \frac{(t_{kl} - 1)p_{kl}^h(\varphi)q_{kl}^h(\varphi)}{t_{kl} \sigma_h} = \left( \frac{(t_{kl} - 1)}{\sigma_h - 1} \right)^{t_{kl}^h w_k^h(\varphi)}
\]

is the portion of tariff revenue contributed to the global mutual fund.
3 Trade with Heterogeneous Firms

In this section, I characterize the equilibrium with trade. Due to the independence of sectors, I only consider sector $h$ and drop the $h$ superscript.

3.1 Equilibrium with trade

The profits firm $\varphi$ earns when exporting from country $i$ to $j$ are

$$\pi_{ij} = \frac{\mu Y_j t_{ij}^{\sigma}}{\sigma} \left[ \frac{\sigma w_i \tau_{ij}}{(\sigma - 1) \varphi P_j} \right]^{1-\sigma} - f_{ij}. $$

Define the threshold $\bar{\varphi}_{ij}$ from $\pi_{ij}(\bar{\varphi}_{ij}) = 0$ as the productivity of the least productive firm in country $i$ able to export to country $j$:

$$\bar{\varphi}_{ij} = \lambda_1 \left( \frac{f_{ij} t_{ij}^{\sigma}}{Y_j} \right)^{1/(\sigma-1)} \frac{w_i \tau_{ij}}{P_j}. $$

(6)

where $\lambda_1 = (\frac{\sigma}{\sigma-1}) (\frac{\sigma}{\mu})^{1/(\sigma-1)}$ is a constant.

Recalling that $Y_k = w_k L_k (1 + \pi)$ so $w_k L_k = \frac{Y_k}{(1+\pi)}$, the price index can be written as

$$P_j = \lambda_2 Y_j^{(\sigma-1)/\gamma} \theta_j $$

(7)

where

$$\lambda_2 = \left( \frac{\gamma - (\sigma - 1)}{\gamma} \right) \left( \frac{\sigma}{\mu} \right)^{(\gamma-1)/(\sigma-1)} \left( \frac{\sigma}{(\sigma - 1)} \right)^{\gamma} \left( \frac{1 + \pi}{Y} \right)$$

$$\theta_j^{-\gamma} = \sum_{k=1}^{N} \left( \frac{Y_k}{Y} \right) (w_k \tau_{ij})^{-\gamma} t_{kj}^{1+\frac{\sigma Y}{1-\sigma}} f_{kj}^{1+\frac{\sigma Y}{1-\sigma}}.$$  

The term $\theta_j$ is a measure of country $j$’s remoteness from the rest of the world such that $\tau_{ij}$ represents physical distance and $t_{ij}$ represents its remoteness as a result of unilateral trade policy.
Using the general equilibrium price index, \( \varphi_{ij} \), I can solve for firm level exports, the productivity thresholds and total world profits:

\[
x_{ij}(\varphi) = \begin{cases} 
\lambda_3 \left( \frac{Y_i}{Y} \right)^{\frac{\sigma-1}{\sigma}} \left( \frac{\theta_j}{w_i \tau_{ij}} \right)^{\sigma-1} \varphi^{\sigma-1}, & \text{if } \varphi \geq \varphi_{ij} \\
0 & \text{otherwise,}
\end{cases}
\]

\[
\varphi_{ij} = \lambda_4 \left( \frac{Y_i}{Y_j} \right)^{\frac{1}{\gamma}} \left( \frac{w_i \tau_{ij}}{\theta_j} \right) \left( f_{ij} f_{ij}^{\sigma} \right)^{\frac{1}{\sigma-1}}
\]

\[
Y_i = (1 + \lambda_5) w_i L_i \\
\pi = \lambda_5
\]

where \( \lambda_3, \lambda_4, \) and \( \lambda_5 \) are constants. It is important to note how tariffs and transport costs enter into the equilibrium firm level of exports and productivity thresholds. These differences translate into the following gravity equation: Total (f.o.b.) \( X^h_{ij} \) in sector \( h \) from country \( i \) to country \( j \) are given by

\[
\begin{align*}
X^h_{ij} = \mu_h \left( \frac{Y_i Y_j}{Y} \right) \left( \frac{w_i \tau_{ij} f_{ij}^{\sigma_h}}{\theta_j} \right)^{-\gamma_h} f_{ij}^\gamma \left[ \frac{\gamma_h}{\sigma_h - 1} - 1 \right].
\end{align*}
\]

Exports are a function of country size (\( Y_i \) and \( Y_j \)), workers’ productivity (\( w_i \)), the bilateral trade cost, variable (\( t_{ij}^h, \tau_{ij}^h \)) and fixed (\( f_{ij}^h \)), and the measure of \( j \)'s remoteness from the rest of the world (\( \theta_j^h \)). Though it appears that tariffs and iceberg transport costs enter the gravity equation differently, recall that the remoteness measure is affected differently by each trade barrier and more analysis is necessary.

\[\lambda_3 = \sigma \lambda_4^{1-\sigma}\]

\[\lambda_4 = \left[ \left( \frac{\sigma}{\mu} \right) \left( \gamma - (\sigma - 1) \right) \left[ 1 + \lambda_5 \right] \right]^\gamma \]

\[\lambda_5 = \frac{\sum_{h=1}^{H} \left( \frac{\sigma_h - 1}{\gamma_h} \right) \frac{\mu_h}{\sigma_h}}{1 - \sum_{h=1}^{H} \left( \frac{\sigma_h - 1}{\gamma_h} \right) \frac{\mu_h}{\sigma_h}}\]
As mentioned earlier, all else equal, the imposition of a tariff results in lower firm profits than the imposition of an identical iceberg transport costs (see Cole (forthcoming) for a proof). Furthermore, this simple difference in only the level of firm profits (recalling that the price consumers pay is identical under both trade barriers) translates into different trade flow elasticities in both the extensive and intensive margin. The corresponding elasticities are as follows:

\[
\text{T tariff: } \vartheta \equiv -\frac{d \ln X_{ij}}{d \ln t_{ij}} = \sigma + \frac{\sigma \gamma}{\sigma - 1} - \sigma = \frac{\sigma \gamma}{\sigma - 1}, \tag{11}
\]

\[
\text{Intensive Extensive}
\]

\[
\text{Iceberg: } \zeta \equiv -\frac{d \ln X_{ij}}{d \ln \tau_{ij}} = \left(\sigma - 1\right) + \left[\gamma - \left(\sigma - 1\right)\right] = \gamma. \tag{12}
\]

\[
\text{Intensive Extensive}
\]

It is straightforward to see by comparing equation (11) with (12), that trade flows are more elastic with respect to changes in tariffs than transport costs,

\[
\vartheta - \zeta = \frac{\gamma}{\sigma - 1}.
\]

What is interesting is that not only the overall trade flow is more elastic with respect to a tariff, but each margin (the intensive and extensive) is more elastic as well. At first glance, this may seem odd since the price paid by consumers is identical under both trade barriers. However, since profits are lower under a tariff, there are less firms in equilibrium. As such, each firm sells more under a tariff despite making lower profit. Furthermore, note that this difference between trade elasticities depends on the elasticity of substitution. This is because

\[\text{Note the claim by Chaney (2008) that the elasticity of trade flows is decreasing in the elasticity of substitution is not only maintained by using tariffs, but is strengthened by it:}\]

\[
\frac{d \vartheta}{d \sigma} = \frac{-\gamma}{(\sigma - 1)^2} < 0.
\]
for highly competitive industries where a firm’s markup is quite low, the ability for a firm to recoup some of its transport costs is also lower. Additionally, the shape parameter of the firms’ productivity distribution plays an important role. When a sector has a high $\gamma$, the smaller, less productive firms are producing relatively more of the sector’s output and since changes in tariffs have a greater impact on whether these firms are producing or not, it will then also have a greater impact on the industry’s aggregate trade flow. Therefore, if one wishes to use estimates of trade flow elasticities derived from data on distance, for instance, in order to assess the impact of tariff policy, the theory suggests that this estimate will underestimate the effect. In particular, this is the case for industries that are not very competitive (low $\sigma$) and are more homogeneous in productivity (high $\gamma$).

4 Conclusion

It is common in the recent trade literature to simply assume iceberg transport costs as a general proxy for many types of trade restrictions (in particular ad valorem tariffs). When perfect competition is assumed the two trade barriers are analogous. However, in the often used model of monopolistic competition, this is no longer the case. By using the Chaney (2008) framework, I illustrate that the trade flows are more elastic in response to changes in tariffs than iceberg transport costs. Since it is quite common to use distance to represent trade restrictions in gravity equations, it is important to understand this difference between tariffs and transport costs when anticipating the affects of tariff policy.

References


A Mathematical Appendix

A.1 Deriving the Gravity Equation

Aggregate Exports in sector $h$ from country $i$ to country $j$ is

$$X_{ij}^h = w_i L_i \int_{\varphi_{ij}}^{\infty} x_{ij}^h(\varphi)dG(\varphi).$$

Using the specific assumption about the distribution $G$, this becomes

$$X_{ij}^h = w_i L_i \int_{\varphi_{ij}}^{\infty} \lambda_3^h \left( \frac{Y_j}{Y} \right)^{(\sigma_h-1)/\gamma_h} \left( \frac{\theta_j}{w_i r_{ij}^h} \right)^{-\sigma_h} \left( \frac{\theta_j}{w_i r_{ij}^h} \right)^{-\sigma_h} \varphi^{\sigma_h-1} \frac{\varphi^{-\gamma_h-1}}{\gamma_h} d\varphi.$$

Solving this integral yields:

$$X_{ij}^h = \left( \frac{Y_j}{Y} \right)^{(\sigma_h-1)/\gamma_h} \left( \frac{\theta_j}{w_i r_{ij}^h} \right)^{-\sigma_h} \left( \frac{w_i L_i}{Y} \right)^{\sigma_h-1} \left( \frac{w_i r_{ij}^h}{\theta_j} \right)^{-\sigma_h} \lambda_4^h \left( \frac{Y_j}{Y} \right)^{1/\gamma_h} \left( \frac{w_i r_{ij}^h}{\theta_j} \right)^{1/\gamma_h} \int_{\varphi_{ij}}^{\infty} \varphi^{\sigma_h-1} \frac{\varphi^{-\gamma_h-1}}{\gamma_h} d\varphi.$$

$$= \left( \frac{Y_j}{Y} \right)^{(\sigma_h-1)/\gamma_h} \left( \frac{\theta_j}{w_i r_{ij}^h} \right)^{-\sigma_h} \left( \frac{w_i L_i}{Y} \right)^{\sigma_h-1} \left( \frac{w_i r_{ij}^h}{\theta_j} \right)^{-\sigma_h} \lambda_4^h \left( \frac{Y_j}{Y} \right)^{1/\gamma_h} \left( \frac{w_i r_{ij}^h}{\theta_j} \right)^{1/\gamma_h} \int_{\varphi_{ij}}^{\infty} \varphi^{\sigma_h-1} \frac{\varphi^{-\gamma_h-1}}{\gamma_h} d\varphi.$$

$$= \sigma_h \left( \frac{Y_j}{Y} \right)^{(\sigma_h-1)/\gamma_h} \left( \frac{w_i L_i}{Y} \right)^{\sigma_h-1} \left( \frac{w_i r_{ij}^h}{\theta_j} \right)^{-\sigma_h} \lambda_4^h \left( \frac{Y_j}{Y} \right)^{1/\gamma_h} \left( \frac{w_i r_{ij}^h}{\theta_j} \right)^{1/\gamma_h} \int_{\varphi_{ij}}^{\infty} \varphi^{\sigma_h-1} \frac{\varphi^{-\gamma_h-1}}{\gamma_h} d\varphi.$$

$$= \mu_h \left( 1 + \lambda_3^h \right) \left( \frac{w_i L_i}{Y} \right)^{(\sigma_h-1)/\gamma_h} \left( \frac{w_i r_{ij}^h}{\theta_j} \right)^{-\sigma_h} \lambda_4^h \left( \frac{Y_j}{Y} \right)^{1/\gamma_h} \left( \frac{w_i r_{ij}^h}{\theta_j} \right)^{1/\gamma_h} \int_{\varphi_{ij}}^{\infty} \varphi^{\sigma_h-1} \frac{\varphi^{-\gamma_h-1}}{\gamma_h} d\varphi.$$

$$= \mu_h \left( \frac{Y_j}{Y} \right)^{(\sigma_h-1)/\gamma_h} \left( \frac{w_i r_{ij}^h}{\theta_j} \right)^{-\sigma_h} \lambda_4^h \left( \frac{Y_j}{Y} \right)^{1/\gamma_h} \left( \frac{w_i r_{ij}^h}{\theta_j} \right)^{1/\gamma_h} \int_{\varphi_{ij}}^{\infty} \varphi^{\sigma_h-1} \frac{\varphi^{-\gamma_h-1}}{\gamma_h} d\varphi.$$

Therefore, total (f.o.b.) $X_{ij}^h$ in sector $h$ from country $i$ to country $j$ are given by

$$X_{ij}^h = \mu_h \left( \frac{Y_j}{Y} \right)^{(\sigma_h-1)/\gamma_h} \left( \frac{w_i r_{ij}^h}{\theta_j} \right)^{-\sigma_h} \lambda_4^h \left( \frac{Y_j}{Y} \right)^{1/\gamma_h} \left( \frac{w_i r_{ij}^h}{\theta_j} \right)^{1/\gamma_h} \int_{\varphi_{ij}}^{\infty} \varphi^{\sigma_h-1} \frac{\varphi^{-\gamma_h-1}}{\gamma_h} d\varphi.$$ (A-1)
A.2 Deriving Elasticities

Totally differentiating (A-1) for a specific sector \( h \) and assuming \( df_{ij} = 0 \) yields the following elasticities:

\[
\frac{d \ln X_{ij}}{d \ln t_{ij}} = -\frac{dX_{ij}/dt_{ij}}{X_{ij}/t_{ij}} = -\frac{t_{ij}}{X_{ij}} \left( w_i L_i \int_{\phi_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial t_{ij}} dG(\varphi) \right) + \frac{t_{ij}}{X_{ij}} \left( w_i L_i x_{ij}(\bar{\varphi}_{ij}) G'(\bar{\varphi}_{ij}) \frac{\partial \bar{\varphi}_{ij}}{\partial t_{ij}} \right)
\]

Intensive margin

Extensive margin

\[
\frac{d \ln X_{ij}}{d \ln \tau_{ij}} = -\frac{dX_{ij}/d\tau_{ij}}{X_{ij}/\tau_{ij}} = -\frac{\tau_{ij}}{X_{ij}} \left( w_i L_i \int_{\bar{\phi}_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial \tau_{ij}} dG(\varphi) \right) + \frac{\tau_{ij}}{X_{ij}} \left( w_i L_i x_{ij}(\bar{\varphi}_{ij}) G'(\bar{\varphi}_{ij}) \frac{\partial \bar{\varphi}_{ij}}{\partial \tau_{ij}} \right)
\]

Intensive margin

Extensive margin

Using the definition of equilibrium individual exports from equation (4), and assuming that country \( i \) is small enough and/or remote enough, so that \( \partial \theta_j/\partial t_{ij} \approx 0 \) and \( \partial \theta_j/\partial \tau_{ij} \approx 0 \), I get

\[
\frac{\partial x_{ij}(\varphi)}{\partial t_{ij}} = -\sigma x_{ij}(\varphi) \quad \text{and} \quad \frac{\partial x_{ij}(\varphi)}{\partial \tau_{ij}} = -(\sigma - 1) x_{ij}(\varphi).
\]

Integrating over all exporters, I get

\[
\text{Elasticity of the intensive margin w.r.t. tariffs} = -\frac{t_{ij}}{X_{ij}} \left( w_i L_i \int_{\phi_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial t_{ij}} dG(\varphi) \right)
\]

\[
= \sigma \frac{t_{ij}}{X_{ij}} \frac{w_i L_i \int_{\phi_{ij}}^{\infty} x_{ij}(\varphi) dG(\varphi)}{t_{ij}}
\]

\[
= \sigma \frac{t_{ij} X_{ij}}{t_{ij}}
\]

\[
= \sigma.
\]

Now, define \( x_{ij} = \lambda_{ij} \varphi^{\sigma-1} \) and note that \( G'(\varphi) = \varphi^{-\gamma-1}/\gamma \). Aggregate exports can be written in the following way

\[
X_{ij} = w_i L_i \lambda_{ij} \int_{\bar{\phi}_{ij}}^{\infty} \varphi^{\sigma-1} \varphi^{-\gamma-1}
\]

\[
= \frac{\gamma}{\gamma - (\sigma - 1)} w_i L_i \lambda_{ij} \bar{\varphi}^{(\sigma - 1) - \gamma}
\]

\[
= \frac{1}{\gamma - (\sigma - 1)} w_i L_i x_{ij}(\bar{\varphi}) G'(\bar{\varphi}) \bar{\varphi}.
\]

\[8\text{The focus of this paper is to compare the effects of iceberg transport costs with that of an } \textit{ad valorem} \text{ tariff, thus analyzing changes in fixed costs would not add anything to the paper and would be exactly that found in Chaney (2008).}\]
I therefore get the simple solution for the elasticity:

\[ \text{Elasticity of the extensive margin w.r.t. tariffs} = \frac{t_{ij}}{X_{ij}} \left( w_i L_i x_{ij}(\bar{\varphi}) G'(\bar{\varphi}) \frac{\partial \bar{\varphi}}{\partial t_{ij}} \right) \]

\[ = \frac{t_{ij}}{X_{ij}} \left( \frac{w_i L_i x_{ij}(\bar{\varphi}) G'(\bar{\varphi}) \varphi}{t_{ij}} \frac{\sigma}{\sigma - 1} \right) \]

\[ = (\gamma - (\sigma - 1)) \frac{t_{ij}}{X_{ij}} \left( \frac{X_{ij}}{t_{ij}} \frac{\sigma}{\sigma - 1} \right) \]

\[ = \frac{\sigma \gamma}{\sigma - 1} - \sigma. \]

Similarly for iceberg transport costs:

\[ \text{Elasticity of the intensive margin w.r.t. iceberg costs} = -\frac{\tau_{ij}}{X_{ij}} \left( w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial \tau_{ij}} dG(\varphi) \right) \]

\[ = (\sigma - 1) \frac{\tau_{ij}}{X_{ij}} \frac{w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} x_{ij}(\varphi) dG(\varphi)}{\tau_{ij}} \]

\[ = (\sigma - 1) \frac{\tau_{ij}}{X_{ij}} \frac{X_{ij}}{\tau_{ij}} \]

\[ = (\sigma - 1). \]