Volcano topography, structure and intrinsic attenuation: Their relative influences on a simulated 3D visco-elastic wavefield

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Abstract

The seismic wavefield recorded on volcanoes can be significantly influenced by path effects. Scattering from the topography and internal volcanic structure can play a dominant role in the recorded seismograms. Intrinsic attenuation is also thought to play an important role in the characteristics of volcano seismic signals. We use 3D numerical modelling of wave propagation in elastic and visco-elastic media including complex velocity models and topography to investigate the scattering and attenuation characteristics of synthetic seismograms. We generate 5 distinct volcano models and simulate wave propagation through these models using shallow and deep double-couple broadband sources. We then analyse 129 synthetic seismograms calculated on the free surface. The synthetic seismograms resemble VT-A and VT-B events. The scattering effect of the topography alone is capable of producing complex seismograms. The introduction of an internal velocity structure increases the duration of these seismograms while the introduction of intrinsic attenuation decreases the duration but not the complexity. We fitted our synthetic seismograms to the diffusion in a half-space model. The measured quality factor is not only sensitive to the attenuation properties but also to the structural properties. In all cases the scattering is more dominant than the intrinsic attenuation.

1. Introduction

Several potential source mechanisms are thought to generate seismicity on active and/or restless volcanoes. These source mechanisms include brittle rock failure, fluid transport, gas slug ascent, choked flow, magmatic activity and the interaction of hot magma with hydrothermal fluids, (see Chouet, 2003 and McNutt, 2005 for reviews of volcano seismology). Seismicity may also be generated by surface events, e.g. rockfall, pyroclastic falls and lahars, (Jolly et al., 2002 and Lavigne et al., 2000). This richness in the potential source mechanism is mirrored in the variety of classes of signals observed on active or restless volcanoes. These signals span a continuum from ultra-long period events to long-period (LP) events and in addition volcano-tectonic signals (McNutt, 2005). Volcano-tectonic signals consist of shear or tensile failure of brittle rock and the spectral characteristics of the waveforms are indistinguishable from local tectonic earthquakes. Interpreting the signals in volcanic environments presents some difficult challenges in seismology. The complexity arises in a volcanic setting because (1) the wavefield is significantly distorted by the complex topography, (2) the volcanic stratigraphy can further distort the wavefield and (3) the presence of magmatic fluids (gas, magma and hydrothermal water) may generate large intrinsic attenuation. Adding to this complexity, the near-field seismic wavefield cannot be ignored, the wavelengths can be large relative to the source-receiver distance and hence the P- and S-wave phases can be intertwined. Also, the signals may have emergent wave-trains due to both source and path effects so separate phase arrivals often cannot be clearly distinguished. The combination of these processes can lead to a diffusive wavefield. Thus, a key question in volcano seismology is what are the relative effects of topography, structure and attenuation on the wavefield and how are they quantifiable?

Numerical simulations of seismic wave propagation on Merapi, Indonesia show that a complex, heterogeneous wavefield can emerge due to the presence of steep topography and that strong surface waves are generated and dominate the coda wavefield (Ripperger et al.,...
Neuberg and Pointer (2000) show that for broadband waveforms, the angle of incidence as well as the back-azimuth is affected by an inclined free surface.

On volcanoes strong lithologically controlled seismic impedance contrasts can occur at layer interfaces depending on the eruptive history. Measurements on rock samples from the Trecase borehole close to Mt Vesuvius show that the mean unfractured P-wave velocity in the top 300 m is less than 2500 m/s, (Bruno et al., 1998). Surface wave dispersion analyses reveal similar results for Vesuvius (De Luca et al., 1997), Stromboli (Chouet et al., 1998), Arenal, Costa Rica (Mora

Table 1
Different volcano models used to generate the synthetic seismograms. Fig. 1 shows the homogenous and velocity profiles along with the location of the synthetic sensors. The visco-elastic attenuation is shown in Fig. 2 where the curve is calculated using Eq. (1).

<table>
<thead>
<tr>
<th>Velocity model</th>
<th>Source depth (km)</th>
<th>Q_p (5 Hz)</th>
<th>Q_s (5 Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1: Homogeneous</td>
<td>1.3</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Model 2: Homogeneous</td>
<td>1.3</td>
<td>80</td>
<td>40</td>
</tr>
<tr>
<td>Model 3: Fractal</td>
<td>1.3</td>
<td>80</td>
<td>40</td>
</tr>
<tr>
<td>Model 4: Fractal</td>
<td>1.3</td>
<td>40</td>
<td>20</td>
</tr>
</tbody>
</table>

Fig. 1. Synthetic volcano model used in this study to investigate the role of topography, internal structure and intrinsic attenuation in distorting the seismic wavefield. 129 sensors are distributed in a 1 km square array across the surface (red dots). The black triangles show the location of a smaller linear array. The P-wave velocity, S-wave velocity and density are shown in the lower panel. See text for a description of the velocity model. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 2. A standard linear solid or Zener body is used to model visco-elastic wave propagation. The P-wave and S-wave quality factors used in our simulations and their dependence on the frequency are shown. The curves are calculated from Eq. (1).
et al., 2006) and Masaya, Nicaragua (Metaxian et al., 1997). Using array analysis of tremor wavefields, Saccorotti et al. (2003) also determined low velocity shallow structures at Kilauea. Hence it seems that near-surface (few hundred metres) low velocities are common in volcanic environments and can play a significant role in distorting the wavefield. For example, Bean et al. (2008) have shown that the presence of a shallow low velocity layer can significantly effect the moment tensor calculated from an inversion of long-period events on Mt Etna even though the smallest wavelength is approximately 4 times longer than the layer thickness.

In addition to the shallow low velocity zone borehole logs have shown variations in velocities and density throughout the volcanic edifice. In particular, borehole sonic logs reveal that exceptionally strong acoustic impedance contrasts can occur at lithological boundaries between tuff, ash and competent basalt (Dolan et al., 1998). As well as these structural heterogeneities, subsurface interface scattering can also play a significant role in distorting the wavefield (Martini and Bean, 2002).

Seismic attenuation has been studied on numerous volcanoes. For example, Del Pezzo et al. (2006) measured the attenuation on Mt Vesuvius using energy envelopes and different diffusive wavefield models. They found the total quality factor $Q$ varies from approximately a minimum of 10 at 2 Hz to over a 1000 for frequencies over 10 Hz. De Gori et al. (2005) produced a tomography model of Mt Etna revealing a large low P-wave quality factor ($Q^p$-wave) region with a value ranging from approximately 30–80 located under the summit and extending in a cylindrical like manner with a radius of 4 km to a depth of 4 km. As with low velocity layers and a variable velocity distribution, attenuation appears to be important on volcanoes.

We have performed full wavefield 3D numerical modelling in the presence of complex topography, complex velocity models and intrinsic attenuation. No analytical solutions can account for this complexity hence numerical methods must be used to model seismic wave propagation in volcanic settings. The output from these simulations is then used to quantify the effect of topography, velocity variations and intrinsic attenuation on synthetic seismograms. A primary aim of this study is to determine the relative contribution of these contributing factors on the wavefield distortion. The numerical scheme and volcano model is discussed in Sections 2 and 3 we present the results of our numerical simulations. The seismograms are then analysed in terms of their scattering parameters in Section 4 and we draw our conclusions in Section 5.

2. Modelling

We use a discrete particle method to model seismic wave propagation in a 3D visco-elastic medium, (O’Brien, 2008). Other methods can be equally used to model seismic wave propagation at
volcanoes, e.g. boundary elements (Neuberg and Pointer 2000) and finite differences methods (Jousset et al., 2004 and Ohminato and Chouet, 1997). The 3D visco-elastic lattice method for the simulation of seismic waves consists of particles arranged on a cubic lattice which interact through a central force term and a bond-bending force along with dashpots for the attenuation. Particle disturbances are followed through space by numerically solving their equations of motion. The method is computationally equivalent in time and memory to a 4th order finite-difference method. For a detailed description of discrete particle methods and their application to seismic wave propagation see Monette and Anderson, (1994), Toomey and Bean, (2000) and O'Brien et al. (in press). We use a Zener body (also called a standard linear solid) which consists of a dashpot with viscosity $\eta$ and spring with stiffness $K_1$ in parallel with another spring with constant $K_o$ as our rheological model. The details of our underlying intrinsic Q model is not of concern in this work, what is important is an assessment of our ability to recover the underlying intrinsic Q. For a discussion of an appropriate medium rheology in volcanic regions see Jousset et al. (2004). The Q curve is given by

$$Q(\omega) = 1 + \frac{\omega^2 \tau_r \tau_\omega}{\omega(\tau_s - \tau_\omega)}$$

(1)

where $\omega$ is the angular frequency and

$$\tau_\omega = \frac{\eta}{K_1}, \quad \tau_s = \frac{\eta}{K_o} + \frac{\eta}{K_1}$$

(2)

and $\tau_r$ is the stress relaxation time and $\tau_s$ the strain relaxation time (Carcione, 2007). The topography boundary condition is implemented as in O’Brien and Bean (2004). We have implemented absorbing boundaries using an exponential tapering of the wavefield on the edges and bottom of the model to mimic an infinite elastic or visco-elastic half-space. The introduction of absorbing boundaries means that energy is removed from the system. The attenuation of the

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**Fig. 4.** Nine synthetic vertical component displacement seismograms from model 3 are shown. Their location is shown in Figs. 1 and 3 as black triangles. The distance to the source is indicated on the right of the seismic traces. The source function is shown in the bottom of the figure (dashed line). All the traces are normalised and the amplitudes are indicated on the left side of the seismogram.
wavefield by the absorbing boundaries is frequency dependent with high frequencies being preferentially damped. We ran a test simulation in a homogeneous infinite elastic medium to check the efficiency of our absorbing boundaries by comparing the numerical solution with the analytical solution. We found that the unwanted reflections from the numerical boundaries were sufficiently damped with the amplitude of the reflected waves 20 grid points from the boundary being less than 2% of the direct arrival.

Table 1 lists the five models used in this work. One of the volcano models (model 3) used in this study is shown in Fig. 1. The topography is taken from a real digital elevation map (Ubinas, Peru) and is used as a generic surface for all models, (the depth is measured against sea level). The location of two seismic arrays is shown in Fig. 1A. The first array is linear with 9 sensors running from south-east to north-west across the summit, black triangles. The second larger array (red dots in Fig. 1A) covers the surface of the volcano with 129 sensors equally spaced 1 km apart in a square geometry. The surface of one quadrangle is removed to show the vertical and horizontal slices of the P-wave velocity structure. The P-wave velocity has a low velocity layer at the surface (2000 m/s) which linearly increases to 3500 m/s over 800 m. The P-wave velocity was then given a fractal distribution with a Hurst exponent of 0.3 with a maximum deviation of 7%, Fig. 1B. The correlation length was set to five times the maximum model length in the North and East directions and twice the maximum model length in the vertical direction. This effectively gives an infinite correlation length over the model dimensions and mimics stratigraphic layering inside the volcano. We imposed a Poisson’s ratio of 0.25 and a density ρ which is related to the velocity through Gardner’s equation (Gardner et al., 1974).

\[ \rho = 1700 + 0.2v_p \]  

(3)

As discussed above the intrinsic attenuation inside volcanoes is very variable so to reduce the parameter space of our model we choose a simple homogeneous Zener body. The frequency dependence of Q is shown in Fig. 2 with Q_P-wave and Q_S-wave set to 80 and 40 at 5 Hz respectively.

We will use our generic volcano models to examine the distortion of the wavefield generated by shallow volcano-tectonic sources. From these events we will measure the contribution of the individual components, namely the topography, structure and attenuation. To achieve this we have run several simulations using variations of the volcano model described above, see Table 1. In all cases the source function is a Gaussian pulse with a half-width of 0.075 s input as a force. We filter the resultant seismograms with a band pass filter from 0.033 Hz to 9 Hz. This was to remove numerical artefacts of unphysical reflections from the finite numerical grid of very large wavelengths and numerical dispersion of small wavelengths. Unless otherwise stated, the source mechanism was a double-couple source located 1.3 km below the summit of the volcano. The moment tensor component M_{xy} = 20^8 N m is the only non-zero component. In reality, small magnitude double-couple events will not radiate at a frequency range covered by our broadband spectrum, however, we use this broadband source to study the scattering characteristics of a relatively small magnitude double-couple event.

![Fig. 5. A synthetic displacement seismogram from model 3 recorded 4299 m from the source is shown. The individual components are given a static displacement shift for the sake of clarity. The initial P-wave onset is visible at 2.5 s but no S-wave or surface wave arrivals are evident. The seismogram has the distinct cigar like shape which is indicative of a diffusive wavefield. The right-hand plots show the particle motion for three different time windows as indicated on the seismic trace. The location of the source is shown as the black dot under the crater summit.](image)
large frequency band. This approach is justified as we are employing power-law (fractal) velocity models to represent our heterogeneity scaling. These models are justified based on borehole log observations (Dolan et al., 1998). As these models contain heterogeneity at all scale sizes, they allow us to study the scattering process, independent of the seismic frequency used in our simulations. As mentioned throughout this section we are using a volcano model which represents some features observed on volcanoes. In reality the velocity and attenuation would be far more complex at all spatial scales and in terms of the relaxation mechanisms. The goal is to investigate the effect of some of these features on the scattered wavefield not to propose these models as a true representation of an actual volcano.

3. Results

The vertical component of the seismic wavefield moving across the surface of the volcano is displayed in Fig. 3 for four different times for model 3 which has infinite intrinsic $Q_P$-wave and $Q_S$-wave value. At 0.9 s the initial vertical radiation pattern from the double-couple source is clearly seen in a spherical pattern on the surface. At 2.7 s the initial P-wave has moved to the edge of the numerical domain while a clear S-wave front is difficult to identify and strong coda wave generation can be clearly seen trailing behind the body waves. After 5.4 s the wavefield on the entire surface clearly appears incoherent with no distinct phases visible. The temporal snapshots illustrate the complex nature of the wavefield.

The vertical displacement seismograms across the linear array are shown in Fig. 4 for model 3. The vertical components across the array show no clear phase arrivals save for the initial P-wave onset which can be identified on some of the traces. The amplitude of this initial P-wave onset is small (relative to the full waveform) and could be easily unrecognizable if the signal-to-noise ratio was not zero. The radiation pattern might influence this result, for example if the linear array was placed along a nodal plane of the double-couple source. However, by visually examining all 129 synthetic stations we determine that this behaviour is a common feature and is not merely a radiation effect. For comparison the source function is plotted in the bottom of the figure and demonstrates the increase in length of the signal due to multiple reflections from the topography and scattering from the internal structure. The particle motions of the seismograms reveals little

![Fig. 6. Time–frequency representation (spectrograms) of the vertical component displacement seismograms shown in Fig. 4. The seismograms are generated in an elastic model (model 3).](image-url)
information following the initial onset as P-waves, S-waves and surface waves are all mixed (Fig. 5). This situation is more complex nearer to the source where the near-field terms cannot be ignored. In general the amplitude falls off with increasing distance as expected but there are some local site effects where the amplitude is stronger than expected, e.g. traces 8 and 9 in Fig. 4. The wavefield becomes incoherent soon after the initial P-wave arrival, as is shown in Fig. 5, where the 3 components of the seismogram located 4299 m from the source are plotted. After approximately 3 s the components are uncorrelated and this is borne out when looking at the particle motions along the trace for different time windows.

The time–frequency representation of the seismograms in Fig. 4 shows a general trend of increasing mean frequency with increasing time, Fig. 6. The longer wavelengths are less sensitive to the topography and structure and therefore are less prone to being trapped by these structures while the smaller wavelengths remain trapped for longer times. This leads to a dominance of higher frequencies at later times. The time–frequency plots highlight the variable nature of the wavefield with different frequencies arriving at variable times with variable power. The synthetic seismograms resemble real seismograms recorded on active volcanoes, for examples see Wegler and Luhr (2001), Sherburn et al. (1998) and McNutt (2005). Considering all 129 synthetic 3 component signals, they can be generally described as an emergent wavefield with a cigar shaped envelope. This behaviour changes with a different source mechanism and source location. If we increase the depth of the source to 3 km below the summit, the P-wave onset becomes more pronounced and we begin to see the development of an S-wave phase arrival. We can distinguish the P-wave and S-wave as we have an infinite signal-to-noise ratio and know the expected arrival times. The emergence of distinct phases for deeper events is well known and these events have been traditionally classified as VT-A events while the shallow source signals are classified as VT-B events. If the shallow source mechanism is changed from a double-couple mechanism to an explosive source, then the initial P-wave onset is much more pronounced and clearly seen across the linear array. This is because the source only generates P-waves and thus there is no large amplitude S-wave source term. Fig. 7 shows four examples of seismograms where the source and depth have been changed using velocity model 3.

When we introduce Q into the model (models 2, 4 and 5) we decrease the length of the signals as the waves are damped and, given our attenuation model, we preferentially damp higher frequency waves. The vertical component seismograms from the linear array for model 4 are shown in Fig. 8. They clearly show the influence of the attenuation as described above when compared with the elastic model, see Fig. 4. The P-wave ballistic arrival increases in amplitude relative to the remaining seismogram as $Q P$-wave is twice $QS$-wave so the S-waves and hence the coda are preferentially damped. The coda rapidly becomes dominated by S-waves due to wave conversions, (Aki, 1992). The spectrum of the signals in Fig. 8 shows spectral peaks and attenuation of the higher frequencies which is more pronounced with increasing time. This is a direct result of the attenuation model. As in model 3, the amplitude falls off with increasing distance with some exceptions where local site effects influence the amplitude. However, this effect is less apparent in the visco-elastic case as the waves are damped by the presence of intrinsic Q. When the source is deeper with an explosive mechanism the resultant seismograms
exhibit the same changes as observed in the elastic case. The dominant effect of including intrinsic attenuation is to change the length of the signals and the frequency content. The temporal dependence of the frequency also changes from attenuation of low frequencies in the elastic case to attenuation of high frequencies in the visco-elastic case. The introduction of $Q$ does not remove the complexity observed in the seismograms but shortens the duration of the signals. Fig. 9 shows a comparison of the vertical seismogram and spectrogram from a station located 4299 m from the source for models 1–5. The most striking differences are the length of the signals and frequency content. Models 1 and 3 highlight the difference of including a more ‘realistic’ internal structure. The wave-train persists for much longer and the higher frequencies are trapped inside the structure for longer. The influence of $Q_{P}$ and $Q_{S}$ is to change the attenuation characteristics and shorten the signal length. Model 5, which has a low $Q_{P}$ and $Q_{S}$, is interesting in that the seismograms appear to resemble a long-period event. Given this resemblance, we investigated whether a more realistic long-period event or hybrid event would emerge from a broadband like source function. We found that while we could replicate LP like signals with low Q values we could not readily replicate hybrid events without specifically designing a source time function or a velocity and attenuation structure. The spectra of two stations located 1479 m and 4590 m from the source are shown in Fig. 10 for all 5 models. As expected the far-field stations in the visco-elastic models are relatively poorer in higher frequencies than the near-field stations given that they propagate further through the visco-elastic medium. For this far-field location and for all models we only find the development of narrow spectral peaks at high frequencies. Whilst the seismograms for model 5 appear LP like, the spectrum shows a low frequency broad spectrum as opposed to the observed narrow spectral peaks associated with LP events. At other far-field locations, for the same setup as shown in Fig. 10, we do observe narrow spectral peaks at low frequencies. Also, when we introduced a deeper source the long frequency broadband pulse was replaced by narrower spectral peaks. This also occurred when the velocity structure was changed to discrete layers as opposed to a gradient and fractal velocity.

The overall picture is that with a shallow double-couple source we see extremely variable behaviour for an elastic homogenous model, an elastic heterogeneous model, a visco-elastic homogenous and a visco-elastic heterogeneous model. In all cases the seismograms consist of...
large amounts of scattered energy and large coda wave-trains relative to the duration of the seismic source. Several different models, whilst appearing intricate merely represent a simplified version of a real volcano, yet produce realistic looking seismograms. The interpretation of seismograms of this type presents a huge challenge in volcano seismology where the source mechanism and location are very poorly constrained. Also, there are very few well constrained fine-scale velocity models on volcanoes. With this limitation in mind in the next section we take a statistical approach and analyse all 129 3 component signals in terms of the scattering properties and relate the measured quantities to the volcano model.

4. Scattering diffusion model

The energy density envelope is calculated from Eq. (4) where $v_i$ is the velocity seismogram in the $i$-direction.

$$E(x,t) = \sum_{i=x,y,z} v_i(x,t)^2 + \text{Hilbert}[v_i(x,t)]^2$$

Fig. 11 shows the energy density for models 1–5 for two different distances filtered in the frequency band of 5–8 Hz. For models 2–5 the energy does not converge for later times but runs parallel for different distances. This is indicative of coda localisation. However, for different receivers and different frequency bands we do observe the energy density converging for different distances (see model 1) but this non-convergence is the normal behaviour. This behaviour has been observed on volcanoes, e.g. Piton Le Fournaise (Aki and Ferrazini, 2000) and Merapi (Friedrich and Wegler, 2005). To measure the scattering properties of our synthetic seismograms we used the diffusive wave field in a half-space model, (Wegler, 2004). The diffusion model is a simplified model which assumes strong scattering, point-like scatters and isotropic scattering. Another choice of model would be the cylinder embedded in a half-space (Wegler, 2004) or the Anderson localisation model, (Weaver, 1994). We found that the homogeneous half-space model performed as adequately as the other models in fitting our synthetic data and hence we chose the simplest.

In this work we wish to focus on what the measured quantities from the seismograms can tell us about our known volcano model.
Eq. (5) describes the distribution in space $x$ and time $t$ of the energy density using the half-space diffusion model.

$$ E(x, t) = \frac{2E_0}{(4\pi)^{3/2}} \frac{1}{d^{3/2}} \frac{1}{t^{3/2}} \exp \left[ -bt - \frac{|x|^2}{4dt} \right] $$

$$ \ln |E(x, t)| = \ln \left[ \frac{2E_0}{(4\pi dt)^{3/2}} \right] - bt - \frac{|x|^2}{4dt} $$

We fit our synthetic energy density data (calculated using Eq. (4)) to Eq. (6) solving for $b$ and $d$ and $E_0$ using a least squares regression for each of the 129 seismograms. The inversion procedure is performed for 8 different frequency bands centred on $[1, 2, 3, 4, 5, 6, 7, 8]$ Hz with a bandwidth of 0.5 Hz. The coefficient of scattering attenuation $\eta_s$ (in $m^{-1}$) and coefficient of intrinsic attenuation $\eta_i$ (in $m^{-1}$) are calculated from

$$ \eta_s = \frac{v_s}{3d} $$

$$ \eta_i = b \frac{v_s}{v_s} $$

where $v_s$ is the S-wave velocity. We use the homogeneous S-wave velocity of 2000 m/s in all calculations. In reality, we do not have detailed access to the S-wave velocity structure so we use a single value as is commonly done with real data.

The dependence of the attenuation coefficients on frequency and distance from the source for model 3 (infinite Q) and model 4 (finite Q) are shown in Fig. 12. Even though model 3 has an infinite Q we find a finite Q using this inversion procedure. This is no surprise as the inversion is providing the best fit without any a priori knowledge of the attenuation or structure and fits all the models with a finite figure.

Fig. 10. For each of our 5 models the spectrum of the vertical component from two stations is shown. The dashed lines are stations located 1479 m from the source while the solid lines are stations located 4590 m from the source. The near-field stations in the visco-elastic medium are richer in higher frequencies. For these locations we do not see the development of narrow spectral peaks at low frequencies. The broadband nature of the source has been retained.

Please cite this article as: O'Brien, G.S., Bean, C.J., Volcano topography, structure and intrinsic attenuation: Their relative influences on a simulated 3D visco-elastic wavefield, J. Volcanol. Geotherm. Res. (2009), doi:10.1016/j.jvolgeores.2009.03.004
scattering and attenuation coefficient. The scattering coefficient $\eta_s$ decreases with distance while the intrinsic attenuation is independent of the distance but is highly spread. The scattering and attenuation coefficients appear to be independent of the frequency but again a large variation in values is observed. The only discernable difference between model 3 and model 4 is that the coefficient of intrinsic attenuation is higher for model 4 and less scattered for all distances and for all frequencies. This is expected as model 3 has no intrinsic attenuation. As the topography and structure are the same for both models we expect no difference between the scattering coefficients. Relating the attenuation coefficients to our quality factor model we use the equations below to determine the dimensionless scattering quality factor $Q_s$ and intrinsic quality factor $Q_i$.

$$Q_s = \frac{\omega}{v_s/\eta_s} = \frac{3\omega d}{v_s^2}$$

(9)

$$Q_i = \frac{\omega}{v_s/\eta_i} = \frac{\omega}{d}$$

(10)

Examining $Q_s$ for models 1–4 we see the same pattern for each model, Figs. 13 and 14. $Q_s$ increases with increasing distance for all 4 models with lower frequencies increasing less than higher frequencies. Comparing models 1 and 3 and models 2 and 4, which differ only in terms of introducing the internal structure, we can postulate that the topography is the dominant factor in determining $Q_s$ in the volcano. In Figs. 13 and 14 there are some clear outliers from some stations for some frequency bands. On inspection we found that they are due to a poor fit with the diffusion model. $Q_s$ increases with increasing frequency indicating the preferential generation and ‘trapping’ of higher frequency waves through multiple scattering. This scattering effect dominates our intrinsic attenuation model, which preferential strips out high frequencies. Therefore, $Q_s$ appears to be measuring the length of time the frequencies remain inside the model and not the specific scattering attenuation. From Figs. 13 and 14 we see that $Q_s$ is independent of the distance but is more scattered for models with no attenuation. We cannot recover $Q_S$-wave from $Q_s$ for both models 2 and 4, the average value of $Q_s$ for all frequencies is less than 40 where we have introduced intrinsic attenuation.

Fig. 11. Energy density of two seismograms recorded at 2049 m (solid line) and 4299 m (dashed line) from the source for the 5 different models. The seismograms were band pass filtered between 5 and 8 Hz. The energy density is calculated using Eq. (4). For late times, the energy density does not converge to the same value but runs parallel for models 2, 3, 4, and 5. This behaviour is indicative of coda localisation.
attenuation. We also cannot recover the curve shown in Fig. 2. In fact the measured Q profile increases with increasing frequency in direct opposition to our Q profile in Fig. 2. It is also worth noting that even when there is no intrinsic attenuation we measure low values of Q for low frequencies. Model 5 showed the same behaviour as model 4 save for a lower value of Q. Using all 129 sensors we investigated if the scattering coefficients had any spatial correlation with the topography but we found no clear correlation. Since Qs clearly increases with distance we did not investigate if there was any link with the topography. To investigate how the energy is distributed in time we have plotted the energy density function (Eq.(5)) using the best fit values for b and d in Fig. 15. The panels (a) and (b) show E(x,t) for every seismogram filtered around 6 Hz for models 3 and 4. Panels (c) and (d) show the arrival time of the maximum of the energy curve for all stations for different frequency bands. For both models the energy arrives at a constant velocity which is higher for model 4. This is easily accounted for as model 4 is visco-elastic and therefore the energy is damped, hence the decrease in the arrival time of the energy maximum. The half-width of the energy curve for different frequencies is shown in panels (e) and (f). The width of the energy distribution is independent of the distance and of the frequency. However, model 3 shows a lot more variation in the half-width for different frequencies. This is borne out in the plot of the histograms of the half-width for all stations and at all frequencies, Fig. 15 g and h.

5. Discussions and conclusions

We have generated a sweep of synthetic seismograms using several different volcano models where we have included some prominent seismological features. These seismograms were then analysed to investigate the scattering and attenuation characteristics. We confirmed, as expected, that the introduction of complex topography significantly distorts the seismic waveform. The introduction of a heterogeneous velocity structure further distorts this waveform. The identification of distinct wave phases depends on the source mechanism and source location with information about direct arrivals being difficult to recover for a shallow double-couple source. These synthetic events correspond in character to the well known VT-B events. The inclusion of intrinsic attenuation decreases the length of the seismograms but does not significantly reduce the complexity of the simulated signals. For low Q values (Q_S-wave~20) we can see the appearance of long period like signals generated from shallow broadband frequency events. Also, the introduction of intrinsic attenuation changes the frequency characteristics of the signals from preferential attenuation of low frequencies to attenuation of the higher frequencies. This is strongly influenced by the characteristics of the visco-elastic body chosen which in this study is a Zener body. Assuming a diffusive wavefield we fitted our synthetic energy density traces calculated from our velocity seismograms using Eq.(4) to the diffusive wavefield in a half-space model. This allowed us to
measure the scattering and attenuation coefficients and to relate them to our specific volcano models.

Using the diffusive wavefield model we find that the scattering quality factor is smaller than the intrinsic Q implying that the scattering is more important than the intrinsic attenuation. The scattering quality factor was similar for all models indicating that the volcano topography is playing a large role in determining the scattering coefficient. This is borne out by the difference observed between the homogeneous and heterogeneous velocity models. It is not surprising that the topography appears to be the dominant scatterer as it has the largest impedance contrast (a perfect reflector in our numerical scheme). It is worth noting that the topography is one of the few variables which can be readily measured in a volcanic region and hence we can potentially use numerical simulations to remove the topographic effects on the wavefield.

With infinite Q, we still observe low values for low frequencies, but with the introduction of attenuation we observe low Q values for all frequencies at all distances from the source. We cannot accurately reproduce the correct attenuation values or frequency dependence. The recovered Q, as measured from our output seismograms is not merely measuring the model Q but is also influenced by frequency dependent scattering enrichment caused by the structural properties (primarily the topography). We recovered an intrinsic Q model which preferentially attenuates high frequencies as opposed to the Zener model used in the simulations. This interpretation of our synthetic data (the scattering Q dominating the intrinsic Q along with preferentially attenuating high frequencies) has been observed in Mt. Vesuvius (Del Pezzo et al., 2006). Whilst the interpretation of Q is not consistent with our model we can determine the presence of intrinsic attenuation. When Q is present the measured Q follows a definite trend and is not widely scattered as in the infinite Q case.

In summary, virtual models of volcanoes can reproduce complex, diffusive wavefields. By using such simulations in conjunction with real data we can constrain the measured attenuation and scattering properties of the seismic wavefield. This in turn may aid our understanding of the role of source versus path effects.

Acknowledgements

The authors wish to acknowledge that this work was carried out in part by the GSI Griffith Research Award and by the 6th framework EU project VOLUME. The SFI/HEA Irish Centre for High-End Computing (ICHEC) is acknowledged for the provision of computational resources.
facilities and support. The authors would like to thank the P. Jousset and an anonymous reviewer whose comments helped improve the manuscript.

References


Fig. 15. Energy density fits to our synthetic data for models 3 and 4 for all sensors and frequency bands. $T_{\text{energy}}$ represents the time of arrival of the maximum energy. $\Delta t_{1/2}$ is the half-width of the energy density curve. The bottom panels show the histogram of values of $\Delta t_{1/2}$. The different symbols represent different frequency bands with 1 Hz being dots, 3 Hz being pluses, 5 Hz being crosses and 8 Hz represented by circles.