Simple Logit and Probit Marginal Effects in R

Alan Fernihough, University College Dublin

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Abstract

This paper outlines a simple routine to calculate the marginal effects of logit and probit regressions using the popular statistical software package R. I compare results obtained using this procedure with those produced using Stata. An extension of this routine to the generalized linear mixed effects regression is also presented.

1 Introduction

A common approach in empirical economic research is to model binary variables using a generalized linear model with a binomial distribution. The advantage of this approach is that it restricts predictions of the dependent variable to values between zero and one, unlike ordinary least squares regression (OLS). One difference of this approach is that the estimated coefficients are not marginal effects, as in OLS, but multiplicative effects. Fortunately, transforming these coefficients into marginal effects is a reasonably straightforward procedure.

In recent years, the open-source statistical program R has exploded in popularity. The primary attraction of R is the extensive repository of packages which have been contributed by researchers across a huge range of disciplines. Currently, the R package repository features 3,369 packages. Surprisingly, to my knowledge there is no general function which easily computes marginal effects from all potential binary dependent models similar to the mfx command as in Stata.¹

The aim of this paper is to present a quick solution to this problem, which is easy to implement.

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¹Some support is offered in both the ‘tonymisc’ and ‘erer’ packages.
2 Binary Dependent Variables

Let $E(y_i|x_i)$ represent the expected value of a dependent variable $y_i$ given a vector of explanatory variables $x_i$, for an observation unit $i$. In the case where $y$ is a linear function of $(x_1, \ldots, x_j) = X$ and $y$ is a continuous variable the following model with $j$ regressors can be estimated via ordinary least squares:

$$y = X'\beta \quad (1)$$

or

$$y = \beta_0 + \beta_1 x_1 + \ldots + \beta_j x_j \quad (2)$$

so the additive vector of predicted coefficients can be obtained from the usual computation $\hat{\beta} = (X'X)^{-1}X'y$. From (1) and (2) it is straightforward to see that the marginal effect of the variable $x_k$, where $k \in \{1, \ldots, j\}$, on the dependent variable is $\frac{\partial y}{\partial x_k} = \beta_k$. In other words, a unit increase in the variable $x_k$ increases the variable $y$ by $\beta_k$ units.

The standard approach to modeling dichotomous/binary variables (so $y \in \{0, 1\}$) is to estimate a generalized linear model under the assumption that $y$ follows some form of Bernoulli distribution. In econometrics, researchers will either use the logistic (logit) or standard normal cumulative (probit) distributions. Thus, the expected value of the dependent variable becomes:

$$y = G(X'\beta) \quad (3)$$

where $G$ is the specified binomial distribution. Since $y \in (0,1)$, the predicted value for observation $i$ ($y_i$) represents the conditional probability that $y_i$ is one, or $\Pr(y_i = 1)$. In the case of the logistic regression the generalized linear model can be specified:

$$\Pr(y_i = 1) = \text{logit}^{-1}(\beta_0 + \beta_1 x_1 + \ldots + \beta_j x_j) \quad (4)$$

where the inverse logit function is used here to map the linear predictions into probabilities. From (3), we see that the marginal effects must be calculated using the chain rule so:

$$\frac{\partial y}{\partial x_k} = \beta_k \times \frac{dG}{dX'\beta} \quad (5)$$

so the marginal effect of variable $x_k$ depends on the derivative: $dG/dX'\beta$, which is either a logistic or normal probability density function, depending on the choice of $G$.

As outlined in Kleiber & Zeileis (2008), there are two main approaches to calculating marginal effects from binary dependent variable models. The first uses
the average of the sample marginal effects, while the other uses average marginal
effects. The average of the sample marginal effects is calculated as follows:

\[
\frac{\partial y}{\partial x_k} = \beta_k \times \frac{\sum_{i=0}^{n} g(X' \hat{\beta})}{n}
\]  

(6)

where there are \( n \) observations in the dataset and \( g \) is the probability density func-
tion for either the normal or logistic distribution. In essence, one can calculate
the marginal effect for a each variable by using the estimated coefficient (corre-
sponding to the inner-part of the chain rule) multiplied by the average value of all
appropriately transformed predicted values.

The second approach calculates the marginal effect for \( x_k \) by taking predicted
probability calculated when all regressors are held at their mean value from the
same formulation with the exception of adding one unit to \( x_k \). The derivation of
this marginal effect is captured by the following:

\[
\frac{\partial y}{\partial x_k} = G(\hat{\beta}_0 + \hat{\beta}_1 \bar{x}_1 + \ldots + \hat{\beta}_k (\bar{x}_k + 1) + \ldots + \hat{\beta}_j \bar{x}_j) - G(\hat{\beta}_0 + \hat{\beta}_1 \bar{x}_1 + \ldots + \hat{\beta}_k \bar{x}_k + \ldots + \hat{\beta}_j \bar{x}_j)
\]  

(7)

where the marginal effect for a variable is computed by subtracting the conditional
predicted probability when all variables are held at their mean values from the same
conditional predicted probability, except with the variable of interest increased by
one-unit (\( \bar{x}_k + 1 \)).

3 Simple Functions of Logit and Probit Marginal Effects in R

Section 2 specified two methods by which marginal effects for either a logit of pro-
bit regression can be calculated. In this section, I outline a basic user-written R-
function which calculates the average of the sample marginal effects, as in equation
(6), and their associated standard errors. Dealing with binary/dummy or factor
variables adds complexity in calculating the average marginal effects of equation
(7). Given the objective of this paper, I do not present a function which calcu-
lates average marginal effects. It is noteworthy that the marginal effects produced
by other statistical software programs, such as Stata, calculate average marginal
effects by default. However, there is no reason to believe that the marginal ef-
effects produced by one method are superior to the other. Similarly, the standard
errors produced by the following R function are via simulation – which captures
uncertainty in both the regression coefficients and the probability density func-
tion. Alternatively, one could compute standard errors using the delta method, as
in Stata. Once again the difference between the two approaches is minimal and
since both methods are “approximations” there is little reason to believe one is
more robust than the other.
A function which calculates the average of the sample marginal effects for either a probit or logit model in R is displayed below. The default number of simulations from which the standard errors are calculated is 1,000. However, the user can change this number using the second argument.

```r
mfx <- function(x,sims=1000){
  set.seed(1984)
  pdf <- ifelse(as.character(x$call)[3]=="binomial(link = \"probit\")",
      mean(dnorm(predict(x, type = "link"))),
      mean(dlogis(predict(x, type = "link"))))
  pdfsd <- ifelse(as.character(x$call)[3]=="binomial(link = \"probit\")",
      sd(dnorm(predict(x, type = "link"))),
      sd(dlogis(predict(x, type = "link"))))
  marginal.effects <- pdf*coef(x)
  sim <- matrix(rep(NA,sims*length(coef(x))), nrow=sims)
  for(i in 1:length(coef(x))){
    sim[,i] <- rnorm(sims,coef(x)[i],diag(vcov(x)^0.5)[i])
  }
  pdfsim <- rnorm(sims,pdf,pdfsd)
  sim.se <- pdfsim*sim
  res <- cbind(marginal.effects,sd(sim.se))
  colnames(res)[2] <- "standard.error"
  ifelse(names(x$coefficients[1])=="(Intercept)",
         return(res[2:nrow(res),]),return(res))
}
```

4 Comparison with Other Software

To demonstrate how the function above works and also the similarities between this function and the `mfx` command in Stata, I perform a basic analysis. To complete this exercise, I use data from the car package in R. These data comprise of individual level information on income, education, gender, age, and language for 3,987 individuals. Creating a binary dependent variable called h.wage – signaling whether an individual earns a 'high wage' – I estimate the probability that an individual is in the ‘high wage’ cohort conditional on their age, education, a dummy variable taking the value one where the individual is a male and two dummy variables to indicate languages spoken other than English.

The code below displays the necessary R syntax and output as if displayed in the R console. The equivalent Stata output is also displayed. For example, the estimated marginal effects for education, i.e. the increase in the probability of being in the high wage category for a one year increase in education, are 4.2%, 4.2%, 4.6% and 4.5% for the probit and logit models estimated using R and Stata respectively. Clearly these values are very alike. The marginal effects for the
other regressors and their standard errors are very similar in each of the four other specifications.

```r
> setwd("C:\\Users\\Alan\\Documents\\My Dropbox\\marginaleffects")
> library(car)
> data(SLID)
> dat1 <- na.omit(SLID)
> dat1$h.wage <- ifelse(dat1$wages>20,1,0)
> p1 <- glm(h.wage ~ education+age+sex+language,data=dat1, family = binomial(link = "probit"))
> mfx(p1)
> mfx(p1)
marginal.effects standard.error
education 0.042139646 0.018387281
age 0.009601096 0.004189945
sexMale 0.134059218 0.058575244
languageFrench -0.015701475 0.028678160
languageOther -0.009959757 0.020615175
> l1 <- glm(h.wage ~ education+age+sex+language,data=dat1, family = binomial(link = "logit"))
> mfx(l1)
> mfx(l1)
marginal.effects standard.error
education 0.04243174 0.020708933
age 0.00947604 0.004632232
sexMale 0.13366522 0.065230162
languageFrench -0.01573230 0.029935596
languageOther -0.01063887 0.021267302
```

Stata Output

```
Marginal effects after probit
y = Pr(hwage) (predict)
= .19364718
```

| variable | dy/dx | Std. Err. | z | P>|z| | [ 95% C.I. ] | X |
|----------|-------|-----------|---|-----|--------------------|---|
| educat'bn | .0460106 | .00226 | 20.39 | 0.000 | .041589 | .050432 | 13.337 |
| age | .0104831 | .00058 | 18.22 | 0.000 | .009355 | .011611 | 37.0981 |
| _Isex_2* | .1460536 | .0131 | 11.15 | 0.000 | .120373 | .171735 | .498119 |
| _Ilang~2* | -.016739 | .02595 | -0.64 | 0.519 | -.067607 | .034129 | .064961 |
| _Ilang~3* | -.0107334 | .01992 | -0.54 | 0.590 | -.049771 | .028305 | .121395 |

Marginal effects after logit
y = Pr(hwage) (predict)
= .18771273

```
| variable | dy/dx | Std. Err. | z | P>|z| | [ 95% C.I. ] | X |
|----------|-------|-----------|---|-----|--------------------|---|
| educat'bn | .0447072 | .00221 | 20.22 | 0.000 | .040373 | .049041 | 13.337 |
| age | .0099842 | .00055 | 18.07 | 0.000 | .008901 | .011067 | 37.0981 |
| _Isex_2* | .1413666 | .01291 | 10.95 | 0.000 | .116064 | .166669 | .498119 |
```

5
5 Generalized Linear Mixed Effects Model

The following output contains a function which calculates marginal effects of the fixed effects of a generalized linear mixed effects model. The output of this function applied to the data used in the previous section is also displayed. This model is estimated using the lme4 package (Bates, 2010).

```r
> library(lme4)
> glmerfx <- function(x,nsims=1000){
+ set.seed(1984)
+ pdf <- mean(dlogis(-log((1-fitted(x))/fitted(x))))
+ pdfsd <- sd(dlogis(-log((1-fitted(x))/fitted(x))))
+ marginal.effects <- pdf*fixef(x)
+ sim <- matrix(rep(NA,nsims*length(fixef(x))), nrow=nsims)
+ for(i in 1:length(fixef(x))){
+ sim[,i] <- rnorm(nsims,fixef(x)[i],diag(vcov(x)^0.5)[i])
+ }
+ pdfsim <- rnorm(nsims,pdf,pdfsd)
+ sim.se <- pdfsim*sim
+ res <- cbind(marginal.effects,sd(sim.se))
+ colnames(res)[2] <- "standard.error"
+ ifelse(names(fixef(x))[1]=="(Intercept)",
+ return(res[2:nrow(res),]),return(res))
+ }
> glme1 <- lmer(h.wage ~ education+age+sex+(1|language),
+ family = binomial(link = logit),data=dat1)
> glmerfx(glme1)
marginal.effects standard.error
education 0.042502043 0.021134699
age 0.009457282 0.004751698
sexMale 0.133529363 0.067310950
```

References


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