Problems in Matrix Theory

Roots of Integer Matrices

An interesting problem in matrix theory is to determine the existence, and form, of a matrix B such that $B^m = A$, for some positive integer m i.e. B is an m-th root of A.
In particular, given a commutative ring R we have the ring $\mathbb{M}_n(R)$ of $n \times n$ matrices with entries in R, we'd like to know, if given $A \in \mathbb{M}_n(R)$ does there exist $B \in \mathbb{M}_n(R')$ with $B^m = A$. Here R' is also a commutative ring with $R \subset R'$ and may necessarily be larger than R. Finding conditions on A to ensure such a B exists is also of interest.

Nonnegative inverse eigenvalue problem

Another interesting problem in matrix theory is the well-studied nonnegative inverse eigenvalue problem (NIEP) which asks for necessary and sufficient conditions on a list of n complex numbers to be the spectra of an $n \times n$ nonnegative ($a_{ij} \geqslant 0,\ \forall i, j$ with $1 \leqslant i, j \leqslant n$). Originating with the work of Perron and Frobenius at the turn of the 20th century the problem of classifying spectra (or lists of eigenvalues) for nonnegative $n \times n$ matrices remains open today for $n \geqslant 5$. Variations on this problem include investigating symmetric matrices (so the spectra are necessarily real), diagonalisable, stochastic and doubly stochastic matrices.

The Pascal Matrix

Consider the problem of finding the multiplicative order of the matrix S_n which is derived from the Pascal matrix, which encodes the binomial coefficients. Insights from the $S = LU$ matrix factorisation, binomial identities, and graph-theoretic methods are of interest here.

Integral Similarity of Matrices

Given two matrices A and B with integer entries, can we decide if these are integrally similar i.e. can we find an invertible matrix P with integer entries such that $P^{-1}AP = B$?

All of the above problems are best studied with at least two courses in Linear Algebra and one in abstract algebra (algebraic structures) taken.