Optimal Contract Orders and Relationship-Specific Investments in Vertical Organizations

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Abstract: This paper characterizes the optimal contracts issued to suppliers when delivery is subject to disruptions and when they can invest to reduce such a risk. When investment is contractible dual sourcing is generally optimal because it reduces the risk of disruption. The manufacturer (buyer) either issues symmetric contracts or selects one supplier as a major provider who invests while the buffer supplier does not. An increased reliance on single sourcing or on a major supplier is optimal under moral hazard. Indeed, we show that order consolidation increases the manufacturer's profits because it serves as an incentive device in inducing investment.

Keywords: Moral Hazard; Vertical Organization; Supply Base Management; Contract Order Size; Relationship-specific Investment; Strategic Outsourcing

JEL Classifications: D23, D86, L24.

1 Introduction

A casual observation of recent development across industries suggests two seemingly contrasting trends in vertical organization. While manufacturers across different industry sectors are outsourcing most of their inputs, they are also relying upon a smaller number of key suppliers.[2] This tendency has been referred to as a "move to the middle" in Clemons et al. (1993). These authors show that this strategic decision allows manufacturers to minimize both the cost and the risk associated with the procurement of their inputs.

In a global outsourcing and in-sourcing survey of executives conducted by Deloitte Consulting LLP, reducing operating costs and improving customer service are ranked as the first two important objectives by most respondents. Moreover, almost half of respondents have terminated an outsourcing contract in regard to the sustainability of a buyer-supplier relationship. Of the terminations, the perceived quality of service is the key factor in the decision (71%) (Deloitte Consulting LLP, 2012).

Many factors affect the quality of service. Of these *"supply chain disruptions and the associated operational and financial risks represent the most pressing concerns facing firms that compete in today's marketplace"* Craighead et al. (2007). As reported in Bill Powell (2012) the CEO of Seagate explained that the computer industry was in need of 175 million hard drives but that suppliers could only supply 125 million units as a result of the natural disaster in Thailand and that this shortfall will have a long lasting impact on the earnings of companies such as Apple and Hewlett-Packard. While this article refers to a disaster of exceptional magnitude, Craighead et al (2007) point out that supply chain disruptions are unavoidable and, as a result, that supply chains are risky.[3]

This paper takes a step in understanding the governance of buyer-supplier relationship in a vertical organizational context with relationship-specific investments. More specifically, we study the design of contract orders by a downstream buyer (a manufacturer) who faces supply hazard defined as the failure of some selected suppliers in meeting the contracting requirement by the buyer. We show that a downstream buyer can strategically segment the contract orders to its suppliers to manage the risk of default and, in the presence of moral hazard, to induce investment.

The papers analyzing optimal outsourcing strategies typically compare dual sourcing to single sourcing. Anton and Yao (1989, 1992) bring to light the collusive feature of bids under dual sourcing and thus its sub-optimality compared to single sourcing. Interestingly though they show that dual sourcing may be optimal when considering investment to reduce the cost since the supplier with the highest cost has an incentive to invest and reduce its cost while it would not do so under single sourcing. Intuitively, as the price paid to the contracted

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[2] In 1998 Boeing managed 3800 direct suppliers and it reduced its base in 2006 to 1200 suppliers. Of these, about 40 to 50 account for two-thirds of their activity (Bernstein, 2006). Ford reduced the number of suppliers from 2,500 to less than 1,000 (McCracken, 2005).

[3] The risk of default and its financial consequences for manufacturers are also analyzed in Kleindorfer and Saad (2005) and Hendricks and Singhal (2003, 2005).

supplier(s) is positively correlated to the highest cost the manufacturer may be better off under dual sourcing.

Li (2013) analyzes a buyer's sourcing strategy consisting of the supply base design and pricing mechanism, considering supplier cost-reduction effort and supplier competition. The supply base design concerns the number of suppliers (one or two) included in the supply base, and the capacity to be invested in each supplier. The pricing mechanism allows for renegotiation. The author assumes that suppliers are capacity constrained. However, orders below a supplier's capacity are guaranteed to be delivered. She shows that dual sourcing fosters competition without discouraging investment.

Perry and Sakovics (2003) consider the sequential offerings of procurement contracts and show that dual sourcing can be optimal provided there is endogenous entry. With fixed entry, dual sourcing is not optimal because it raises the expected profits gathered by the winner of the first contract who would not accept anything less than the winner of the second contract. However, for that same reason, dual sourcing promotes entry which, in turn, may lower the expected price below that of single sourcing.

In a framework of generalized second-price auctions with pre-auction investment, Gong et al. (2012) studies procurement contracts where a buyer can either divide full production among multiple suppliers or award the entire production to a single supplier. The authors focus on the effect of using multiple suppliers on investment incentives and show that the optimality of split-award depends on the socially efficient number of firms at the investment stage. Their findings suggest that sole-sourcing is buyer-optimal when that number is greater than one, while split-award lowers the buyer procurement cost when the number is one.

Despite the important insights provided by these authors, the linkages of a buyer's design for optimal market orders size in the presence of supply hazard and moral hazard is not much addressed. This paper attempts to fill this gap.

We study the problem of a (risk neutral) manufacturer who purchases an input from at most two (risk averse) suppliers. While the suppliers are not capacity constrained, neither can guarantee that their order will be delivered on time satisfying all specifications. We consider that default risk rises with the size of the order. Intuitively, in the event of a disruption, the larger the order is, the greater the risk that it won't be completed on time. We also assume the suppliers can invest to improve the quality of their service. Investing reduces the supplier's absolute and marginal exposure to risk (namely, the marginal increment in the risk of failure resulting from a slight order increase). To highlight the role of moral hazard, we study both the case when investment is contractible and where it is not.

When investment is contractible the manufacturer strategically arranges the market orders so as to curtail the risk of default from the suppliers. Dual sourcing is optimal unless contracting costs are very high. These costs are contingent on the suppliers' alternative contract opportunities. Our findings show that the manufacturer may request that both, none or only one supplier invest(s) depending on the size of the investment. Two symmetric equilibria

exist where either both or none of the suppliers invest and where each receive identical orders. In these the probability of a delivery failure is minimized. For a non-empty range of parameters the manufacturer proposes different contracts to the two identical suppliers. One is selected as a major supplier. He is requested to invest and supplies a larger proportion of the input. The other is used as a buffer supplier and does not invest. As the cost associated with a supply disruption increases this strategy is optimal for a wider range of parameters. Moreover, as this cost rises the manufacturer increases her reliance on the main supplier because his marginal exposure to risk is lower.

When investment is not contractible we show that consolidation increases the manufacturer's profits as it alleviates the moral hazard issue. This result hinges on the fact that larger orders are associated with more risk. Thus risk-averse suppliers have a greater incentive to invest when receiving large orders to curtail the risk these are associated with. Consequently, the moral issue is less severe when orders are large and, as a result, the expected monetary transfer to the supplier decreases with the order size.

Our results suggest that the move towards consolidation may be a sign that moral hazard concerns have been exacerbated in recent years. If so the improvement in information and communication technologies which has enabled the manufacturers to have more control over the suppliers did not successfully reduce the monitoring costs. It may be that, in recent years, inputs have become increasingly sophisticated and/or that the need to have on-time deliveries has become more of an imperative. As a result, it has become more important that suppliers invest in non-contractible such as innovation (cf. Bakos and Brynjolfsson, 1993)).

Finally, this paper is also related to a series of influential papers on firm organization which highlight the choice faced by firms in purchasing from an affiliate or from an independent supplier, where the latter gives rise to a hold-up problem when contracts are incomplete (see Antràs and Helpman, 2004, 2008 among others).[4] Clearly, outsourcing only takes place for tasks below a certain complexity threshold due to better communication and less opportunistic behavior among affiliated parties. The insight provided here suggests that higher technology requirements complicates the relation with the supplier and may make it optimal to vertically integrate. Despite the clear findings on the relation between outsourcing and technological complexity, the role of contract design by the size of market orders as a determinant of outsourcing upper parts of the value chain has attracted little attention in the economic literature.[5]

The rest of the paper is organized as follows. Section 2 presents the model.

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[4]Following Antràs and Helpman (2004, 2008), the literature that relates the intensity of the sourced input to technology transfer costs includes Grover (2007) showing the results from Antràs and Helpman (2004, 2008) only hold for a certain range of technological complexity of the input, and Costinot et al. (2011) reinterpreting the source of contractual frictions as arising from the non-routineness of tasks.

[5]Antràs and Rossi-Hansberg (2009) suggest that past literature has focused too much on hold-up inefficiencies as the main drivers of the internalization decision and underline the importance of the effects of the non-appropriable nature of knowledge on the internalization decision of firms.

Section 3 deals with sourcing under complete information. Section 4 studies the other scenario where the supplier's effort in investment is not contractible. Section 5 discussed the robustness of our model and some extensions. Finally, section 6 concludes.

2 The model

A manufacturer must purchase a quantity Q of a specific input to be delivered at a specific time. For simplicity assume that $Q = 1$. She faces 2 identical risk averse suppliers (supplier 1 and 2). Each can produce up to 1 unit. Their production costs are common knowledge and we assume that it costs cq for each supplier to produce and deliver q units.

We consider that production is subject to some uncertainty which jeopardizes either the quality or the timely delivery of orders or both. To simplify matters we assume that the supplier will either *succeed* or *fail*. Success means that the supplier delivers his order on time and all units meet the required specifications. Failure means that the supplier only delivers up to $q_L < \frac{1}{2}$ units that match the specifications and the rest of the order (if initially larger than q_L) is either not completed or does not comply with the requirements and cannot be used.

Once contracted, suppliers can make a relation-specific investment to increase the chances of achieving timely, satisfactory deliveries. Let $p_i(q)$ denote the probability of success:

$$p_i(q) = \begin{cases} \alpha(q_i) \text{ if } I^i = I, \\ \beta(q_i) \text{ if } I^i = 0. \end{cases} \tag{1}$$

Assume that $\alpha(q) = \beta(q) = 1$ for $q \leq q_L$ and that

$$\alpha(q) = \frac{1}{1 - q_L} \left[1 - q + \alpha \left(q - q_L \right) \right] \text{ and } \beta(q) = \frac{1}{1 - q_L} \left[1 - q + \beta \left(q - q_L \right) \right] \tag{2}$$

for any $q \geq q_L$. Finally we have $0 < \beta < \alpha < 1$. Figure 1 below represents both functions.
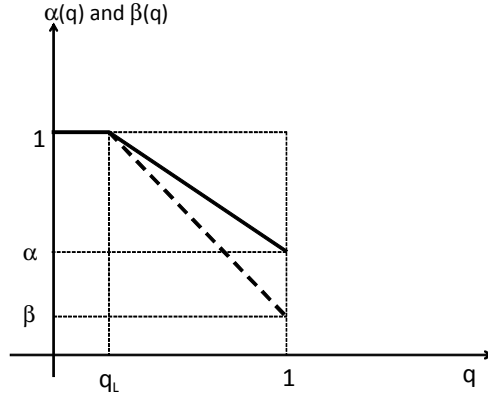
Figure 1: The functions $\alpha(q)$ (plain line) and $\beta(q)$ (dotted line).

The functions $\alpha(q)$ and $\beta(q)$ reflect the hypothesis we make on risk and on the value of the investment.

(i) The risk associated with orders larger than q_L increases with the order size. In other words, as deliveries are subject to a deadline the more units a supplier is responsible for, the less flexibility he has to complete large orders on time if something goes wrong.

(ii) Investing decreases the absolute exposure to risk: $\alpha(q) \geq \beta(q)$ for any q.

(iii) Investing decreases the marginal exposure to risk:

$$\frac{d}{dq}\alpha(q) > \frac{d}{dq}\beta(q) \text{ for } q > q_L. \tag{3}$$

The larger the order the greater the benefits of investing.

Failure triggers two types of costs. The first, denoted by $\phi \geq 0$, is a fixed cost born by the supplier. It represents for instance the cost associated with fixing or replacing a machine. The second, labelled $\Phi(q - q_L)$, is proportional to the number of units that are needed to fulfill the order. We make the following assumption.

Assumption: $\Phi(0) = 0$, $\Phi(x) > cx$, the first derivative is such that $\Phi'(0) = c$ and $\Phi'(x) > c$ for all $x > 0$, finally $\Phi''(x) \geq 0$.

An example for the function $\Phi(.)$ which we will use in some instances is

$$\Phi(q - q_L) = c(q - q_L) + \frac{c}{2}(q - q_L)^2. \tag{4}$$

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We consider that $\Phi(q - q_L)$ may either be born by the manufacturer or by the supplier.

The timing of the game is such that first the manufacturer proposes a contract to each supplier which they can accept or reject. When investment is not verifiable each contracted supplier decides whether to invest or not. Finally production takes place and each may fail or succeed. We solve for the sub-game perfect equilibrium of the game.

3 Contractible Investment Levels and Optimal Risk Management.

The case of contractible investment is motivated considering that manufacturers often rely on industry accreditation to decide on which supplier enters their supply base. To be chosen it is often requested that suppliers must comply with certain specific requirements. Such a requirement can take the form of relationship-specific investment in specified technologies for either cost reduction, quality improvement or to address potential compatibility issues.[6] In this section we analyze the optimal contracting strategy (dual or single sourcing) and what investments are required when the management of risk is the manufacturer's only concern.

Let $P > c$ denote the proceeds to the manufacturer from selling her output.[7] When investment is contractible the contract to supplier i ($i = 1, 2$) is of the form (I^i, q_i, t_S^i, t_F^i) where $I^i \in \{I, 0\}$, $q_i \in [0, 1]$ is the quantity to be supplied and finally where t_S^i is the transfer upon success and t_F^i is the transfer upon failure. Let $\Pi^M(q_1, q_2)$ denote the manufacturer's profit when issuing orders (q_1, q_2) and let Π^S denote a supplier's expected profit. The manufacturer designs contracts that solve

$$\max \Pi^M(q_1, q_2) = P - \sum_{i=1,2} \left[p_i(q_i) t_S^i + (1 - p_i(q_i)) t_F^i \right] - C_F \qquad (5)$$

where C_F is the cost of failure. We have $C_F = 0$ when the supplier pays $(\phi + \Phi(q - q_L))$, otherwise we have

$$C_F = \sum_{\substack{i=1,2 \\ j \neq i}} p_i(q_i) \left(1 - p_j(q_j)\right) \Phi(q_j - q_L) + (1 - p_1(q_1)) (1 - p_2(q_2)) \Phi(1 - 2q_L). \qquad (6)$$

He selects the orders such that

$$q_1 + q_2 = 1 \qquad (7)$$

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[6] For instance, Bakos and Brynjolfsson (1993) report that Boeing requested that its suppliers adopt the same computer-aided design system (CATIA) when it manufactured the 777 aircraft.

[7] In an early version we considered that the manufacturer only gets P when none of the contracted suppliers fails. Provided P was large enough, all results were similar.

By imposing this constraint we implicitly assume that the manufacturer does not want to be stuck with obsolete inventory. Evidence for such a behavior can be found in Gans (2007) which gives as an example the strategy of the company Dell. As many other manufacturers it orders upon realization of the demand so as to have the exact number of units it requires.

Finally the monetary transfers and orders must ensure that the supplier is better-off accepting the contract that is that $\Pi^S \geq \underline{\pi}$ where $\underline{\pi}$ is a supplier's reservation profits. We have

$$\Pi^S = p_i(q_i)\pi\left(t_S^i - cq_i\right) + (1 - p_i(q_i))\pi\left(t_F^i - cq_L - c_F\right) - I^i, \qquad (8)$$

where

$$c_F = \begin{cases} \phi \text{ when the manufacturer bears the cost } \Phi(q_i - q_L), \\ \phi + \Phi(q_i - q_L) \text{ when she does not,} \end{cases} \qquad (9)$$

and where $\pi(.)$ is an increasing concave function that accounts for risk aversion.

When investment is contractible the contracts are efficient: the suppliers are fully insured and receive their reservation utility.

Lemma 1: Optimal transfers.

The optimal transfers are given by

$$t_S^i = cq_i + r^i \text{ and } t_F^i = cq_L + c_F + r^i \qquad (10)$$

with $i = 1, 2$ where

$$r^i = \begin{cases} \pi^{-1}(\underline{\pi} + I) \text{ if } I^i = I, \\ \pi^{-1}(\underline{\pi}) \text{ if } I^i = 0. \end{cases} \qquad (11)$$

and where $c_F \in \{\phi, \phi + \Phi(q_i - q_L)\}$ depending on who bears the cost $\Phi(q_i - q_L)$. Therefore the manufacturer always pays the full cost of failure. If not directly, she pays it as a reimbursement via the transfer upon failure.

(The proof is straightforward and follows from the fact that $t_S^i - cq_i = t_F^i - cq_L - c_F$ and $\Pi^S = \underline{\pi}$.)

We now consider the choices between high and low investment, between single sourcing and dual sourcing and, in the case of the latter, we calculate the optimal orders.

- **Single sourcing**.

Single sourcing corresponds to a situation where the manufacturer issues one contract and the order size is fixed to $q_1 = 1$ (while supplier 2 receives no order and no monetary transfer). The only decision the manufacturer must make is whether $I^1 = 0$ or 1.

Corollary 1: *Under single sourcing the manufacturer is better off if the supplier invests if and only if*

$$(\alpha - \beta)\left[\Phi(1 - q_L) + \phi - c(1 - q_L)\right] \geq \triangle r, \tag{12}$$

where

$$\triangle r = \pi^{-1}(\underline{\pi} + I) - \pi^{-1}(\underline{\pi}). \tag{13}$$

The proof is straightforward.

Inequality (12) is very intuitive. Given the assumption regarding the function $\Phi(.)$ we know that the left had side of (12) is always positive. Basically this inequality compares the benefits from reducing the probability of failure with the cost of investment. If the investment does not substantially improve orders delivery (α close to β) or if the cost associated with failure is not large ($\Phi(1 - q_L) + \phi$ close to $c(1 - q_L)$), then the manufacturer is less likely to request $I^1 = I$.

- **Dual sourcing.**

Note that under (7) the manufacturer must only decide upon the quantity it orders from supplier 1. Let $\Pi^M(q) \equiv \Pi^M(q, 1 - q)$. The analysis proceeds as follows. First we characterize the optimal orders associated with each level of investment (I or 0) (Lemma 2 and 3). Given these orders, we characterize the optimal investment strategy under dual sourcing (Proposition 1). Finally we compare dual sourcing to single sourcing (Proposition 2).

When $q_1 \neq q_2$ we will refer to the *"buffer supplier"* as the supplier who receives $q_i = \min\{q_1, q_2\}$.

Lemma 2: *Orders within the range $]0, q_L[$ are not optimal. In other words a buffer supplier must produce at least q_L units. When his order equals q_L he does not invest ($I = 0$).*

Proof: Any order q within the range $]0, q_L]$ will be delivered for sure. By slightly increasing it the manufacturer reduces the risk that the other supplier fails to deliver his order and her overall profits increase. Indeed, for any $q < q_L$ we have

$$\frac{d\Pi^M}{dq} = (1 - p_2(1 - q))\left[\Phi'(1 - q - q_L) - c\right] \tag{14}$$
$$-p_2'\left[\Phi(1 - q - q_L) - c(1 - q - q_L) + \phi\right] > 0,$$

where $p_i' \equiv \frac{dp_i(q)}{dq} < 0$. Finally since for orders of size q_L the probability of success is equal to 1, thus there is no reason to request that the supplier invests as this would only increase the transfer to be paid.

From lemma 2 it follows that any $q_1 > 1 - q_L$ is not optimal as it would lead supplier 2 to produce less than q_L. Therefore, under dual sourcing, the optimal order from supplier 1 $q^* \in [q_L, 1 - q_L]$.

Lemma 3: Optimal order sizes. *When both suppliers invest or when none of them invests it is optimal to set $q^* = \frac{1}{2}$ which minimizes the probability of failure $p^F = 1 - p_1(q)p_2(1-q)$. When only one supplier invests it is optimal to request that he delivers a quantity $q^*(q_L)$ such that $q^*(q_L) \in \left]\frac{1}{2}, 1 - q_L\right]$ for all $q_L < \frac{1}{2}$ and $q^*(\frac{1}{2}) = \frac{1}{2}$.*

Proof*: See Appendix.*

Given the optimal orders for each possible investment level we can now determine the optimal investment levels.

Proposition 1: Optimal investment strategy and orders under dual sourcing.

Consider any orders $q \geq q_L$ and let $\triangle r = \pi^{-1}(\underline{\pi} + I) - \pi^{-1}(\underline{\pi})$. For any $q_L \leq \frac{1}{2}$ there exists $\underline{r}(q_L)$ and $\overline{r}(q_L)$ such that $\underline{r}(q_L) \leq \overline{r}(q_L)$ where the equality holds at $q_L = \frac{1}{2}$, such that it is optimal to request that

(i) both suppliers invest when $\triangle r \in [0, \underline{r}(q_L)]$ and each receives an order $q = \frac{1}{2}$,

(ii) only one supplier invests when $\triangle r \in [\underline{r}(q_L), \overline{r}(q_L)]$ and his order is $q^* \in \left]\frac{1}{2}, 1 - q_L\right]$ while the other supplier provides $(1 - q^*)$.

(iii) none of the suppliers invest when $\triangle r \geq \overline{r}(q_L)$ and each receives and order $q = \frac{1}{2}$.

Proof: The proof in Appendix is done for the case where the supplier pays the cost of failure but it can easily be extended to the other case.

The main conclusion from proposition 1 is that for any $c > 0$, $\phi \geq 0$ and $q_L < \frac{1}{2}$ there exists a non-empty range for the parameter $\triangle r$ over which it is optimal for the manufacturer to treat the identical suppliers differently and use one as a buffer.

It is interesting to analyze how the manufacturer deals with an increase in ϕ to understand how meaningful the investment in technology is.

Corollary 2: *As ϕ increases (and for any $q_L < \frac{1}{2}$) the interval $\triangle r = [\underline{r}(q_L), \overline{r}(q_L)]$ shifts up and widens. This means that, as ϕ increases the manufacturer is more likely to require investment by at least one supplier and she relies on a buffer supplier for a wider range of parameters. Moreover, when only one supplier invests, the order to the buffer supplier is non-increasing in ϕ (that is $\frac{d(1-q^*)}{d\phi} \leq 0$).*

Proof: See Appendix.

Figure 2 illustrates the above result when we consider that the function $\Phi(.)$ is given by (4).
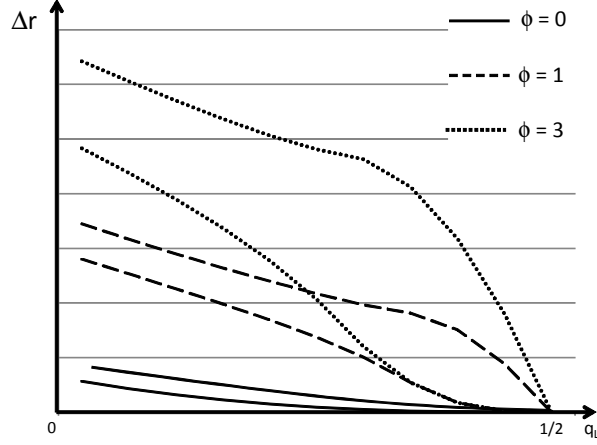
Figure 2: This figure depicts $\triangle r = [\underline{r}(q_L), \overline{r}(q_L)]$ as a function of q_L for $\phi \in \{0, 1, 3\}$.

That the manufacturer is more likely to request that at least one of the suppliers invests as ϕ increases is to be expected. As the cost associated with failure increases relative to the monetary compensation needed to support investment, it becomes optimal to demand that the suppliers invest. The fact that the interval $\triangle r$ widens as ϕ increases is maybe less expected and shows that using one supplier as a buffer is indeed a strategy to reduce risk. Moreover, as ϕ increases the buffer supplier's order is reduced to the point where it equals q_L meaning that his delivery is not associated with any risk. Conventional wisdom may lead one to expect that the manufacturer should decrease the order to the main supplier as ϕ increases since large orders are associated with more risk. We show that the opposite is true. Of the two suppliers it is best to request more from the one who has a lower marginal exposure to risk. This outcome extends to all situations where a marginal increase in the order size marginally increases the probability of failure but less so when the supplier invested in technology improvement. Thus it holds whenever (3) holds.

Proposition 2: Single sourcing versus dual sourcing.

Assume that it is optimal to request investment from at least one supplier. The choice between dual sourcing and single sourcing depends solely on the suppliers' reservation profits, that is on $\underline{\pi}$. Unless these profits are large, dual sourcing is optimal.

Proof: For any $\underline{\pi} > 0$ the manufacturer's profits are discontinuous at $q = 1$ and we have

$$\Pi(1) - \lim_{\varepsilon \to 0} \Pi(1 - \varepsilon) = \pi^{-1}(\underline{\pi}).$$

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Furthermore, as we decrease the order size to the main supplier from $1 - \varepsilon$ to $1 - q_L$ the profits increase. Thus, for $\underline{\pi}$ relatively small, the manufacturer's profit increase as he moves away from single sourcing.

Figures 3 and 4 below give a visual representation of $\Pi^M(q)$ taking into account the optimal investment levels. Figure 3 represents the profits when $\triangle r \in [\underline{r}(q_L), \overline{r}(q_L)]$ and figure 4 assumes $\triangle r \leq \underline{r}(q_L)$.
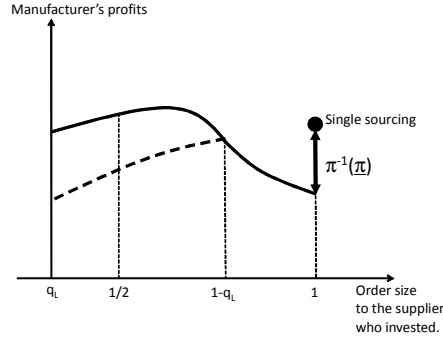

Figure 3: The manufacturer's profits when a single supplier invests. The dotted line shows a situation where ϕ is such that $q^* = 1 - q_L$ under dual sourcing.

Assume $\triangle r \geq \underline{r}(q_L)$ so that it is optimal to request that only one supplier invests under dual sourcing. The only discontinuity in the manufacturer's profits occurs at $q = 1$. It is due to having to pay $r^2 = \pi^{-1}(\underline{\pi})$ when contracting an extra supplier as the manufacturer reduces the order to the main supplier from 1 to $1 - \varepsilon$. The greater $\underline{\pi}$ the less likely the manufacturer will be to rely on dual sourcing.
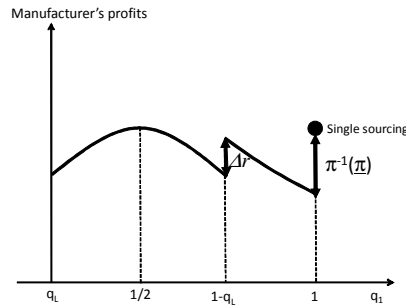

Figure 4: The manufacturer's profits when a both suppliers invest.

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Assume now that $\triangle r < \underline{r}(q_L)$. In that case it is optimal to request that only one supplier invests as long as $q_1 \geq 1 - q_L$ and thus $q_2 \leq q_L$ and it is optimal to have both suppliers investing when $q_i \in]q_L, 1 - q_L[$ for $i = 1, 2$. The manufacturer's profit function is then discontinuous at $q_1 = 1$ for the same reason as before. However it is also discontinuous at $q_1 = 1 - q_L$ as the manufacturer must now compensate one more supplier for investing. The size of the second discontinuity is given by $\triangle r$. However, since $\triangle r < \underline{r}(q_L)$, the manufacturer's profits are greater when both invest and both produce $q = \frac{1}{2}$ as opposed as having one investing and producing $q_1 = 1 - q_L$. While both, the value of $\underline{\pi}$ and that of $\triangle r$ when comparing single to dual sourcing, it remains true that single sourcing is not optimal for low values of $\underline{\pi}$ since the manufacturer's profits increase to the left of $q_1 = 1$. Thus, in either cases, if $\underline{\pi}$ is low then dual sourcing is optimal.

The value of $\underline{\pi}$ depends, among other things, on the number of manufacturers, the number of suppliers and the level of competitiveness in the industry. As competitiveness increases among suppliers, the reservation profits are more likely to be low and dual sourcing becomes optimal. This may explain why relying on several suppliers was common practice in the US where the economy is assumed to be more competitive.

3.1 On the Use of a Buffer Supplier.

When the function $\Phi(.)$ is given by (4) we can analyze the case where the manufacturer requires that a single supplier invests in greater details. This case is interesting because it reflects what is observed in some industries. We can prove the following.[8]

(i) The manufacturer encourages investment (by at least one supplier) more so under single sourcing than under dual sourcing since

$$\overline{r}(q_L) < (\alpha - \beta) \left[\Phi(1 - q_L) - c(1 - q_L) + \phi \right]. \tag{15}$$

(ii) The manufacturer is more inclined to set $q^* = 1 - q_L$ (and thus ask the buffer to supply no more than q_L) when she bears the cost associated with a supply disruption..

(iii) For $\phi > 0$, the variable q_L must be large enough for the manufacturer to set $q^* = 1 - q_L$. Formally, there exists $\overline{q_L} \leq \frac{1}{2}$ such that $q^* = 1 - q_L$ if and only if $q_L > \overline{q_L}$. When the supplier pays $\Phi(q - q_L)$ we have

$$\overline{q_L} = \frac{1}{2} - \sqrt{\frac{2\phi(\alpha - \beta)}{3c(1 - \alpha)}}, \tag{16}$$

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[8] Please see the Appendix for a formal proof of all points.

otherwise it is the value that solves

$$\phi(\alpha - \beta) = (1-\alpha)\frac{c}{2}\left[\frac{1-2\overline{q_L}}{1-\overline{q_L}}\right]^2 [3(1-\overline{q_L}) + \overline{q_L}(1-\beta)(1-2\overline{q_L})]. \quad (17)$$

While we prove the above points when the function $\Phi(.)$ is given by (4), there is a sense that these must hold in general. If we consider point (i) in particular. Note that under single sourcing the manufacturer is not able to use the order size to monitor the risk. Thus single sourcing is inherently associated with more risk and thus with higher costs. Therefore the manufacturer should be more inclined to require investment when contracting a single supplier. Indeed, when she contracts two suppliers and does not request investment on their side, the manufacturer is able to split orders in two (setting $q = \frac{1}{2}$ for each) so as to reduce the risk of failure.

The tables below gives the outcome of simulations regarding the optimal order to the main supplier as a function of q_L for different values of ϕ.[9] The buffer produces $1 - q^*$.

	$\phi = 0$	$\phi = 1$	$\phi = 3$
q_L	q^*	q^*	q^*
0	0.542	0.654	0.843
0.1	0.531	0.648	0.838
0.25	0.518	0.656	0.75
0.4	0.506	0.6	0.6
0.5	0.5	0.5	0.5

Table 1: Optimal order to the supplier who invests
when the manufacturer bears the cost $\Phi(q - q_L)$.

	$\phi = 0$	$\phi = 1$	$\phi = 3$
q_L	q^*	q^*	q^*
0	0.531	0.616	0.791
0.1	0.525	0.632	0.857
0.25	0.516	0.691	0.75
0.4	0.506	0.6	0.6
0.5	0.5	0.5	0.5

Table 2: Optimal order to the supplier who invests
when the suppliers bear the full cost of failure.

Here are a few observations we can make given the above tables.

- The difference in the marginal exposure to risk means that the manufac-
  turer relies more on the main supplier as the cost associated with a supply

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[9] We also take $c = 1$, $\alpha = 0.3$ and $\beta = 0.1$ but these values have no impact on the patterns observed provided $\alpha > \beta$.

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disruption. increases. Indeed we can clearly see that $\dfrac{dq^*}{d\phi} > 0$ as we did prove formally (see corollary 2). It is also clear that the manufacturer demands at least as much from its main supplier when she bears the cost of failure. Orders are equal only when $q^* = 1 - q_L$.

- It is not clear how q^* varies with q_L. We can see that for $\phi = 0$ the optimal order decreases with q_L and in that case we always have $q^* < 1 - q_L$ unless of course $q_L = 0.5$. For any $\phi > 0$ we have $q^* = 1 - q_L$ when q_L is large enough.

The figure below summarizes some of the findings above as it gives a visual representation of the optimal order to the supplier who incurred the investment.
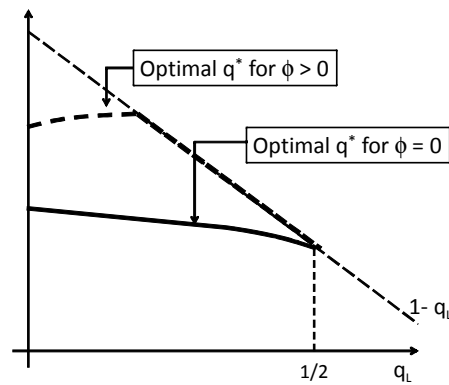

Figure 5: Optimal order to the main supplier under dual sourcing when he is the only one to invest.

4 Optimal contracts under moral hazard.

We consider now a situation where investment is no longer verifiable. Due to the nature of the transfers in this case the analysis is complex. We specifically address the following questions.

Question 1 Under moral hazard is the manufacturer more, or less, inclined to rely on dual sourcing?

Question 2 Is she more, or less, inclined to use a supplier as a buffer?

Question 3 What does the current trend towards consolidation in some industries tell us about moral issues in these sectors?

Under moral hazard, in addition to the participation constraint, an incentive constraint must hold to guarantee that the suppliers will invest when it is desirable for them to do so. The constraint is given by

$$\alpha(q_i)\pi(t_S^i - cq_i) + (1 - \alpha(q_i))\pi(t_F^i - c_F) - I \geq \tag{18}$$
$$\beta(q_i)\pi(t_S^i - cq_i) + (1 - \beta(q_i))\pi(t_F^i - c_F),$$

where c_F is given by (9). The above constraint can be re-written as

$$[\alpha(q_i) - \beta(q_i)] \left[\pi(t_S^i - cq_i) - \pi(t_F^i - c_F) \right] \geq I. \tag{19}$$

When there is no moral hazard issue the suppliers are fully insured so that $\pi(t_S^i - cq_i) = \pi(t_F^i - c_F)$. The greater the discrepancy between $\pi(t_S^i - cq_i)$ and $\pi(t_F^i - c_F)$, the more inefficient the contract is and the lower the returns for the manufacturer. It is important to note that the order size is a key determinant of the level of inefficiency that must be introduced to guarantee incentive compatibility.

Lemma 4: *When investment is no longer contractible, the transfers implementing $I^i = I$ on the part of supplier i $(i = 1, 2)$ are given by*

$$\begin{cases} t_S^* = cq_i + r_S, \\ t_F^* = c_F + r_F(q_i), \end{cases} \tag{20}$$

where

$$r_S = \pi^{-1}\left(\underline{\pi} + \frac{1 - \beta}{\alpha - \beta}I \right) \tag{21}$$

and

$$r_F(q_i) = \pi^{-1}\left(\underline{\pi} - \frac{1 - q_i + \beta(q_i - q_L)}{(\alpha - \beta)(q_i - q_L)}I \right). \tag{22}$$

and where $q_i > q_L$ is the order submitted to supplier i.

Proof: Both the participation and incentive constraints bind.

To implement $I^i = I$ the transfer upon a successful delivery consists of a cost plus contract, as before, but the bonus is now greater. When the supplier fails to deliver his order the transfer still covers the cost but the bonus is now lower than what it used to be with contractible effort and it depends on the order size. The penalty incurred decreases with the order size. The penalty is lower for larger orders because larger order are associated with greater risks. This may sound counter-intuitive but it reflects the fact that risk averse suppliers are more willing to invest when receiving a larger order because they dislike the risk these are associated with. Thus less transfer distortions is needed to guarantee incentive compatibility as the order size increases. Note finally that, as before, the manufacturer ends up paying the entire cost associated with a supply disruption., that is $\Phi(q - q_L) + \phi$.

Note that we have

$$\alpha(q_i)\left(\underline{\pi} + \frac{1 - \beta}{\alpha - \beta}I \right) + (1 - \alpha(q_i))\left(\underline{\pi} - \frac{1 - q_i + \beta(q_i - q_L)}{(\alpha - \beta)(q_i - q_L)}I \right) = \underline{\pi} + I. \tag{23}$$

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However, due to the risk aversion the function $\pi^{-1}(.)$ is convex and we have, for all $q_i > q_L$

$$\alpha(q_i)r_S + (1 - \alpha(q_i))r_F(q_i) > \pi^{-1}(\underline{\pi} + I). \qquad (24)$$

The transfer payment is greater under moral hazard and it increases the more risk averse the suppliers are.[10]

Proposition 3: *The expected fee, given by $R(q) = \alpha(q)r_S + (1 - \alpha(q))\, r_F(q)$, decreases with q, the order size.*

Proof: See Appendix.

Proposition 3 reflects the fact that smaller orders are associated with greater inefficiencies. Since $[\alpha(q_i) - \beta(q_i)]$ increases with q_i it follows that transfers associated with larger orders guarantee incentive compatibility via a more efficient outcome. The key element for this result does not relate to the fact that investment is a discrete variable $\{0, I\}$. In any context where investment is continuous, and where the probability of success $p_i(q_i, I)$ increase with I, this result would hold in as long as $\frac{\partial p_i}{\partial I_i}$ increases with q_i. In other words this results hinges on the assumption that the marginal increase in the probability of success from investing a little more increases with the order size. This captures situations where larger suppliers have more to gain from investing.

Before we answer the three questions we need to verify what order size emerges in equilibrium under moral hazard.

Lemma 5: *When $I^1 = I^2 = I$ setting $q = \frac{1}{2}$ can still form an equilibrium. When only one supplier has an incentive to invest and compared to the case of symmetric information, the manufacturer wants to reduce the order to the supplier who did not invest, meaning that he submits larger orders to the supplier who invests and is more inclined to set $q^* = 1 - q_L$.*

Proof: See appendix.

We may now answer the questions we addressed.

Proposition 4: *Under moral hazard the manufacturer is more likely to*

(i) Rely on a single supplier.

(ii) Offer contracts such that only one of the two suppliers has an incentive to invest and where the other one is used as a buffer.

Therefore the observed move towards the middle (i.e. towards a consolidation of orders) observed in some industries may be a sign that moral hazard issues were exacerbated.

Proof: See Appendix.

The improvement in information and communication technologies has improved the ability for manufacturers to enlist and manage suppliers. In that sense one may argue that it could reduce moral hazard issues. However, in recent years, some industries have increased their reliance on and demand for high tech products from some of their suppliers and valued just-in-time deliveries to reduce costs. Requiring higher quality, better fitting and a perfect coordination may have exacerbated moral hazard issues.

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[10] Note that $r_F(q)$ is not defined at $q = q_L$ because at this point the incentive contraint does cannot bind. However it is not rational to induce investment from a supplier who gets an order less or equal to q_L.

5 Robustness and extensions

- Throughout the paper we have considered that the manufacturer may or may not pay the cost $\Phi(q - q_L)$. The result below expresses her preference in that respect.

Corollary 3: *The manufacturer is indifferent between paying the full cost associated with a supply disruption. directly and compensating the supplier who fails under single sourcing or when she requires that the buffer supplier produces no more than q_L. When both produce more than q_L the manufacturer is better off when the supplier bears the cost and she reimburses the expenses via the transfer.*

Proof: Let $\Pi^M(q) \equiv \Pi(q)$ when the manufacturer bears part of the cost of failure and let $\Pi^M(q) \equiv \widehat{\Pi}(q)$ when she does not. The sign of $\Pi(q) - \widehat{\Pi}(q)$ is given by

$$\Phi(1 - q - q_L) + \Phi(q - q_L) - \Phi(1 - 2q_L) \leq 0$$

where the inequality follows from the fact that $\Phi(.)$ is convex thus $\Phi(x + y) \geq \Phi(x) + \Phi(y)$. Note finally that under single sourcing and with a buffer supplier the product of probabilities is zero.[11]

- In this paper we have considered that demand is deterministic and set it to $Q = 1$. If we allowed the demand to be uncertain all results would hold provided we keep the assumption that orders are submitted after demand is realized. This assumption receives support in Gans (2007).

- Finally, if we allowed investment to be continuous and let $p_i(q_i, I)$ be increasing with I the main results would still hold. Evidence for this can be found in the Appendix where we use a specific function for the $p_i(q_i, I)$. In that case note that the incentive constraint would write as follow. Let I^* be the investment that the manufacturer wishes to implement:

$$I^* \in \arg\max_I p_i(q_i, I)\pi(t_S^i - cq_i) + (1 - p_i(q_i, I))\pi(t_F^i - c_F) - C(I),$$

where $C(I)$ is the cost associated with this investment and it is increasing. Provided $p_i(q_i, I_i)$ is concave in I_i we may have $C(I) = I$ as before. However if $p_i(q_i, I_i)$ was linear in I_i then we would need the function $C(I)$ to be convex to guarantee that the above problem is concave. In any case, the above requires that

$$\left.\frac{\partial p_i}{\partial I}\right|_{I^*} \left[\pi(t_S^i - cq_i) - \pi(t_F^i - c_F)\right] \geq C'(I^*) > 0.$$

Note that if we assume that $\frac{\partial p_i}{\partial I}$ increases with q_i, that is if we assume that the marginal benefit from investment increases with the order size,

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[11] One can verify that under single sourcing the manufacturer's profits that can be written as $\Pi(1) + r_j^2$ and $\widehat{\Pi}(1) + r_j^2$ depending on whether she bears the cost of failure or not.

then the same outcome arises, namely that the level of inefficiency that must be introduced to guarantee incentive compatibility decreases with the order size.

6   Conclusion

In recent years manufacturers have increased their reliance on suppliers. While they used to produce most of their inputs there has been a tendency to rely more and more on outsourcing. This move has, with no doubt, been facilitated thanks to improvements in information and communication technologies (ICT). These have helped manufacturers develop a supply base and helped with the logistic and management of orders. However, while manufacturers bought more from the outside, some have also reduced the number of suppliers they deal with. Orders have been consolidated. Few suppliers are entrusted with larger orders and are referred to as "risk sharing partners".

This paper considers a situation where orders are subject to a default risk and where this risk increases with order size. The suppliers can make an investment to reduce both, their absolute exposure to risk as well as their marginal exposure to risk. When this investment is contractible we found that dual sourcing is optimal unless the suppliers' reservation profits are substantially high. Whether suppliers receive equal orders or not depends on the investment level. In some circumstances it is optimal for the manufacturer to rely on one of the supplier as a buffer who does not invest in technology and receives smaller orders. This particular strategy is optimal for a wider range of parameters as the cost associated with a supply disruption increases.

Orders consolidation and an increased reliance on a single supplier becomes more profitable as we introduce moral hazard. Indeed, we show that issuing larger orders can serve as an incentive devise. It is because larger orders are associated with more risk that these induce risk averse suppliers to invest in technology.

We conclude by suggesting that despite the improvements in ICT, the move towards consolidation may be an indication that moral hazard may have become more of an issue in some industries. One reason may be that inputs are increasingly sophisticated and require more expertise. Or it may be that "on time deliveries" have become essential for such industries.

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8 Appendix

- **Proof of lemma 3.**

Let $\Pi^M(q) \equiv \Pi(q)$ when the manufacturer bears part of the cost of failure and let $\Pi^M(q) \equiv \widehat{\Pi}(q)$ when she does not. Given the transfers, the manufacturer's profits can be written as

$$\Pi(q_1, q_2) = P - \sum_{i=1,2} \left[p_i(q_i)cq_i + r^i \right] - (cq_L + \phi)\left(2 - p_1(q_1) - p_2(q_2)\right) \quad (25)$$
$$- \sum_{\substack{i=1,2 \\ j \neq i}} p_i(q_i)\left(1 - p_j(q_j)\right)\Phi(q_j - q_L) - \left(1 - p_1(q_1)\right)\left(1 - p_2(q_2)\right)\Phi(1 - 2q_L),$$

and

$$\widehat{\Pi}(q_1, q_2) = P - \left(2 - p_1(q_1) - p_2(q_2)\right)\phi - r^1 - r^2 - c(q_1 + q_2) \quad (26)$$
$$- \sum_{i=1,2}\left(1 - p_i(q_i)\right)\left[\Phi(q_i - q_L) - c(q_i - q_L)\right].$$

Consider any $q \in [q_L, 1 - q_L]$. To simplify the notation let $p_1 \equiv p_1(q)$ and $p_2 \equiv p_2(1 - q)$, let $p_i' = \frac{dp_i}{dq} < 0$ $(i = 1, 2)$ and finally let $\Phi'(x) = \frac{d\Phi}{dx} > 0$ and $\Phi''(x) = \frac{d^2\Phi}{dx^2} > 0$. Note that since the probabilities are linear we have $\frac{d^2 p_i}{dq^2} = 0$ $(i = 1, 2)$. We have

$$\frac{d\Pi}{dq} = -\phi\left(p_2' - p_1'\right) + \left[p_1' p_2 + (1 - p_1)p_2'\right]\left[\Phi(q - q_L) - c(q - q_L)\right] \quad (27)$$
$$- \left[p_1'(1 - p_2) + p_1 p_2'\right]\left[\Phi(1 - q - q_L) - c(1 - q - q_L)\right]$$
$$+ \left[p_1'(1 - p_2) - (1 - p_1)p_2'\right]\left[\Phi(1 - 2q_L) - c(1 - 2q_L)\right]$$
$$+ p_1(1 - p_2)\left[\Phi'(1 - q - q_L) - c\right] - (1 - p_1)p_2\left[\Phi'(q - q_L) - c\right]$$

We then have

$$\frac{d^2\Pi}{dq^2} = 2\left[\Phi'(1 - q - q_L) - c\right]\left[p_1'(1 - p_2) + p_1 p_2'\right]$$
$$+ 2\left[\Phi'(q - q_L) - c\right]\left[p_1' p_2 + (1 - p_1)p_2'\right]$$
$$- \Phi''(1 - q - q_L)p_1(1 - p_2) - \Phi''(q - q_L)p_2(1 - p_1)$$
$$+ 2p_1' p_2'\left[\Phi(1 - 2q_L) - \Phi(1 - q - q_L) - \Phi(q - q_L)\right].$$

The above is a sum of negative term as $\Phi'(x) - c > 0$ and where the last term is negative since $\Phi(.)$ is convex so that

$$\Phi(x + y) > \Phi(x) + \Phi(y)$$

and where we have $x = q - q_L$ and $y = 1 - q - q_L$. Thus the objective function is concave and the stationary point is a maximum.

When the manufacturer does not pay the cost of failure we have

$$\frac{d\widehat{\Pi}}{dq} = p_1' \left[\Phi(q - q_L) - c(q - q_L) \right] - p_2' \left[\Phi(1 - q - q_L) - c(1 - q - q_L) \right] \quad (28)$$
$$- (1 - p_1) \left[\Phi'(q - q_L) - c \right] + (1 - p_2) \left[\Phi'(1 - q - q_L) - c \right] - \phi \left[p_2' - p_1' \right].$$

We then have

$$\begin{aligned}
\frac{d^2\widehat{\Pi}}{dq^2} &= 2p_1' \left[\Phi'(q - q_L) - c \right] + 2p_2' \left[\Phi'(1 - q - q_L) - c \right] \\
&\quad -\Phi''(1 - q - q_L)(1 - p_2) - \Phi''(q - q_L)(1 - p_1).
\end{aligned}$$

The above is a sum of negative term. Thus the objective function is concave and the stationary point is a maximum. Whether $\Pi^M = \Pi$ or $\widehat{\Pi}$ the following results apply.

- When $I^1 = I^2$ we have $\left.\frac{d\Pi^M}{dq}\right|_{q=q_L} > 0$ and that $\left.\frac{d\Pi^M}{dq}\right|_{q=1-q_L} < 0$. Moreover, it is obvious that $\left.\frac{d\Pi^M}{dq}\right|_{q=\frac{1}{2}} = 0$ and thus both functions reach a unique maximum at $q = \frac{1}{2}$.

  The probability of failure is $p^F = 1 - \alpha(q)\alpha(1 - q)$ when both invest and $p^F = 1 - \beta(q)\beta(1 - q)$ when none invests. These expressions reach a minimum at $q = \frac{1}{2}$.

- Assume $I^1 = I$ and $I^2 = 0$, in that case we have $\left.\frac{d\Pi^M}{dq}\right|_{q=q_L} > 0$ but we may either have $\left.\frac{d\Pi^M}{dq}\right|_{q=1-q_L} < 0$ in which case the solution is interior or $\left.\frac{d\Pi^M}{dq}\right|_{q=1-q_L} > 0$ which implies that the optimal order is $q_1 = 1 - q_L$. In any case, one can show that $\left.\frac{d\Pi^M}{dq}\right|_{q=\frac{1}{2}} > 0$ meaning that there exists a unique maximum $q^*(q_L) > \frac{1}{2}$ and such that $q^* = 1 - q_L$ if $\phi \left(\frac{\alpha - \beta}{1 - q_L} \right)$ is large enough. Indeed notice that for $\phi = 0$ and $q_L < \frac{1}{2}$ we have $\left.\frac{d\Pi^M}{dq}\right|_{q=1-q_L} < 0$ so the optimal order is always below $1 - q_L$ in that case. When $q_L = \frac{1}{2}$ we obviously have $q^* = \frac{1}{2}$.[12]

- **Proof of proposition 1.**

The proof is done considering the case where the suppliers bears the full cost of failure. It extends easily to the other case. Let $\widehat{\Pi}(q_1; I^1, I^2)$ denote the equilibrium profits when supplier 1 produces q_1, and supplier i invests I^i with $i = 1, 2$. It can be calculated from (26). Let $\alpha^* = \alpha(q^*)$ and $\beta^* = \beta(1 - q^*)$ where $q^* \in \left[\frac{1}{2}, 1 - q_L \right]$ is the optimal order to the supplier who invested. For notation purposes let

$$\Omega(x - q_L) = \Phi(x - q_L) - c(x - q_L). \quad (29)$$

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[12] A symmetric argument holds when $I^1 = 0$ and $I^2 = I$.

The function $\Omega(x - q_L)$ is increasing in x.

We have

$$\widehat{\Pi}(\frac{1}{2}; I, I) \geq \widehat{\Pi}(\frac{1}{2}; 0, 0) \Leftrightarrow r_H - r_L \leq G_1(q_L) \tag{30}$$

with

$$G_1(q_L) = \left[\alpha(\frac{1}{2}) - \beta(\frac{1}{2})\right] \left[\Omega(\frac{1}{2} - q_L) + \phi\right] \tag{31}$$

$$\widehat{\Pi}(\frac{1}{2}; I, I) \geq \widehat{\Pi}(q^*; I, 0) \Leftrightarrow r_H - r_L \leq G_2(q_L) \tag{32}$$

with

$$\begin{aligned}
G_2(q_L) &= \left[2\alpha\left(\frac{1}{2}\right) - \alpha^* - \beta^*\right]\phi - 2\left(1 - \alpha\left(\frac{1}{2}\right)\right)\Omega(\frac{1}{2} - q_L) \\
&\quad + (1 - \alpha^*)\,\Omega(q^* - q_L) + (1 - \beta^*)\Omega(1 - q^* - q_L),
\end{aligned} \tag{33}$$

and finally

$$\widehat{\Pi}(q^*; I, 0) \geq \widehat{\Pi}(\frac{1}{2}; 0, 0) \Leftrightarrow r_H - r_L \leq G_3(q_L) \tag{34}$$

where

$$\begin{aligned}
G_3(q_L) &= \left[\alpha^* + \beta^* - 2\beta\left(\frac{1}{2}\right)\right]\phi + 2\left(1 - \beta\left(\frac{1}{2}\right)\right)\Omega\left(\frac{1}{2} - q_L\right) \\
&\quad - (1 - \alpha^*)\,\Omega(q^* - q_L) - (1 - \beta^*)\Omega(1 - q^* - q_L).
\end{aligned} \tag{35}$$

Clearly, from the statement in proposition 1 we have $\overline{r}(q_L) = G_3(q_L)$ and $\underline{r}(q_L) = G_2(q_L)$. Thus, we must show that for any $q_L \in \left[0, \frac{1}{2}\right]$ we have

$$G_3(q_L) \geq G_2(q_L)$$

where the equality holds at $q_L = \frac{1}{2}$ only since we have $q^* = 1 - q_L = \frac{1}{2}$ at $q_L = \frac{1}{2}$.

Note that

$$G_2(q_L) + G_3(q_L) = 2G_1(q_L).$$

Therefore, for any $q_L \in \left[0, \frac{1}{2}\right]$ the value at $G_1(q_L)$ is the middle point between $G_2(q_L)$ and $G_3(q_L)$ and we have

$$G_3(q_L) - G_1(q_L) = G_1(q_L) - G_2(q_L).$$

Furthermore, for any given $q_L \in \left[0, \frac{1}{2}\right]$ we have

$$G_3(q_L) - G_1(q_L) = G_1(q_L) - G_2(q_L) = \widehat{\Pi}(q^*; I, 0) - \widehat{\Pi}(\frac{1}{2}; I, 0), \tag{36}$$

where the expression for $\widehat{\Pi}(q; I, 0)$ can be calculated from (26). Since the function $\widehat{\Pi}(q; I, 0)$ reaches a maximum at q^* (which is only equal to $\frac{1}{2}$ when $q_L = \frac{1}{2}$) we necessarily have

$$\widehat{\Pi}(q^*; I, 0) \geq \widehat{\Pi}(\frac{1}{2}; I, 0),$$

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which implies that

$$G_3(q_L) - G_1(q_L) = G_1(q_L) - G_2(q_L) \geq 0$$

where the equality only holds when $q_L = \frac{1}{2}$. Thus $G_3(q_L) \geq G_1(q_L) \geq G_2(q_L)$. The proof of points (i), (ii) and (iii) in proposition 1 follows from (32) and (34).

- **Proof of corollary 2.**

Here again we concentrate on the case where $\Pi^M = \widehat{\Pi}$ but the extension to the other case is straightforward.

First we wish to show that

$$\frac{d\overline{r}(q_L)}{d\phi} > \frac{d\underline{r}(q_L)}{d\phi} > 0.$$

Recall that $\overline{r}(q_L) = G_3(q_L)$ and $\underline{r}(q_L) = G_2(q_L)$. Note that from (36) we have

$$G_2(q_L) = G_1(q_L) - \widehat{\Pi}(q^*; I, 0) + \widehat{\Pi}(\frac{1}{2}; I, 0). \tag{37}$$

Although q^* depends on ϕ (provided $q^* < 1 - q_L$) we need not worry about $\frac{dq^*}{d\phi}$ since we have $\frac{\partial G_2}{\partial q^*} = -\frac{\partial \widehat{\Pi}(q^*; I, 0)}{\partial q^*} = 0$. Thus,

$$\frac{d\underline{r}(q_L)}{d\phi} = \frac{dG_2}{d\phi} = 2\alpha\left(\frac{1}{2}\right) - \alpha^* - \beta^* > 0, \tag{38}$$

where $\alpha^* = \alpha(q^*)$ and $\beta^* = \beta(1 - q^*)$. Similarly, we have

$$G_3(q_L) = G_1(q_L) + \widehat{\Pi}(q^*; I, 0) - \widehat{\Pi}(\frac{1}{2}; I, 0), \tag{39}$$

and once again $\frac{\partial G_3}{\partial q^*} = \frac{\partial \widehat{\Pi}(q^*; I, 0)}{\partial q^*} = 0$ so that

$$\frac{d\overline{r}(q_L)}{d\phi} = \frac{dG_3}{d\phi} = \alpha^* + \beta^* - 2\beta\left(\frac{1}{2}\right) > 0.$$

It is then straightforward to show that $\frac{d\overline{r}(q_L)}{d\phi} > \frac{d\underline{r}(q_L)}{d\phi}$.

Finally, concerning the second point, the optimal q^* solves $\dfrac{d\widehat{\Pi}}{dq} = 0$. Differentiating this equation with respect to ϕ leads to:

$$\text{sign of } \frac{dq^*}{d\phi} = \text{sign of } \frac{\partial \widehat{\Pi}}{\partial \phi} = \frac{\alpha - \beta}{1 - q_L} > 0.$$

at any $q^* < 1 - q_L$.

- **On the use of a buffer supplier.**

Point (i) Given that $\overline{r}(q_L) = G_3(q_L)$ given by (35) above, we must show that

$$G_3(q_L) < (\alpha - \beta) \left[\Phi(1 - q_L) - c(1 - q_L) + \phi \right].$$

Let

$$G_0(q_L) = (\alpha - \beta) \left[\Omega(1 - q_L) + \phi \right]$$

We have

$$(\alpha - \beta) - (\alpha^* + \beta^*) + 2\beta \left(\frac{1}{2} \right) = \frac{(\alpha - \beta)(1 - q^*)}{1 - q_L} > 0.$$

Using the above we have

$$
\begin{aligned}
G_0(q_L) - G_3(q_L) = {} & \frac{(\alpha-\beta)(1-q^*)}{1-q_L}\phi + (\alpha - \beta)\,\Omega(1 - q_L) \\
& -2\left(1 - \beta\left(\tfrac{1}{2}\right)\right)\Omega\left(\tfrac{1}{2} - q_L\right) \\
& +(1 - \alpha^*)\Omega\left(q^* - q_L\right) + (1 - \beta^*)\Omega\left(1 - q^* - q_L\right).
\end{aligned}
$$

The first term is positive and increases with ϕ. Moreover we have

$$\frac{d(G_0 - G_3)}{d\phi} = \frac{(\alpha-\beta)(1-q^*)}{1-q_L} > 0.$$

Thus, if we can prove that $G_0(q_L) - G_3(q_L) > 0$ for $\phi = 0$, it will be positive for $\phi > 0$. At $\phi = 0$ we have

$$
\begin{aligned}
G_0(q_L) - G_3(q_L) = {} & \frac{c}{2(1 - q_L)} (\alpha - \beta) \left[(1 - q_L)^3 - 4\left(\tfrac{1}{2} - q_L\right)^3 \right] (40) \\
& + \frac{2c}{(1 - q_L)} \gamma \left(\tfrac{1}{2} - q_L\right)^3,
\end{aligned}
$$

where

$$\gamma = \frac{(1 - \alpha)\left(\sqrt{1 - \beta} - \sqrt{1 - \alpha}\right)\left(\sqrt{1 - \alpha} + 3\sqrt{1 - \beta}\right)}{\left(\sqrt{1 - \alpha} + \sqrt{1 - \beta}\right)^2} > 0.$$

The second term of (40) is positive as well as the first since one can easily verify that

$$(1 - q_L)^3 - 4\left(\tfrac{1}{2} - q_L\right)^3 > 0$$

for any $q_L \in \left[0, \tfrac{1}{2}\right]$, therefore, $G_0(q_L) - G_3(q_L) > 0$.

Point (ii) and (iii) When it is optimal for the manufacturer to request that only one firm invests we have q^* solves (41) or (42) depending on whether the manufacturer bears the cost $\Phi(q - q_L)$ or not. When she bears the cost $(\Pi^M = \Pi)$ we have

$$
\begin{aligned}
\frac{d\Pi}{dq} = 0 \Leftrightarrow {} & \tfrac{c}{2}(1 - \beta)(1 - q^* - q_L)^2 \left[3\alpha(q^*) + \frac{1 - \alpha}{1 - q_L}(1 - q^* - q_L) \right] \quad (41) \\
& -\tfrac{c}{2}(1 - \alpha)(q^* - q_L)^2 \left[3\beta(1 - q^*) + \frac{1 - \beta}{1 - q_L}(q^* - q_L) \right] \\
& +\tfrac{c}{2}(1 - \beta)(1 - \alpha)(1 - 2q_L)^2(2q^* - 1) + \phi(\alpha - \beta)(1 - q_L) = 0,
\end{aligned}
$$

and when she does not bear the cost $(\Pi^M = \widehat{\Pi})$ we have

$$\frac{d\widehat{\Pi}}{dq} = 0 \tag{42}$$

$$\Leftrightarrow \tfrac{3c}{2}(1-\beta)(1-q^* - q_L)^2 - \tfrac{3c}{2}(1-\alpha)(q^* - q_L)^2 + \phi(\alpha - \beta) = 0. \tag{43}$$

To prove point (ii) note that $\left.\frac{d\Pi}{dq}\right|_{1-q_L} > \left.\frac{d\widehat{\Pi}}{dq}\right|_{1-q_L}$. For point (iii) and to find the value for $\overline{q_L}$.one must solve $\left.\frac{d\Pi}{dq}\right|_{q^*=1-q_L} = 0$ and $\left.\frac{d\widehat{\Pi}}{dq}\right|_{q^*=1-q_L} = 0.$

- **Proof of proposition 3.**

The expected fee $R(q)$ is given by

$$R(q) = \alpha(q)r_S + (1 - \alpha(q)) r_F(q).$$

We have

$$\frac{dR}{dq} = -\left(\frac{1-\alpha}{1-q_L}\right)[r_S - r_F(q)] + (1 - \alpha(q))\frac{dr_F}{dq}. \tag{44}$$

To evaluate the second term we need to calculate $\frac{dr_F}{dq}$. We know that at the solution, the incentive constraint binds:

$$[\alpha(q) - \beta(q)][\pi(r_S) - \pi(r_F(q))] = I.$$

Differentiating both sides with respect to q we get (after some simplifications)

$$\frac{dr_F}{dq} = \frac{(\alpha - \beta)(1 - \alpha(q))[\pi(r_S) - \pi(r_F(q))]}{(1-q_L)(\alpha(q) - \beta(q))\frac{d\pi(r_F(q))}{dq}} > 0.$$

Using the above in (44) we get

$$\frac{dR}{dq} < 0 \Leftrightarrow \frac{d\pi(r_F(q))}{dq} > \frac{(\alpha - \beta)(1 - \alpha(q))}{(1-\alpha)(\alpha(q) - \beta(q))}\frac{[\pi(r_S) - \pi(r_F(q))]}{[r_S - r_F(q)]}.$$

We have

$$\frac{(\alpha - \beta)(1 - \alpha(q))}{(1-\alpha)(\alpha(q) - \beta(q))} = 1,$$

thus the above can be written as

$$\frac{dR}{dq} < 0 \Leftrightarrow \frac{d\pi(r_F(q))}{dq} > \frac{[\pi(r_S) - \pi(r_F(q))]}{[r_S - r_F(q)]},$$

which is true since $\pi(.)$ is concave and $r_F(q) < r_S$ for any q.

- **Proof of Lemma 5.**

- Let us start with the case where $I^1 = I^2 = I$.[13] Let $\Pi^*(q)$ and $\widehat{\Pi}^*(q)$ denote the manufacturer's profits under moral hazard depending on whether she bears part of the cost of failure or not. Let us concentrate on the case where she bears the cost. (The extension to the other case is straightforward.) We have

$$\Pi^*(q) = \Pi(q) + 2\pi^{-1}(\underline{\pi} + I) - R(q) - R(1 - q),$$

for any $q \in [q_L, 1 - q_L]$, where $\Pi(q)$ is the manufacturer's profit under symmetric information given by (25) with $q_1 = q$ and $q_2 = 1 - q$ and where

$$R(x) = \alpha(x)r_S + (1 - \alpha(x))\, r_F(x).$$

The derivative of the above is given by

$$\frac{d\Pi^*(q)}{dq} = \frac{d\Pi(q)}{dq} - \left.\frac{dR(x)}{dx}\right|_q + \left.\frac{dR(x)}{dx}\right|_{1-q}.$$

We know that $\left.\frac{d\Pi(q)}{dq}\right|_{q=\frac{1}{2}} = 0$ and the two last terms cancel out as we set $q = \frac{1}{2}$. Therefore the first derivative equals zero at $q = \frac{1}{2}$. The second derivative is given by

$$\frac{d^2\Pi^*(q)}{dq^2} = \frac{d^2\Pi(q)}{dq^2} - \left.\frac{d^2R(x)}{dx^2}\right|_q - \left.\frac{d^2R(x)}{dx^2}\right|_{1-q}.$$

We have

$$\frac{d^2R}{dx^2} = 2\left(\frac{1 - \alpha}{1 - q_L}\right)\frac{dr_F(x)}{dx} + (1 - \alpha(x))\frac{d^2r_F(x)}{dx^2}.$$

The function $r_F(x) = \pi^{-1}(g(x))$ where

$$g(x) = \underline{\pi} - \frac{1 - q_i + \beta(q_i - q_L)}{(\alpha - \beta)\,(q_i - q_L)}I.$$

Both $\pi^{-1}(.)$ and $g(x)$ are increasing so that $\frac{dr_F(x)}{dx} > 0$. However $g(x)$ is concave while $\pi^{-1}(.)$ is convex therefore the sign of $\frac{d^2r_F(x)}{dx^2}$ is not easy to determine. However, simulations using for instance a CARA profit function show that we have $\frac{d^2\Pi^*(q)}{dq^2} < 0$.

- Assume now that $I^1 = I$ and $I^2 = 0$.

We now have

$$\Pi^*(q) = \Pi(q) + \pi^{-1}(\underline{\pi} + I) - R(q),$$

---

[13] When none of the suppliers invest we do not have a moral hazard issue and we know that $q = \frac{1}{2}$ is the unique equilibirum.

for any $q \leq q_L$. Therefore

$$\frac{d\Pi^*(q)}{dq} = \frac{d\Pi(q)}{dq} - \frac{dR(x)}{dx}\bigg|_q.$$

Recall that $\frac{dR(x)}{dx} < 0$. Therefore at q^* such that $\frac{d\Pi(q)}{dq} = 0$ we now have $\frac{d\Pi^*(q)}{dq} > 0$ suggesting that the new solution is greater than q^*. This means that when $q^* = 1 - q_L$ then the new solution will also be equal to $1 - q_L$. To verify the second order condition one must evaluate

$$\frac{d^2\Pi^*(q)}{dq^2} = \frac{d^2\Pi(q)}{dq^2} - \frac{d^2R(x)}{dx^2}\bigg|_q.$$

Here again it can be shown that the above is negative when considering a CARA profit function.

- **Proof of proposition 4.**

Let $\left\{\Pi_1, \Pi_{1/2}, \Pi_{q^*}\right\}$ denote the expected profits the manufacturer raises when there is no moral hazard under single sourcing, dual sourcing with equal orders and dual sourcing where only one supplier invests.

Similarly let $\left\{\Pi_1^*, \Pi_{1/2}^*, \Pi_{q^\#}^*\right\}$ denote the expected profits the manufacturer raises under moral hazard under single sourcing, dual sourcing with equal orders and dual sourcing where only one supplier invests and he must deliver $q^\# > q^*$ (as shown above). We have

$$\Pi_1^* = \Pi_1 - R(1) + \pi^{-1}(\underline{\pi} + I),$$

$$\Pi_{1/2}^* = \Pi_{1/2} - 2R(\tfrac{1}{2}) + 2\pi^{-1}(\underline{\pi} + I),$$

and

$$\Pi_{q^\#}^* = \Pi_{q^*} - R(q^\#) + \pi^{-1}(\underline{\pi} + I).$$

First we show that the manufacturer is more likely to use single sourcing under moral hazard. Assume that he is indifferent between single sourcing or dual sourcing when there is no moral hazard. In other words assume that $\Pi_1 = \Pi_{1/2}$ or that $\Pi_1 = \Pi_{q^\#}$. We then have

$$\Pi_1^* - \Pi_{1/2}^* = \left(R(\tfrac{1}{2}) - R(1)\right) + \left(R(\tfrac{1}{2}) - \pi^{-1}(\underline{\pi} + I)\right) > 0$$

since $R(q)$ is decreasing and $R(q) > \pi^{-1}(\underline{\pi} + I)$ for all $q > q_L$. We also have

$$\Pi_1^* - \Pi_{q^\#}^* = R(q^\#) - R(1) > 0.$$

Second we show that when using dual sourcing the manufacturer will be more inclined to have only one supplier invest. Assume that $\Pi_{1/2} = \Pi_{q^*}$, we then have

$$\Pi_{q^\#}^* - \Pi_{1/2}^* = \left(R(\tfrac{1}{2}) - R(q^\#)\right) + \left(R(\tfrac{1}{2}) - \pi^{-1}(\underline{\pi} + I)\right) > 0.$$

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