Moment-tensor inversion of LP events recorded on Etna in 2004 using constraints obtained from wave simulation tests

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The persistent occurrence of long period (LP) events at Mt. Etna became apparent with the installation of the first fixed broad-band seismic network in late 2003. Repeating similar LP events from Nov. '03 to Sept. '04 indicate a non-destructive source process. We perform moment tensor (MT) inversions on a stacked high S/N ratio representative LP signal, conducting a grid search for the source geometry and L2-inversion for the source time function. Results indicate a NNW-SSE oriented resonating sub-vertical crack as the most probable source. This result is consistent with deformation and GPS observations. Crucial to this result are constraints imposed by detailed 3D full waveform numerical simulations in a heterogeneous tomographic model with topography, and in particular a detailed assessment of the influence of very near surface velocity structure on LP signals. Pulsating gas injection is hypothesised as the most likely LP trigger. Citation: Lokmer, I., C. J. Bean, G. Saccorotti, and D. Patanè (2007), Moment-tensor inversion of LP events recorded on Etna in 2004 using constraints obtained from wave simulation tests, Geophys. Res. Lett., 34, L22316, doi:10.1029/2007GL031902.

1. Introduction

Long-period (LP) events with typical oscillation periods of 0.2–2 s are frequently observed at active volcanoes [e.g., Chouet, 2003, and references therein]. These LP events are considered to be manifestations of acoustic vibrations and/or inertial oscillations of a fluid-filled cavity. The fluid may have a hydrothermal or magmatic origin, and a number of models addressing the oscillating features have been proposed so far [Aki et al., 1977; Ferrick et al., 1982; Chouet, 1986, 1988; Neuberg et al., 2000]. Mapping the location and extent of these sources, and investigating their temporal evolution are fundamental goals toward a better understanding of the dynamics of active magmatic systems [Chouet, 2003].

Following the recent advances in computing power, the source process of LP signals is currently investigated using full-waveform inversion [e.g., Legrand et al., 2000; Kumagai et al., 2002; Nakano et al., 2003; Kumagai et al., 2005]. The most critical aspect of the modelling procedure is the accurate calculation of Green’s functions. With a single exception [Kumagai et al., 2005], in all aforementioned studies the Green’s functions were calculated for a homogeneous half-space. Typical LP wavelengths are short enough to allow structural and topographic complexities to significantly affect wave propagation. In general, such scale sizes are smaller than the resolution limits commonly achieved in tomographic velocity models of volcanic interiors, thus implying that the homogeneous half-space Green’s functions may not be accurate enough for providing a realistic prediction of ground motion at individual recording sites.

In November 2003, the installation of the first permanent broad-band network allowed the identification of LP activity as the most common seismic signature at Mt. Etna. Since then, LP signals are detected at a rate of several hundreds per day, in association with either quiescent or eruptive periods [Saccorotti et al., 2007; Lokmer et al., 2007].

This study addresses the quantitative analysis of LP signals recorded at Etna in 2004. The Green’s functions are calculated for a heterogeneous velocity model of Patanè et al. [2002] with DEM topography. In two previous studies [Saccorotti et al., 2007; Lokmer et al., 2007], we addressed the location of these signals, in turn analysing their oscillating behaviour from which we obtained constraints about the physical state of the fluids involved in the source process. In this work, we use full-waveform modelling of broad-band seismograms to infer the time history of the equivalent force systems acting at the source, in turn constraining its geometry. This study represents the first moment-tensor inversion of LP sources at Etna, and opens the way to the rigorous definition of the structure of its shallow plumbing system.

2. Data

In this study we analyse LP signals recorded by the permanent INGV broad-band network (Figure 1) between November 2003 and September 2004. LP events were accompanied by very long period (VLP) pulses, recorded only at ECPN station near the summit crater. The energy peak for all events spans the 0.3 – 1 Hz frequency band. The majority of events exhibits similar waveforms with a high cross-correlation coefficient (>0.85) at this frequency range. Saccorotti et al. [2007] identified 4 families of similar events (1759 events in total). Locating the largest 123 events from all 4 families, with peak-to-peak amplitudes between 50 and 120 μm/s, they found that all of them share a common location within the location uncertainties, centred at a depth of about 550 m below the summit crater [Saccorotti et al., 2007, Figure 12]. Maximum-likelihood epicenters obtained by the above authors are shown in Figure 1. Moreover, the 4 families...
are similar to each other [see Saccorotti et al., 2007, Figure 5], thus suggesting that all the events were generated by the same source process. In order to create a dataset for a moment-tensor inversion, we used these previously located 123 largest events and for each individual family we aligned seismograms using preliminary time picks and least-squares adjustment of differential travel times (details are given in the work of Saccorotti et al. [2007]). Once aligned, traces are stacked (averaged) in order to retrieve family-representative seismograms with high Signal-to-Noise-Ratio (SNR). As stacked seismograms were again similar to each other, the same procedure was applied recursively to the family-representative seismograms and thus a single dataset was obtained, representative of the complete LP database.

3. Method

[7] Seismic sources in volcanic environment are often modelled by a set of equivalent forces acting at the source, including the force couples (moment-tensor, MT hereinafter) and single forces (SF) resulting from mass transport [Ohminato et al., 1998; Kumagai et al., 2002; Chouet et al., 2003; Nakano et al., 2003]. These forces, i.e. their source-time histories and magnitudes, can be obtained in the frequency domain by solving the following, for each frequency:

\[ d = Gm. \]  

(1)

where \( d \) contains all data components for all stations, \( G \) is a matrix containing Green’s functions and their spatial derivatives and \( m \) contains moment tensor and single force components. We use the least squares approach to solve the overdetermined inverse problem described by equation (1). The misfit, \( R \), between observations (left-hand side of equation (1)) and synthetic seismograms (right-hand side of equation (1)) is estimated from the following equation:

\[ R = \frac{(d - Gm)^T(d - Gm)}{d^Td}. \]  

(2)

[8] C. J. Bean et al. (The influence of near-surface volcanic structure on long-period (LP) seismic signals and source inversions: Simulated examples from Mt. Etna, submitted to Journal of Geophysical Research, 2007, hereinafter referred to as Bean et al., submitted manuscript, 2007) demonstrate that careful attention must be paid to near surface velocity structure when inverting for LP sources. In particular whole-volcano tomography models typically do not have sufficient near surface (few hundred metres) resolution. Performing MT inversions of synthetic data, Bean et al. (submitted manuscript, 2007) showed that in the case of uncertainties in the uppermost part (c. 400 m) of the velocity structure, inverting LP signals for MT + SF leads to an apparently stable but erroneous solution, yielding a wrong source orientation and a strong spurious single force. In contrast, they show that an acceptable solution can be obtained by following the approach of Nakano and Kumagai [2005], where waveforms are inverted for a set of pre-assumed possible source geometries.

[9] Consequently, as our near surface velocity structure is poorly constrained, we invert our signals for the 3 most likely source geometries capable of generating LP events: a pipe, a crack and an isotropic source. We use the two sets of equations given in the work of Nakano and Kumagai [2005], for the Cartesian components of the moment tensors for a crack and a pipe, respectively, expressed as functions of the angles \( \theta \) and \( \phi \), defined as in Figure 2a. Equation (1) then becomes:

\[ d = G M_0 f(\lambda/\mu, \theta, \phi), \]  

(3)

where \( M_0 \) represents the scalar seismic moment, and \( f \) is a column vector containing 6 functions of Lamé’s constants \( \lambda \) and \( \mu \), and spherical angles \( \theta \) and \( \phi \). Vector \( f \) is independent of frequency, so it can be directly calculated for certain values of \( \theta, \phi, \lambda \) and \( \mu \). Thus, the inversion procedure is reduced to inverting for a single parameter, \( M_0(\omega) \), for a certain value of Poisson’s ratio and particular source orientation. A search for the most likely solutions is then performed in the \( \theta-\phi \) space by calculating the misfit using equation (2) for each \((\theta, \phi)\) pair. A range of solutions which are the most likely source candidates is then confined by the lowest 10% values of the misfit function, \( R(\theta, \phi) \). From this set of solutions, a mean and a standard deviation of the angles \( \theta \) and \( \phi \) are estimated, thus providing us with information on the source orientation and the stability of the solution. We assume a Poisson ratio of 1/3, a value appropriate for volcanic rock at high temperatures [Murase and McBirney, 1973].

[10] Bean et al. (submitted manuscript, 2007) showed that, when inverting for a pre-assumed source geometry, uncertainties in the velocity model produce a much higher error in the amplitude of the source-time function (STF),
than in the source orientation. In order to estimate the error in the STF, we use a bootstrap approach described by Cesca and Dahm [2007]. During the inversion process, the STF spectrum is obtained, along with variances for the real and imaginary part of spectral coefficients at each frequency. In order to calculate the error of the STF in the time domain, we perturb individual spectral coefficients using samples from a zero-mean Gaussian distribution having the same variance as that derived from the inversion. This perturbed STF is then transformed to the time domain; iterating the procedure for a number (100) of times allows us to retrieve a mean time-domain STF along with its standard deviation.

[11] So far, nothing has been mentioned about the search for the “best” source location. In a number of papers [Ohminato et al., 1998; Kumagai et al., 2002; Chouet et al., 2003; Nakano and Kumagai, 2005], a grid search was performed to find “the best location and the best source mechanism”. While this approach can be justified in the case of VLP signals, a good instrumental near-field coverage and slight uncertainties in the velocity model, we test such a procedure for the inversion of LP events on Etna, where there is only one station in the near-field (Figure 1), the source is shallow and the uppermost part of the velocity model unknown. The synthetic test is presented in the following section.

4. Grid Search – Synthetic Test

[12] In order to test the influence of the low-velocity superficial layers on the reliability of a source mechanism and its location retrieved by moment-tensor inversion, we performed the inversion on synthetic datasets calculated for two different models: (1) a homogeneous model with DEM topography and (2) a hypothetical layered Etna model with a 400 m thick superficial layer following the topography, embedded on the top of a homogeneous velocity structure. Values of P- and S-wave velocities for the homogenous model of 3500 m/s and 2000 m/s, respectively, are adopted from the upper part of tomographic model of Patane` et al. [2002]. The velocity for the layer is derived by a weighted average of the velocity model from Saccorotti et al. [2004], i.e. \(v_P = 2000 \text{ m/s} \) and \(v_S = 1200 \text{ m/s}\). Densities are estimated from the polynomial fit of density vs. P-wave velocity, based on the dataset given by Corsaro and Pompilio [2004], giving 2530 kg/m\(^3\) and 2420 kg/m\(^3\), for homogeneous and superficial layer respectively. The source mechanism is representative of a vertical tensile crack with the crack normal oriented in the \(x\)-direction (\(\theta = 90, \varphi = 0\)), while a STF is a modified Ricker wavelet with a central frequency 0.7 Hz, matching the most energetic frequency band of the observed data. Synthetic seismograms were generated by discrete elastic lattice scheme described by O’Brien and Bean [2004], for the source location \(S1\) in Figure 1, which corresponds to the centre of the location centroid of the observed LP activity obtained by Saccorotti et al. [2007].

[13] Synthetic signals for both homogeneous (blue) and layered model (red) for all the stations are shown in Figure 2b. Each pair of signals is normalised with respect to a signal calculated for a homogeneous model. Notice that signal durations are strongly affected by the superficial layer for all the stations, except ECPN which is located in the near-field above the source location. Also, it can be seen that different signals exhibit different amplification, which would have a great impact on the moment-tensor solution. However, the first motions are preserved; so to ensure reliability of the solution, we invert a portion of signals only, as depicted by the shaded area in Figure 2b.
In order to perform the inversion and the grid search in space, the Green’s functions are calculated for a homogeneous model with topography, for seven sources $S_1 - S_7$ shown in Figure 1, bounding the confidence region of LP source locations. First, we performed the inversion for both the homogeneous and layered models and the true source position, for a crack, a pipe and an isotropic source, respectively, and in both cases we obtained a minimum residual for the crack source geometry. Thus, the source geometry is successfully recovered regardless of large uncertainties in the velocity structure.

Next, we performed a grid search over all the sources and angles $\theta$ and $\phi$ (in increments of 5 degrees), for both velocity models. The results of the inversion for a homogenous model gave us, as expected, the true source position and its orientation, as well as successfully recovering the STF (blue trace in Figure 2c). The results for the layered model are given in the Table 1 and the STF for the true source position is shown in Figure 2c (red trace). It can be seen that the STF obtained from the inversion has an amplitude about twice as large as the true STF, but it resembles it quite well in its most energetic part. In Table 1, results are sorted in ascending order with respect to the minimum of the misfit function. Although the sources $S_4$ and $S_6$ have smaller residuals than the true source $S_1$, the $S_1$ solution gives a crack orientation closest to its real value. Such observations suggests that there is a trade-off between the source position and the source mechanism, introduced by unmodelled heterogeneities in the uppermost part of the volcano, which is impossible to resolve if the stations distribution is unfavourable. Hence we perform a moment-tensor inversion for real data, assuming a single source location ($S_1$), in the centre of the LP location cluster.

Our synthetic test also suggests that the duration and shape of the STF can be determined from near-field stations, while the first arrivals recorded at further stations provide constraints on the source orientation. Since an inversion depends on the network configuration, we deem that such synthetic experiments are necessary for a successful inversion of shallow sources in volcanic settings. However, actual volcano topography must be included in such modelling.

### 5. MT Inversion of Real Data

Following the synthetic test outlined in the previous section, we performed a moment-tensor inversion of the real data for a source location at position $S_1$, using a tomography velocity model given by Patané et al. [2002], which lacks detailed near surface velocity control. Minimum residuals for a crack, a pipe and a purely volumetric source were $R = 0.50$, $R = 0.60$ and $R = 0.72$, respectively, thus suggesting a crack as a most likely geometry of the source. The fit of the waveforms, the misfit function, $R(\theta, \phi)$, and the retrieved STF are shown in Figure 3. The inversion was robust, i.e. insensitive to the exclusion of stations for which data and synthetics are slightly out of phase. The shape of the STF is fully determined by the signal recorded at ECPN station, while other stations contributed to determining the crack orientation.

As seen from Figure 3c, the main peak of the STF corresponds to its minimum where the magnitude of the seismic moment is about $5 \cdot 10^{11}$ Nm. Volumetric change of the crack can be estimated from $M_0 = \mu \Delta V$, where $\mu$ denotes

![Figure 3. Moment-tensor inversion of real data. (a) Waveforms fit. Thick and thin lines denote data and synthetics, respectively. (b) Residual as a function of crack normal inclination and azimuth. (c) STF and its standard deviation.](image-url)
the rigidity of the medium and $\Delta V$ is incremental volume change. Although the rigidity of the medium can differ considerably from the rigidity of the source region, we use it as a rough estimation of the volume change within the source. The obtained value of 50 m$^3$ is comparable to a value for the VLP source (~100 m$^3$) estimated by [Saccorotti et al., 2007], who suggested an interaction between the two systems (LP-VLP). The principal peak of the STF corresponds to deflation of the source, a result which is in agreement with Lokmer et al. [2007] who proposed a model where the excess gas injected from the magmatic column into a fluid-filled crack is repeatedly discharged into the surrounding medium.

[19] Retrieved inclination and azimuth of the crack normal are $\theta = 72 \pm 5^\circ$ and $\varphi = 35 \pm 9^\circ$, respectively, thus suggesting a crack striking NNW-SSE in the summit region (Figure 1), slightly inclined from the vertical towards the WSW. This is consistent with the trend of dykes formation at shallow depths beneath the summit [Bonaccorso and Davis, 2004]. Moreover, it corresponds to the orientation of a dyke constrained by joint seismological-deformation modelling of Patané et al. [2005]. A zone of weakness with fractures propagating SSE-ward is additionally supported by a significant trend of ESE-ward flank spreading initiated during the 2001 eruption, recorded by tilt measuring and confirmed by GPS and SAR data [Allard et al., 2006].

[20] It must be stressed that the small misfit change (0.5 vs 0.6) is the only evidence supporting our choice of a crack instead of a cylindrical geometry. In the case of a pipe-like solution, the source would be represented by a sub-vertical structure practically lying on the plane of the crack; this source would also (1) permit upward migration of volcanic fluids, and (2) be equally consistent with the structural constraints outlined above. Hopefully, future experiments taking advantage of larger deployments will help to better distinguish between these two competing hypotheses.

6. Concluding Remarks

[21] We performed a moment-tensor inversion of stacked LP signals recorded at Mt. Etna representative of the period of LP activity from November 2003 to September 2004. We calculated Green’s functions using full waveform numerical simulations in a heterogeneous velocity model of Patané et al. [2002] and topography of Mt. Etna. Following Bean et al. (submitted manuscript, 2007), who suggest using as few free source parameters as possible in the case of considerable uncertainties in the near surface velocity model, and Tarantola [2006], who proposes using all available a priori information, we performed our inversion for 3 pre-assumed source geometries capable of generating LP signals: a crack, a pipe and a pure volumetric source. The result suggests a crack centred at about 500 m below the summit, striking in a NNW-SSE direction, inclined about 20$^\circ$ from the vertical towards the WNW, a result which is in agreement with dyke propagation trends and the zone of weakness revealed by other studies [e.g., Bonaccorso and Davis, 2004]. The LP perturbation lasts approximately 4 seconds, suggesting a very short duration excitation mechanism, possibly gas “pulsing” [Lokmer et al., 2007].

[22] Synthetic tests reveal the extreme sensitivity of the STF to near surface velocity structure and network configuration. Such test should be an integral part of source inversions and could be used for planning the configuration of seismological networks.

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