Seismic image quality beneath strongly scattering structures and implications for lower crustal imaging: numerical simulations

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SUMMARY
The aim of this paper is to analyse how the scattered wavefield in a highly heterogeneous structure influences the image quality of structures below it and how the velocity control during processing of data from highly heterogeneous areas is important in the imaging problem. Statistical 2-D two-layer models are built based on deep borehole data. The layers are characterized by a fractal dimension (Hurst exponent) and by the standard deviation of the velocity fluctuations over a constant background velocity value. Shot gathers are generated in these models with a 2-D elastic and viscoelastic finite difference simulator. The synthetic data are first stacked and migrated using an average background velocity function, as would be done in a realistic situation during which only a smooth version of the velocity field can be resolved. Second, the full heterogeneous velocity models are applied to the migration scheme. The final sections are compared using a semblance-based technique to quantify the amount of contamination from an upper heterogeneous structure of a lower heterogeneous structure. In data that are not properly migrated, multiple scattering in the upper layer seems to dominate the lower part of the section. This contamination is controlled by the standard deviation of the velocity fluctuations and it is reduced when viscoelasticity is introduced in the simulations. Counterintuitively, a low $Q$ value may improve deep imaging. When the seismograms are migrated with the real complex velocity field, this contamination is reduced to acceptable levels. Residual contamination cannot be corrected completely because of the single-scattering assumption inherent to all migration techniques.

Key words: finite difference methods, heterogeneity, lower crust, scattering, seismic velocities, statistical methods.

1 INTRODUCTION
Over the past 20 years or so considerable effort has been invested in deep reflection seismic imaging of the lithosphere in projects such as COCORP, COCRUST, BIRPS, ECORS, DEKORP, BELCORP, Lithoprobe and CROP. This has led to a significant improvement in our knowledge of the lithospheric structure and, to a lesser extent, our understanding of lithospheric processes. Although by no means ubiquitous in the crystalline crust, a systematic pattern of transparent or, more precisely, largely featureless upper crust and reflective lower crust emerged early in this work [i.e. BIRPS Atlases (Klemperer & Hobbs 1992; Snyder & Hobbs 1999); DEKORP Atlas (Meissner & Bortfeld 1990)]. Reviews of the findings of these studies are given by Brown (1991), Meissner et al. (1991), Sadowiak et al. (1991), Dyment & Bano (1991) and Blundell (1990). There have been many geological arguments put forward to explain lower crustal reflectivity patterns (i.e. strain, fabrics, fluids and underplating). Some of these have been put into question by more recent work, which investigated the role of wave scattering in seismic imaging (Levander & Holliger 1992; Holliger et al. 1994; Levander et al. 1994a,b; Holliger & Robertsson 1998). Impressive qualitative similarities can be achieved between real and synthetic seismic sections for models that do not require the presence of fluids or highly strained (horizontally smeared) lower crustal rocks. In these investigations, crustal models were built statistically, based on detailed observations of lower crustal rocks in outcrop. In this paper, we take a broadly similar approach in order to quantify the degree to which wave scattering in the upper crust contaminates lower crustal images. Our major aim is to see how, in the

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presence of a highly heterogeneous structure, the scattered wavefield influences the image quality and reliability of the structures below the scattering layer. Although in this paper we work at a crustal scale, our results also have implications for imaging below any highly heterogeneous structure, for example, basalt. Imaging below basalts is a major problem faced in hydrocarbon exploration in many parts of the world. Our statistical models for the upper crystalline crust are based in part on superdeep borehole data. Our approach is numerical throughout, using 2-D elastic and viscoelastic finite difference wave simulations. The synthetic data are processed as if they were real data and the final unmigrated and migrated sections are analysed using a semblance-based technique, which measures the similarity between corresponding portions of the sections, to quantify the amount of contamination due to scattering in the upper crust. After an introduction to the general problem and a description of our models, we show our final sections and compare them qualitatively and quantitatively. We analyse the elastic and then the viscoelastic case. We then analyse the effect of migration for the different scenarios. We show how the velocity control in migration is important in order to reduce the contamination from overlying strongly scattering structure.

2 SYNTHETIC MEDIA

The synthetic media on which we perform our simulations are stochastic and are constrained by geostatistical inversion of borehole sonic logs. We employ stochastic modelling as we want to incorporate a very high degree of heterogeneity, which cannot be known in a deterministic sense. Borehole logs provide in situ information about upper crustal velocity fluctuations and take into account both fractures and lithology. In particular, we use measures obtained from deep borehole data in crystalline rock. Without exception, these velocity fluctuations exhibit a power-law scaling, surprisingly with a ‘universal’ scaling exponent independent of locality, leading to a 1/f noise spectrum (Dolan et al. 1998). However, the apparent standard deviation of velocity fluctuations does vary significantly both with locality (Holliger et al. 1994) and often with increasing depth within a given borehole (Frenje & Juhlin 1998). The typical average value for the apparent standard deviation is \( \approx 4 \) per cent, while the scaling (Hurst) exponent is usually \( H \approx 0.2 \). Equivalent measures of apparent standard deviation for purely lithology-related velocity fluctuations in exposed lower crustal rocks can be significantly less, \( \approx 1.5 \) per cent (Bean 1996). We might expect lithology-related velocity heterogeneity to be less than measured velocity heterogeneity in boreholes as the latter also include fractures. By correlating sonic, resistivity and gamma logs in the Deep Cajon Pass Borehole in Southern California, Leary (1991) concluded that fractures dominated the velocity fluctuations over the depth range of the borehole (2.5 km). It follows that we can expect larger standard deviations in velocity fluctuations in the upper crust relative to the lower crust, due to fracture closure with depth. Hence we expect the upper crust to be a more efficient seismic scatterer than the lower crust as scattering is controlled by velocity deviations from a nominal mean value. In the models that follow we have a heterogeneous upper crust overlaying a less scattering lower crust. The aim of this study is to investigate the effects of a highly heterogeneous upper crust on lower crustal imagery, without focusing on the origin of this heterogeneity.

3 STOCHASTIC MODELS

In order to investigate the effects of scattering in the upper crust on mid- to lower crustal images, we generate four different two-layer models. The upper layer in each model is a 10 km thick stochastic layer, based on the typical statistical parameters obtained from borehole logs. The lower layer in each model is also 10 km thick and represents the mid- to lower crust ‘target’ layer. It is also statistically defined but is ‘less scattering’ than the upper crust (see Section 2). All models have a Poisson ratio of 0.25. Material densities are derived using the expression \( \rho = 1700 + 0.2\rho_F \) (Sheriff & Geldsarf 1995). All models are 20 km wide. The models are described in more detail in Sections 3.1–3.4. Power spectral estimates of borehole logs reveal that most spectra exhibit power-law behaviour (Holliger 1996; Dolan & Bean 1997). The scaling exponent (Hurst exponent, \( H \)) seems insensitive to lithology and setting. Typically, \( 0 < H < 0.3 \), indicating that upper crustal velocity fluctuations are instances of flicker noises (i.e. \( H = 0 \) case). In contrast, the apparent standard deviation (\( \sigma_p \)) of velocity fluctuations in well logs does vary significantly with both lithology and setting and may be a controlling parameter in scattering excitation. All the upper crustal models in this paper have \( H = 0.2 \) but varying values of \( \sigma_p \). In this work, correlation lengths, \( L \), are taken to be effectively infinitive, i.e. larger than model dimensions (Dolan et al. 1998; Bean et al. 1999), and \( L_x = L_z \).

3.1 Model 1

The top layer has a Hurst exponent \( H = 0.2 \), an average \( P \)-wave velocity of 5770 m s\(^{-1} \) and a standard deviation, \( \sigma_p \), of 8 per cent (Fig. 1a). The combination of a low Hurst exponent and a relatively high standard deviation makes this quite a strong scattering structure. The values are taken from analyses on the Nirex PRZ boreholes, West Cumbria, UK (Nirex UK Ltd 1993). The standard deviation value is in the upper range of the values found by Holliger (1996). The bottom layer has an average velocity of 6000 m s\(^{-1} \), a standard deviation \( \sigma_p \) of 3 per cent [which is closer to the global average standard deviation reported by Holliger (1996) for metamorphic rocks], and a Hurst exponent in the persistent regime (\( H = 0.7 \)). Although we did not find any evidence for crustal heterogeneity in the persistent regime (i.e. \( H > 0.5 \)) from the well logs, we have chosen this value to make the bottom layer more correlated per unit distance. The lower crust is generally assumed to be more correlated than the upper crust.

3.2 Model 2

The top layer is the same as in Model 1 (\( H = 0.2 \), \( \sigma_p = 8 \) per cent and average \( v_p = 5770 \) m s\(^{-1} \)). We consider exactly the same top layer, that is, the same realization of the same distribution. The bottom layer in Model 2 is homogeneous with a \( P \)-wave velocity of 6000 m s\(^{-1} \) (Fig. 1b).

3.3 Model 3

In Model 3, the top layer is a homogeneous layer with a velocity of 5770 m s\(^{-1} \). The bottom layer is the same as in Model 1, that is, \( H = 0.7 \), \( \sigma_p = 3 \) per cent and an average \( v_p = 6000 \) m s\(^{-1} \) (Fig. 1c).

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3.4 Model 4

Model 4 is the same as model 1, except the top layer is less scattering with the standard deviation in the velocity fluctuations reduced to 3 per cent (Fig. 1d). Both models have the same random number seed.

4 SYNTHETIC SEISMIC SECTIONS

4.1 Introduction

20 shot gathers were generated using a finite difference elastic and viscoelastic wave simulator, eighth order in space and second order in time on a staggered grid (Igel 1993; NicCanna 1997). The stochastic models were surrounded at the edges by an absorbing boundary layer, including the top layer to prevent the generation of Rayleigh waves (Levander et al. 1994a). The source is an explosive Ricker wavelet with a peak frequency of 12 Hz. For a mean model (both layers) velocity of 5884 m s\(^{-1}\) and the source frequency, the mean wavelength is 490 m. Therefore, there are approximately 20 wavelengths per layer, one way. Due to necessary stability criteria for the finite difference simulations, there must be at least four grid nodes per smallest wavelength. The following relationship between the time increment, \(dt\), the grid spacing, \(dx\), and the maximum P-wave velocity, \(V_P\), must hold (Igel 1993; NicCanna 1997):

\[
V_P \frac{dt}{dx} \leq 0.424.
\]  

The choice of model dimensions, grid spacing, source frequency, model velocity and time increment are motivated by the need to satisfy these stability criteria and produce a model comparable in dimensions to real seismic surveys. 20 sources were placed at intervals of 640 m from 1840 to 14 000 m, with receiver intervals of 80 m. The resulting seismic sections are about 15 km long and recorded to 6.8 s two-way time (TWT). The data were processed as if they were real data using an industry standard package (ProMAX\textsuperscript{TM}). They were corrected...
for normal moveout (NMO) using the average layer velocity, muted, gain-corrected and stacked. As random noise was not included in the simulations, random noise attenuation techniques were not necessary. There are no surface multiples because of the absorbing boundary, and deconvolution was excluded from the processing sequence. Two different types of migration were tested on the data—post-stack time-stacked migration and, subsequently, pre-stack depth Kirchhoff migration (see Section 7). The frequency–wavenumber (F–K) post-stack migration was tested first as a standard processing procedure. It is the fastest implementation of migration and is based upon the approach outlined by Stolt (1978). F–K migration involves a Fourier transform from the time–space (T–X) domain to the F–K domain, where a mapping operation is performed using a single velocity value. The migrated image is generated by performing an inverse Fourier transform to the T–X domain. The Stolt migration uses rms velocities. The primary advantage of this approach is speed. Kirchhoff migration is commonly implemented following the technique described by Schneider (1978) for the integral solution of the scalar wave equation. It is implemented as either time or depth migration. In the depth implementation, Kirchhoff migration estimates traveltimes from a velocity model, using ray tracing or finite differences on the Eikonal equation. We used a Kirchhoff depth migration algorithm that performs a migration by applying a Green function to each CDP location using a traveltime map. Traveltimes relate the time, or optionally the amplitude, from each surface location to a region of points in the subsurface. This migration uses a vertically and laterally varying interval velocity field.

4.2 Limitations of this study
Before showing and analysing the results from our modelling in the next sections, we outline the limitations of this study of which we are aware. The acquisition geometry used yields a low interval velocity field. The frequency–wavenumber (F–K) migration involves a Fourier transform from the time–space (T–X) domain to the F–K domain, where a mapping operation is performed using a single velocity value. The migrated image is generated by performing an inverse Fourier transform to the T–X domain. The Stolt migration uses rms velocities. The primary advantage of this approach is speed. Kirchhoff migration is commonly implemented following the technique described by Schneider (1978) for the integral solution of the scalar wave equation. It is implemented as either time or depth migration. In the depth implementation, Kirchhoff migration estimates traveltimes from a velocity model, using ray tracing or finite differences on the Eikonal equation. We used a Kirchhoff depth migration algorithm that performs a migration by applying a Green function to each CDP location using a traveltime map. Traveltimes relate the time, or optionally the amplitude, from each surface location to a region of points in the subsurface. This migration uses a vertically and laterally varying interval velocity field.

4.3 Synthetic section for model 1
The resulting post-stack migrated seismic section (Stolt algorithm) z-component from Model 1 is plotted in Fig. 2(a). The strong scattering layer produces a distinct ‘diffractive’ pattern. There is no identifiable structure such as the boundary between the two layers, which should occur at 3.4 s TWT. The arrival times from the reflection at the top boundary is quite strong.

4.4 Synthetic section for model 2
The resulting migrated seismic section, z-component of displacement, from Model 2 is plotted in Fig. 2(b). One would expect the seismic section from this model to be as ‘diffractive’ as before, which it is, but it is remarkable how similar it is to the section from Model 1. In this model, the bottom layer was chosen to be homogeneous, to distinguish it from Model 1 where we have the same top layer but a non-homogeneous bottom layer. Nevertheless, the two final sections—after the same processing sequence—look very similar between 3.4 and 6.8 s, corresponding to expected arrival times from the bottom layer, despite the multifold stacking and migration. This is quantitatively analysed in Section 6.2.

4.5 Synthetic section for model 3
The resulting migrated seismic section z-component for Model 3 is plotted in Fig. 3(c). This seismic section at times greater than 3.4 s TWT looks very different from the one from Model 1 (see Fig. 3a), despite the fact that layer 2 is the same in both models. A top mute has been applied before 3.4 s TWT to eliminate the S arrivals in the homogeneous top layer.

4.6 Synthetic section for model 4
The resulting migrated seismic section z-component for Model 4 is plotted in Fig. 3(d). It is interesting to note that there is evidence of the boundary between the two layers, at approximately 3.5 s TWT, even though this reflector shows little continuity and variation in amplitude.

5 QUALITATIVE INTERPRETATION OF RESULTS
The four models and corresponding synthetic seismic sections allow us to make some qualitative statements about deep crustal imaging in a strongly scattering environment. First, Fig. 3(c) indicates that laminar lower crustal seismic character can emerge in the absence of lower crustal laminar structure. The underlying structure in Fig. 1(c) is statistically isotropic. Levander et al. (1994a) and Bean et al. (1999) have also reported analogous behaviour for heterogeneous fields. This is not only related to migration or decrease in fold at the edges, both factors that enhance this effect: this ‘layering’ is also very evident in the PRS (primary reflectivity section) for model 3, shown in Fig. 4. The PRS, obtained by convolving the vertical reflection coefficients with the source waveform, represents an ideal perfectly migrated section. Lateral continuity, which is not present in the original statistically isotropic model, can still be recognized. This may in part be due to differing horizontal and vertical resolutions, approximately λ and λ/4, respectively, for the PRS.

A close inspection of Figs 2(a) and (b) reveals that both sections are almost identical in the 3.4–6.5 s window. As the time interval 3.4–6.8 s corresponds to the expected arrival...
times from the homogeneous layer in Fig. 1(b), we might infer that multiple scattering in the upper layer in Model 1 (Fig. 1a) is dominating the lower part of Fig. 2(a). That is, our signal-to-noise ratio for arrivals from the lower crust is extremely low. Although large, the standard deviation of velocity fluctuations in the upper layer in Model 1 (8 per cent) is not unrealistic: the average $P$-wave velocity and the standard deviation were taken from borehole data (see Section 3.1). Finally, by reducing the value of the standard deviation in the upper layer to 3 per cent in Model 1 (see Model 4, Fig. 1d), we see an improvement in the signal-to-noise ratio on the seismic section in Fig. 3(d). Fig. 3(d) shows a hint of the expected laminar structure seen in Fig. 3(c) between 3.4 and 6.5 s, yet there still appears to be a significant degree of contamination due to scattering in the upper layer. In the next section we will try to quantify these contamination effects using a semblance-based technique to compare the bottom part of the sections from different models.

6 QUANTIFYING CONTAMINATION BY SCATTERING

6.1 Cross-correlation method

We are interested in quantifying the degree to which scattering from the upper layer contaminates our image of the lower layer. We do this by comparing pairs of seismic sections corresponding to the various models. For any two models that we are comparing, we calculate the cross-correlation coefficient.
Figure 3. Resulting migrated seismic sections, z-component displacement data. (c) Model 3; (d) Model 4. A post-stack Stolt F-K migration has been applied to the data. Average amplitude decay curves on a dB scale are plotted on the right for each section. Amplitude decay correction is applied to the data based on these curves.

Figure 4. Primary reflectivity section (PRS) for Model 3, 4.0–6.8 s TWT window, corresponding to the bottom layer (see text).
between all pairs of traces corresponding to the same common depth point (CDP) within a 4–6 s window (corresponding to the bottom layer). Cross-correlation values of \( \sim 1 \) imply a high degree of similarity; values of \( \sim 0 \) imply no correlation. As this is a normalized value (between \( -1 \) and 1), it is not influenced by the amplitude but is a measure of the similarity of the phase component. Measuring the similarity between seismic traces in this way is comparable to the ‘semblance’ seismic attribute. The results from Model 3 are excluded from the calculations as the velocity distribution of the upper layer in this model is homogeneous. Consequently, the waves arriving at the lower layer will have different phase characteristics and cannot be compared to the other models.

6.2 Elastic case

6.2.1 Model 1 versus Model 2

The results of the cross-correlation method are given in Fig. 5. They show a perfect match between the seismograms from Model 1 and Model 2 (that is, heterogeneous versus homogeneous bottom layer, Fig. 5a). When the top layer is a strong scattering structure, multiple scattering from the heterogeneous top layer seems to dominate.

6.2.2 Model 1 versus Model 4

The cross-correlation between the seismograms from Model 1 (\( \sigma = 8 \) per cent, \( H = 0.2 \)) and Model 4 (the same velocity model, but Model 1 has larger velocity fluctuations in the top layer, Fig. 5b) shows no correlation (average value zero). The top layer in Model 4 is a less strongly scattering structure than in Model 1. The contamination from the upper layer in this case is therefore weaker.

6.2.3 Interpretation of the results

It seems that multiple scattering in the top layer ensures that waves—scattered by the top layer—continue to be recorded at times/depths corresponding to the bottom layer, which is not properly imaged. The cross-correlation coefficient plots for corresponding traces show that if the top layer is a strongly scattering structure, multiple scattering can dominate lower parts of the section, controlled by the standard deviation of velocity fluctuations in overlying layers. The multiple scattering and associated coda formation interfere with and obscure the arrivals from the underlying layer. In the case where the upper layer has a standard deviation in velocity fluctuations of 8 per cent, information from the lower layer is completely obscured (see Fig. 5a). For a 3 per cent standard deviation, there is a signal from the lower layer present in the seismograms. In this particular case, the signal-to-noise ratio can be estimated from Fig. 5, where ‘noise’ refers to contamination from the upper structure. We interpret a correlation coefficient of 0.5 to represent approximately a signal-to-noise ratio of 1 on the basis that a correlation coefficient of 1 represents the case where the noise is suppressing any signal from the lower layer (noise stronger than signal) and a correlation coefficient of 0 is the case where the two lower structures are properly imaged as being different (signal stronger than noise). Hence, for a 3 per cent standard deviation in upper layer velocity fluctuations, our signal-to-noise ratio is low. An upper crust standard deviation of 3 per cent is lower than the global average (Holliger 1996).

6.3 Viscoelastic case

The absence of intrinsic attenuation is a significant difference between the simulations shown thus far and real data. As absorption may be an important mechanism for removing and/or damping scattered waves, intrinsic attenuation is introduced into the simulation, with a \( Q_p \) value of 60, homogeneously
distributed throughout the entire model. Numerous studies have shown that $S$ waves are more easily attenuated than $P$ waves (e.g. Anderson et al. 1965; Anderson & Hart 1978). Hence the quality factor for shear waves, $Q_S$, in our models is about 80 per cent of the $P$-wave value; this compares well with the observation that shear waves are more attenuated than $P$ waves (Igel 1993; Carcione et al. 1988; Carcione 1993). Recordings in deep boreholes show that intrinsic $Q$ can increase rapidly with depth (Abercrombie 1998). The same $Q$ value throughout the model is probably not realistic; it nevertheless yields a relative difference between the elastic and viscoelastic cases, giving a qualitative result. Repeating the same procedure as that employed for the elastic case, the cross-correlation coefficient has been calculated between seismograms occupying the same position in a 4–6 s time window. The results are plotted in Fig. 5(c).

6.3.1 Model 1 versus Model 2

Comparing the plot with that for the elastic case (which showed a perfect match with values close to 1, Fig. 5a), the cross-correlation coefficients are lower between seismograms from Model 1 and Model 2 (range 0.5–0.8, Fig. 5c).

The same procedure was repeated for $Q_P=600$, which is a more realistic value for the crust (Hobbs 1990). The values obtained are in between the two previous cases, as expected (values for Model 1 versus Model 2 are in the range 0.8–0.9).

6.3.2 Interpretation of the results

When viscoelasticity was introduced into the simulations there was a reduction in the degree of contamination in the lower part of the seismic image (compare Fig. 5c to Figs 5a and b). Other simulations for Model 1, generating $P$ waves only, excluding the $S$ waves, were run in the elastic and viscoelastic cases. In Fig. 6, snapshots of the wavefield 3.2 s after the explosion are compared for (a) $P$ and $S$ waves, elastic case, (b) $P$ waves only, elastic case, (c) $P$ and $S$ waves, viscoelastic case ($Q_P=60$), and (d) $P$ waves only, viscoelastic case ($Q_P=60$). Comparing Figs 6(a) and (b), it is clear that the scattered waves in the top layer are mostly $S$ waves: excluding $S$ waves from the simulation, the scattered wavefield is notably reduced. This

Figure 6. Snapshot of the wavefield propagating in a Model 1-type medium, z-component displacement. (a) $P$ and $S$ waves, elastic case; (b) $P$ waves only, elastic case; (c) $P$ and $S$ waves, viscoelastic case; (d) $P$ waves only, viscoelastic case. The black circle indicates the shot point position.
dominance of S waves in the scattered wavefield is consistent with the results from scattering studies in earthquake seismology (Sato & Fehler 1998). S waves are taken to be more intrinsically absorbed than P waves (Anderson et al. 1965; Anderson & Hart 1978; Carcione et al. 1988; Carcione 1993). The snapshot of the full wavefield in the viscoelastic case (Fig. 6c) is somewhat similar to the P-wave-only snapshot shown in Fig. 6(b): S waves are preferentially suppressed. The two snapshots in Figs 6(b) and (d) look very similar, showing how absorption is not as effective on P waves. The shear waves are relatively more attenuated than the P waves, a consequence of the lower S quality factor. The long-path scattered arrivals are suppressed relative to the primaries. Therefore, counterrIBUTIvely, in the presence of S-wave scattering, a low intrinsic Q upper crust may help lower crustal imagery, especially if the upper crust is also strongly scattering.

7 VELOCITY CONTROL ON MIGRATION

7.1 Migration with full velocity field

Our models have a complex structure, with velocities varying continuously from one gridpoint to the next over an average background value. The post-stack Stolt migration was applied to the data with the average background velocity function, mimicking a realistic situation. In the real world, a processor would not be able to determine the velocity field in fine detail, but some smoothly varying velocity function or an average velocity would usually be available with which to stack and migrate the data. This value corresponds to a maximum in the velocity analysis spectra for our synthetic data. We wish to test how sensitive our results are to the velocity field used in the migration. This is especially important as our underlying velocity fluctuations are broad band. With this aim in mind, we also used the full velocity field to stack and migrate the data. We compare the results from different models with the cross-correlation method, exactly as seen before for the post-stack migration data with the average velocity function. We show the cross-correlation coefficient plots for data migrated with post-stack F–K Stolt migration (same algorithm as before) and a pre-stack depth Kirchhoff migration, the two extremes of migration. The former is shown in order to compare these plots with those in Fig. 5, the latter to analyse the best possible processed section, fully and accurately migrated and converted to depth. Stolt migration is a constant-velocity migration that does not handle large vertical and lateral velocity variations, but the algorithm considered uses Stolt's stretching technique to account for those variations. Pre-stack Kirchhoff migration uses a vertically and laterally varying interval velocity field accounting well for varying velocities. It was applied to the shot-gather domain. The migration aperture, which limits the horizontal distance energy can move in the migration, was the maximum possible aperture. The algorithm calculates the maximum by default during the calculation (20 566 m). The Green function was produced from the maximum amplitude ray tracing method using a 40° angle from the vertical to trace rays. The cross-correlation coefficient plots are shown in Fig. 7.

Figure 7. Cross-correlation coefficient versus seismogram position for Model 1 versus Model 2. Post-stack migration is applied with the real velocity field for (a) the elastic case and (c) the viscoelastic ($Q_p=60$) case; pre-stack depth migration is applied with the real velocity field for (b) the elastic case and (d) the viscoelastic ($Q_p=60$) case.

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Fig. 7(a) shows the cross-correlation values obtained from traces in Models 1 and 2 (elastic case) after post-stack migration using the actual velocity model. With the average velocity, we obtained a mean cross-correlation coefficient value of 0.98 (Fig. 5a); using the full model the mean value drops to 0.65 (Fig. 7a), and to 0.06 if a pre-stack depth migration with the actual model is applied (Fig. 7b). Figs 7(c) and (d) show the same for the viscoelastic case ($Q_P = 60$), where the values change from 0.64 in the case of post-stack migration with the average velocity function (Fig. 5c) to 0.60 for the post-stack migration with the real model velocities (Fig. 7c) to 0.04 for the pre-stack migration case (Fig. 7d). These results are summarized in Table 1.

In Fig. 8 we show cross-correlation plots for horizontal slices in the same 4–6 s TWT window for Models 1 and 2 (elastic case) for (a) post-stack average background velocity migration, (b) post-stack migration with the full velocity field, (c) pre-stack depth migration with background velocity and (d) pre-stack depth migration with the actual velocity field. As seen for the plots between traces from different models, the cross-correlation values decrease progressively in the plots. Starting with an average value of approximately 1 in the post-stack migration with background velocity case, the values drop to nearly zero when the data are migrated with the actual velocity field.

### 7.2 Interpretation of the results

In this section we investigate the effects of velocity on migration. Multiple scattering contributes to the energy distribution in observed seismograms, but its contribution is not as strong as we expected from the results in Section 6. What at first looks like multiple scattering contamination is in fact due to improper data migration. If the seismograms are migrated with the real full velocity field, the cross-correlation values

<table>
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<th>Migration</th>
<th>Average cross-correlation value, elastic case</th>
<th>Average cross-correlation value, viscoelastic case ($Q_P = 60$)</th>
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<td>0.64</td>
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<td>Post-stack, actual velocity model</td>
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<tr>
<td>Pre-stack, actual velocity model</td>
<td>0.06</td>
<td>0.04</td>
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</table>

![Figure 8](image.png)

Figure 8. Cross-correlation coefficient between corresponding horizontal lines in the same time window (4–6 s) for Model 1 versus Model 2, elastic case. Post-stack migration is applied with the average background velocity value in (a) and with the real velocity field in (b). Pre-stack migration is applied with the background velocity value in (c) and with the real velocity field in (d).
between Model 1 and Model 2 drop dramatically, indicating the importance of velocity control in migration in the presence of broad-band velocity scaling. The values close to zero obtained in the cross-correlation plots do not indicate that there is no scattering contamination. The information we can obtain from these plots is that the seismograms are different. They reveal nothing about the amount of uncorrected contamination. There is some residual scattering contamination caused by multiple scattering. This cannot be corrected completely because of the single-scattering assumption inherent to all of the migration techniques. Model 2 has a homogeneous bottom layer: no seismic amplitudes should be recorded at times corresponding to it. The fact that there are amplitudes indicates that there is still some 'leakage' from the top layer, which is not corrected by the migration. This is clear in Fig. 9, where we compare two Model 2 sections, (a) after post-stack time migration with the background velocity function and (b) after pre-stack depth migration with the actual velocity field. The latter has been converted from depth to time for comparison purposes. The correlation coefficient obtained between the two models migrated with the complex velocity field has low values: the residual scattering contamination is not very strong. If it was, it would suppress any information from the bottom layer and the two sections would be similar (higher cross-correlation values).

Figure 9. Model 2, resulting migrated seismic sections, z-component, elastic case. Post-stack time migration is applied with the average background velocity value in (a) and pre-stack depth migration with the actual velocity field is applied in (b). The section has been converted from depth to time for plotting purposes. Average amplitude decay curves on a dB scale are plotted to the right of the sections. Amplitude decay correction is applied to the data.
8 CONCLUSIONS

We analysed finite difference synthetic seismic data for different two-layer models whose layers are statistically defined based on deep borehole data. They have been processed as real data and the final sections analysed using a semblance-based method to determine the degree of contamination on the lower layer caused by scattering in the upper layer. Scattering in the upper layer ensures that waves continue to be recorded at times/depts corresponding to the bottom layer, which is then not properly imaged. The degree of contamination is controlled by the standard deviation of the velocity fluctuations in the upper layer. When viscoelasticity is applied in the simulations there is a general reduction in the degree of contamination in the lower part of the seismic sections. Scattered waves are mostly $S$ waves. Quality factor $Q_S$ for shear waves is smaller than for $P$ waves in the model. Hence, the multiply scattered arrivals are suppressed relative to the primaries. Thus, counterintuitively, in regimes of strong scattering a low intrinsic $Q$ upper crust may improve lower crustal imaging, at least where the scattered wavefield is predominantly $S$ waves. The velocity control in migration is shown to be of extreme importance. We have shown that a very detailed knowledge of the velocity field and pre-stack depth migration are required to reduce contamination of our lower crustal images to acceptable levels. Some contamination still remains, caused by multiple scattering, but its amplitudes are low relative to singly scattered arrivals. This contamination cannot be corrected completely because of the single-scattering assumption inherent to all the migration techniques. Our primary focus in this work was on the sensitivity of the ‘character’ of lower crustal reflectivity to overlying upper crustal structure. On the other hand, our tests have shown that discrete individual reflectors with very high reflection coefficients (e.g. $R \approx 0.1$) can be clearly imaged if ‘buried’ in our lower crustal models, albeit with degraded lateral continuity. On real seismic sections, such discrete events are often seen in the upper mantle. There is no inconsistency between our results and these real observations. We conclude that in a regime with a strongly scattering upper crust, excellent velocity control and pre-stack depth migration are required to obtain acceptable signal-to-noise ratio levels on lower crustal arrivals. Counterintuitively, a low $Q$ environment may also improve the signal-to-noise ratio for deep arrivals.

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