CENTRE FOR ECONOMIC RESEARCH

WORKING PAPER SERIES

2003

Auction Versus Private Treaty

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WP03/06

February 2003

DEPARTMENT OF ECONOMICS
UNIVERSITY COLLEGE DUBLIN
BELFIELD DUBLIN 4

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February 14, 2003

1 Introduction

Selling not standardized goods such as jewelry, works of art and houses, by English auction is a revenue maximizing choice. The superiority of auctions in terms of maximizing a seller's revenue has been largely acknowledged in the literature. One reason for this superiority is that, in quite general environments, auctions, such as the English auction, are efficient. They guarantee that the item on sale will end up in the hands of the one who values it the most. Moreover the auction price (i.e. the seller's revenue) is such that no other bidder would be willing to pay as much or more. Thus, in the presence of serious buyers, using an efficient auctions is a rational choice. Whether this remains true when a seller may fail to gather any serious buyer is not clear. Allowing for some inefficient allocations, by potentially selling to bidders having lower values, is one way for the seller to attract some buyers. Thus it is one way to reduce the risk of not selling at all. Most of the literature considers monopolistic sellers who face some demand. Thus the consequences of a substantial risk of no-sale has received little attention. This paper attempts to fill this gap by considering competing sellers offering horizontally differentiated products. In particular, the paper offers a rational explanation for resorting to inefficient allocations under market share uncertainty.

This paper was motivated by observing the housing market in Dublin (Ireland). High quality houses in Dublin are generally sold by English auction. Towards the end of the year 2000 a sudden change was observed. Auctions were abandoned and private treaty was used more and more often. The reason proposed by a national newspaper was the following. The sharp increase in the price of high quality houses led many owners to sell their property but at the same time reduced the number of potential buyers. In such an environment, auctions were performing particularly poorly at achieving a sale. From a theoretical point of view, it appeared that the fear of not selling led sellers to abandon an efficient mechanism for one (private treaty) offering the possibility to meet and deal with a possibly reduced number of interested buyers: the first ones who come to view the house.

The literature considering competing sellers (see McAfee (1993), Peters (1997)) departs very much from the issue we will address here. Such papers analyze the properties of decentralized competition in an auction context. They examine the convergence of the reserve prices in second price auctions which reflect prices in a Walrasian equilibrium. Thus, this literature is not related with the issue we intend to understand.

In his empirical study focusing on the housing market Lusht (1996) shows that, in an active market, auctions extract higher prices than private negotiations. However, he acknowledges the fact that he considers an active market. In the conclusion, he mentions the risk of no sale and suggests that this risk could explain the use of private negotiation. In the early literature allowing for endogenous entry, the importance of guaranteeing fruition is also mentioned in Harstad (1990). In his paper, the author compares the revenue from different types of common value auctions under free entry. Under free entry, gathering more bidders can lower a seller's revenue. Indeed, an auction attracting fewer participants gives each of them a greater probability of winning. Thus, in equilibrium each participant will settle for a lower expected profit upon winning. This means that, provided the seller attracts some interested bidders, less can be better than more. This remains true provided selling is sufficiently likely. Harstad (1990) considers a situation where the risk of not selling at all sets a constraint on the optimal number of bidders. The situation considered in this paper is one in which more bidders is always better than less. The risk of not selling affects the extend to which a seller may want to preserve efficiency.

The trade-off between efficient allocation and selling at all is at the center of Gilbert and Klemperer (2000). Using a theoretical model they prove that the fear of no sale can lead a monopoly to ration demand. In their model, demand is subject to buyers undertaking some initial investment and subject to this investment being successful. They show that a seller can benefit from committing to a fixed price, potentially lower than the market clearing price, provided this price leads both buyers to invest. In an auction environment their result suggests that a seller can increase his revenue by allowing for inefficient allocations if doing so increases entry. In what follows we consider competing sellers instead of a monopoly. Competition brings to light some interesting results that a monopoly setting fails to capture. First it highlights the key role of the degree of product differentiation. This parameter determines what should be the primary interest for the seller: selling at all or extracting rents. Second it shows how competition can lead all sellers to resort to inefficient allocations. In such a situation the potential benefits from inefficient allocations vanish but not their cost.

The model considered is very simple. I consider a situation where 2 sellers, each with an item to sell, face 2 potentially interested buyers. The items on sale are horizontally differentiated. Under this assumption, each buyer shows a preference for one item over the other. This is modelled using the traditional Hotelling model where the 2 sellers are located at the extremity of a line of symbolic length equal to 1. The 2 buyers are located between the 2 sellers. A buyer located closer to seller 1 has a higher willingness to pay for seller 1's item. The position of a buyer is private information. The game analyzed is sequential. First, both sellers decide simultaneously and non-cooperatively on which mechanism to use and announce it. Second, the buyers decide simultaneously and non-cooperatively on which seller to attend. We assume that there is only one period and that a buyer can attend one seller at most. We restrict attention to 2 mechanisms only referred to as auction and private treaty. The auction is assumed to be an English auction. Private treaty is modeled as a potentially inefficient auction. With probability α all interested buyers will submit an offer which are then confronted until one drops out, in which case the outcome is the same as the auction. With probability $(1-\alpha)$ the seller deals with at most one buyer to whom he sells the house for the lowest acceptable price. This buyer is potentially the one who values the good the least. Thus, private treaty is potentially inefficient. If a seller has no interested buyer in his market he must keep the good which is of no value to him. I do not incorporate here some cost of using any of the 2 mechanisms. The reason is that I intend to find determinant for inefficient allocations that are not cost-related. In Dublin, when the sellers abandoned auction, there was no increase in the relative cost of one mechanism. It did not become more expensive to auction relative to using private treaty. Thus, the reason for the change lies somewhere else.

The results are rather intuitive and interesting. First, the degree of product differentiation intervenes in an unusual and crucial way. In a traditional Hotelling model, the more differentiated the products are the more reluctant the buyers are to purchase their less favored item. This reduced mobility of buyers enables to sellers to exert some monopoly power. Using this logical argument, we could expect that a lower mobility enables the seller to use the efficient auction mechanism. Interestingly, in what follows, the market share (and thus mobility) is independent of product differentiation. Thus, the reasoning is slightly different. As we shall see that the degree of product differentiation determines what should be the primary preoccupation of the seller: market share or rent extraction. As intuition suggests, when products are more homogeneous, market share should come first. By opposition, as the degree of product differentiation increases, rent extraction is the priority. In equilibrium we then observe the following. For low degrees of product differentiation, there are 2 possible equilibrium configurations. Either private treaty is the only dominant strategy equilibrium or the two symmetric equilibria in which auction or private treaty are used by both buyers co-exist. In any case, the best reply to private treaty is never auction. This is so because auction would decrease a seller's market share when his opponent is allowing for inefficiency. And when the degree of product differentiation is low, a seller never wants to decrease his market share. For higher degrees of product differentiation, there are again 2 possible equilibrium configurations. Either auction forms a dominant strategy equilibrium or the two symmetric equilibria in which auction or private treaty are used by both buyers co-exist. In any case, the best reply to auction is never private treaty. This is so because private treaty would increase a seller's market share when his opponent is using auction at the expense of efficiency. And when the degree of product differentiation is high enough, a seller never wants to extend his market share if it means losing on efficiency. When the consumers are evenly spread over the market, the equilibrium in which private treaty is played forms a prisoner's dilemma. Indeed, as the sellers both use the same mechanism (whether it is auction or private treaty) they share the market in halves. Thus, they both lose on efficiency when they both use private treaty as opposed to both using auction. By opposition, the dominant strategy equilibrium in which auction is adopted is Pareto efficient. Finally the parameter α plays a role for intermediate values of the degree of product differentiation. The two symmetric equilibria arise for lower values of α. In one case, when product differentiation is low enough, it is because the best reply to auction ceases to be private treaty when the latter is too inefficient. In the other case, when product differentiation is more important, it is because the best reply to private treaty ceases to be auction when the latter induces a too large market share loss.

The following section describes the model. The game is then solved by backwards induction to identify the subgame perfect equilibria. We first consider the buyers whose decisions are perfectly anticipated by the sellers. Finally, before we conclude, we look at all market equilibria and analyze their property.

2 The model

Consider a market with 2 consumers and 2 sellers (1 and 2). Each seller has a single item on sale (e.g. a house). These items are horizontally differentiated. Consumers are characterized by their taste ($\theta \in [0,1]$) which gives a measure for their willingness to pay for each item. A consumer with taste θ is willing to pay $v_1(\theta)$ and $v_2(\theta)$ for seller 1 and seller 2 item respectively. Assume

$$v_1(\theta) = v - t\theta,$$

$$v_2(\theta) = v - t(1-\theta),$$

where v and t are positive. The variable t is a measure of product differentiation. It is common knowledge. Graphically, this situation can be represented as a Hotelling situation with sellers located at the extremities of a line of length 1. We assume that the lowest valuation for each item $(v-t)$ is, in this model, the highest price that guarantees a sale. We will then assume that $(v-t)$ is the seller's reservation price..

The seller can decide on how to sell their item. Two possibilities will be considered. First he can hold an English auction. To simplify we consider the following rule. The seller starts with a price equal to $(v-t)$. Then (if there is more than a single buyer) the seller raises the price. Buyers can stay in or drop out as the price raises. If they drop out, they give up the possibility of trading. The price raises until a single buyer remains. He gets the item and pays the price at which his last competitor dropped out. If the seller has committed to use an auction and if there is a single buyer initially, this buyer is the winner and pays $(v-t)$.

As an alternative, the seller can choose to use a private treaty. We will model this using a parameter $\alpha \in [0,1]$. In practice, one can interpret α as the probability that the house remains on sale long enough *after* the first visit (if any) for any other interested buyer to see it. (This parameter could also refer to the seller's possibility of being patient.) In a context with 2 buyers, the following happens. With probability α all (at most both) interested buyers will make an offer. The seller will confront their offers until one drops out. With probability $(1-\alpha)$ the item will be sold to the first buyer to show up (if any). We will assume that if both are interested, the closest consumer meets the seller first with probability $\beta \geq \frac{1}{2}$.

Each buyer privately observes his own taste. However, it is common knowledge that tastes are i.i.d. according to a distribution function $F(\theta)$ defined over $[0,1]$. Let $F(.)$ be differentiable and let $f(\theta)$ denote the density function. When attending the auction, the buyers are able to assess how many competitors they face (although they do not need this information to know at what price they should drop out). We finally make the assumption that α and β are common knowledge.

In what follows I want to abstract from cost related explanations and will therefore not include the cost of either mechanisms in the analysis. We also do not consider that a seller benefits from some reputation by considering equal transport costs.

The timing of the game is the following. First, Nature draws each buyer's taste (refer to as a type). Second, both sellers simultaneously announce what selling mechanism they intend to use. Third, the buyers decide which seller to deal with. To simplify I will assume that a buyer can only deal with 1 of the 2 sellers. Finally, trade (if any) takes place.

3 The buyers' game

A buyer attending an auction will have to calculate a bid unless he is alone. Given the auction's rule, it is trivial to show that a dominant strategy for the buyer consists in dropping out whenever the price reaches his true valuation (see Milgrom (1989)). This decision maximizes forms a dominant strategy equilibrium. If the buyer is alone he will pay $(v-t)$ for the item and has no strategic decision to take. A buyer attending a private treaty will follow a similar strategy if needed. With probability $(1-\alpha)$ the item is sold to the buyer coming first. In that case, the first to see it should propose a price equal to $(v-t)$ as it is the lowest acceptable price. He will not be asked to increase it and cannot improve his gain. With probability α the private treaty mechanism is similar to an auction. If there is only one interested buyer he will just propose a price equal to $(v - t)$ which will be accepted and paid. If both buyers are interested, offers will raise until they reach the lowest valuation. The buyer having the highest valuation will get the house for that price.

Consider now the decision on which seller to attend. Let $S = (s_1, s_2)$ denote the strategies used by seller 1 and 2 respectively. We have $S \in \{(A, A), (A, P), (P, A), (P, P)\}$ where A stands for auction and P for private treaty.

Proposition 1: *For all possible sellers' strategies, there will always exist a treshold value $\theta_S^* \in [0, 1]$ such that the following strategy forms a symmetric Nash equilibrium: all buyers with a valuation $\theta \leq \theta_S^*$ deal with seller 1, while all buyers with a valuation $\theta \geq \theta_S^*$ deal with seller 2.*

Proof: see Appendix 1.

The variable θ_S^* is crucial as it determines each seller's market share.

Lemma 1: In the model considered, the equilibrium market shares only depend on the variables α and β. In particular, they are independent of the degree of product differentiation[1]. (Proof: see Appendix 1)

The main assumption triggering this result is that minimum acceptable price is $(v-t)$. Under this assumption, a buyer' surplus is always proportional to t. Moreover, we did not allow for any reputation effect by which one seller would benefit from a lower transport cost. This result is interesting. In the traditional Hotelling model, as products become more different, consumers are more reluctant to attend suppliers further away. This gives their closest supplier more monopoly power and enables him to charge higher prices. In what follows, although product differentiation plays a crucial role it is not via this traditional argument. Instead market share depends only on which mechanisms are offered and on the variables α and β.

Lemma 2: At $\alpha = 1$ and/or $\beta = 1$, $\theta_{AA}^* = \theta_{PP}^*$.

Lemma 3: Under strategies (A, P) and (P, A), the market share to the seller using private treaty has the following properties:

(i) It decreases with α, and equals θ_{AA}^* at $\alpha = 1$.

(ii) It decreases with β, and equals θ_{AA}^* at $\beta = 1$.

[1]This also holds under quadratic transport costs.

When $\alpha = 1$, private treaty is the same as auction since all interested buyers get to confront their offers. When $\beta = 1$, the buyer with the highest valuation will be the one getting the item, just as in the auction process. Given those two points, lemma 2 is obvious. Lemma 3 highlights a key feature of this analysis. It shows that a seller can benefit from an inefficient sale. Indeed, as a seller allows the lower valuation buyer to get the item, he increases his market share. As we shall see, the gain in market share can, in some instances, more than compensate the loss in efficiency.

Lemma 4: If $F(1-\theta) = 1 - F(\theta)\ \forall\theta$, we have:

$$\theta^*_{AA} = \theta^*_{PP} = \frac{1}{2}$$

and

$$\theta^*_{PA} = 1 - \theta^*_{AP}.$$

The sellers share the market in halves when both use the same mechanism when the median is situated at $\frac{1}{2}$. In general, market shares depend on how the population is spread. Thus, market equilibria are affected by the types' distribution. In what follows we will focus at a market where consumers are evenly spread.

4 The sellers' game

The game being symmetric, we can focus at seller 1 only. Seller 1's profit can always be written as an expression which is proportional to v. Therefore there is no loss in generalities to consider the profit as a function of T, where $T = \frac{t}{v}$ and $T \in [0, 1]$ by construction. Define the function $\Pi(\theta, \alpha)$ as:

$$\Pi(\theta, \alpha) = a(\theta)(1 - T) + \alpha T b(\theta), \tag{1}$$

where

$$a(\theta) = F(\theta)(2 - F(\theta)) \tag{2}$$

and

$$b(\theta) = (F(\theta))^2 \left[(1 - \theta) + \int_0^\theta \left(\frac{F.(x)}{F(\theta)} \right)^2 dx \right]. \tag{3}$$

Let $\pi^S(s_1, s_1)$ denote the seller's profits under strategy (s_1, s_2). We have:

$$\pi^S(A, s_2) = v.\Pi\left(\theta^*_{As_2}, 1\right),$$

and

$$\pi^S(P, s_2) = v.\Pi\left(\theta^*_{Ps_2}, \alpha\right).$$

Note that the parameter β, reflecting the probability of dealing with the highest type under private treaty only matters indirectly to the seller. This is so because when the seller deals with the first buyer he meets, the price equals $(v - t)$ whoever gets the item. In a more general model, where the seller potentially sells the object to the highest bidder within a subset of all interested buyers, such a parameter would matter.

The profit's expression can be understood as follows. The expression $a(\theta)$ is the probability of fruition. It is the probability that seller 1 will face *at least* one buyer. If this is the case, she will get the minimum acceptable price for sure. Let us now turn to $b(\theta)$. The expression $(F(\theta^*_S))^2$ is the probability with which both buyers attend seller 1. Under auction, this would allow her to get rents in addition to the minimum acceptable price. Those rents can be extracted under private treaty only when all interested buyers confront their offers. These rents are given by the expression in brackets in (3). Note that if the items for sale are identical $(T = 0)$ the sellers get no informational rents. Thus some degree of product differentiation is needed for the seller to extract rents.

Lemma 5: Expression (1) shows that the degree of product differentiation of product differentiation determines whether the seller should care about selling at all or about extracting rents. As T converges to $T = 1$, guaranteeing fruition becomes less important. Instead, the seller should focus on extracting rents.

This lemma is just an observation. However it is of critical importance to understand the results that follow. Before we search for the market equilibria, note that this game is symmetric. Thus there will always exist at least one equilibrium in pure strategies. Unfortunately, the comparison of the seller's revenue under different strategies is, in general, complicated. Therefore we will analyze the situation under the following assumptions.

- **Assumption 1**: $\forall\theta$, $F(1-\theta) = 1 - F(\theta)$.

Thus as stated in Lemma 4 we have: $\theta^*_{AA} = \theta^*_{PP} = \dfrac{1}{2}$and

$$\theta^*_{PA} = 1 - \theta^*_{AP}.$$

- **Assumption 2**: Let $\theta_\alpha \equiv \theta_{PA}$ and $\widehat{\theta}_\alpha \equiv \theta_{AP}$. Assume that the distribution function is such that the functions $g_A(\alpha)$ defined as

$$g_A(\alpha) = \frac{a(\theta_\alpha) - a(\theta_1)}{a(\theta_\alpha) - a(\theta_1) + b(\theta_1) - \alpha b(\theta_\alpha)}$$

is increasing in α, and such that the function $g_P(\alpha)$ defined as

$$g_P(\alpha) = \frac{a(\widehat{\theta}_1) - a(\widehat{\theta}_\alpha)}{a(\widehat{\theta}_1) - a(\widehat{\theta}_\alpha) + b(\widehat{\theta}_\alpha) - \alpha b(\widehat{\theta}_1)}$$

is decreasing in α.

This assumption holds in particular under the uniform distribution. It states the following. Let T^A_α (resp. T^P_α) denote the degree of product differentiation such that seller 1 is indifferent between auction and private treaty when seller 2 uses auction (resp. private treaty). Under assumption 2, T^A_α is increasing in α, while T^P_α is decreasing in α. Thus, assumption is similar to a monotonicity condition on the degree of product differentiation.

4.1 Best reply to auction

Assume that seller 2 is using an Auction. If seller 1 uses the same mechanism, they share the market. If he switches to private treaty, he loses on efficiency but gains market share. The best reply to auction will depend on the degree of product differentiation and the efficiency loss measured via α.

Proposition 2: Under assumption 2, there exist $T_{1A} \in]0, 1[$ *and* $T_{2A} \in]0, 1[$ *with* $T_{1A} < T_{2A}$*, and* $\alpha_A \in]0, 1[$ *such that the best reply to auction is:*

-Private treaty for all α*, when* $T < T_{1A}$

-Auction for all α *when* $T > T_{2A}$

-Auction for $\alpha < \alpha_A$ *and Private treaty for* $\alpha \geq \alpha_A$ *when* $T \in [T_{1A}, T_{2A}]$.

Proof: See Appendix 2.

The graphical representation represents the result described in proposition 2.

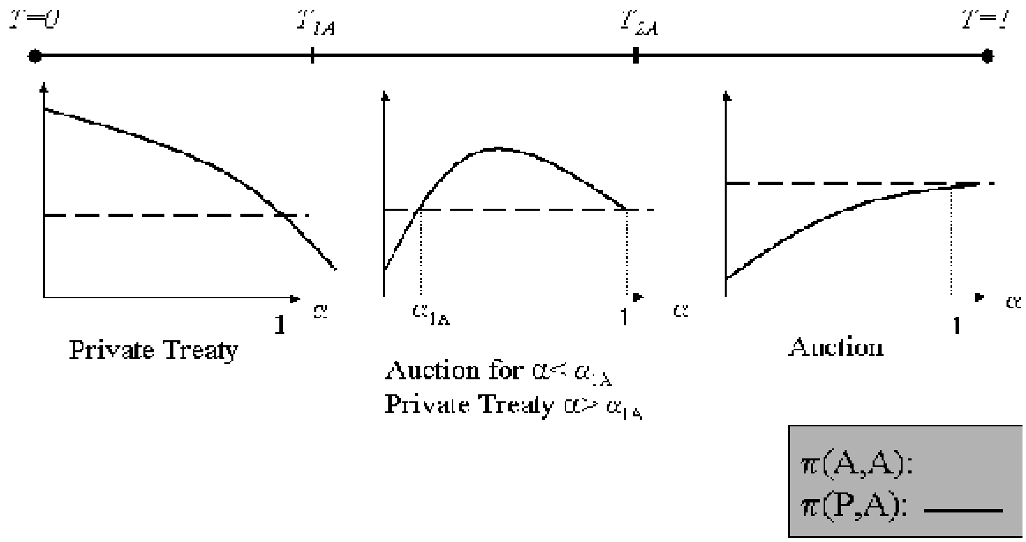

Figure 1: Best reply to auction.

When products are sufficiently different, it never pays to increase market share. It only does as products become closer substitutes. The parameter α plays a crucial role when $T \in [T_{1A}, T_{2A}]$. In that case, private treaty is a best reply to auction only when not too inefficient. That is when the gain in market share is not dissipated by the inefficiency.

4.2 Best Reply to Private Treaty

Assume seller 2 is using private treaty. If seller 1 uses the same mechanism, they share the market. If seller 1 switches to auction, he will lose market share. Once again, the best reply to private treaty depends on T and on α.

Proposition 3: Under assumption 2, there exist $T_{1P} \in]0,1[$ and $T_{2P} \in]0,1[$ with $T_{1P} < T_{2P}$, and $\alpha_P \in]0,1[$ such that the best reply to private treaty is:

-Private treaty for all α, when $T < T_{1P}$

-Auction for all α when $T > T_{2P}$

-Private treaty for $\alpha < \alpha_P$ and auction for $\alpha \geq \alpha_P$ when $T \in [T_{1P}, T_{2P}]$.

Proof: See Appendix 3.

The following graphs represent the best replies as T increases.

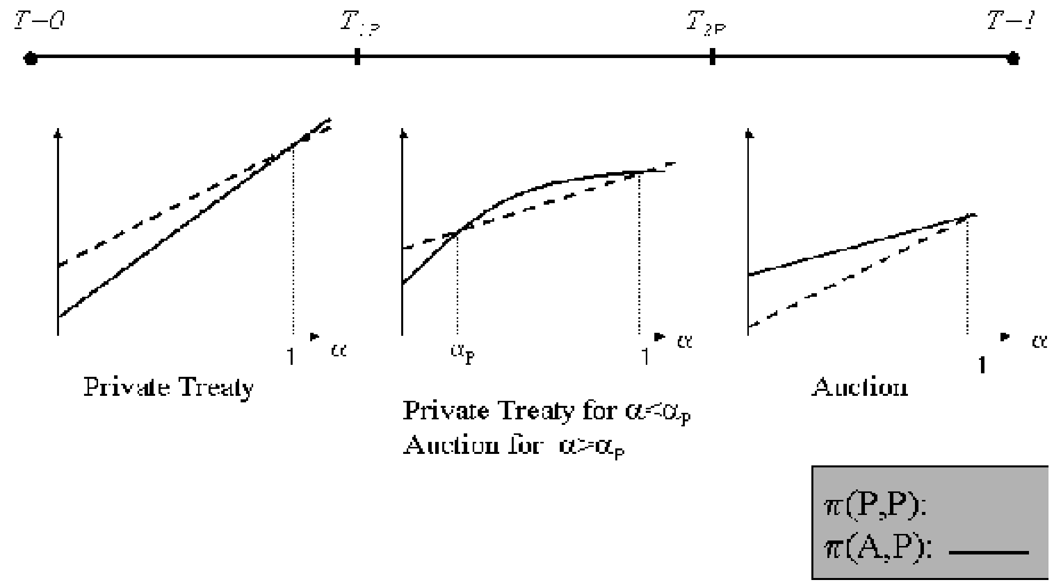

Figure 2: Best reply to private treaty.

Once again, when products are sufficiently different, gaining efficiency compensates the loss in market share. The role of α is now slightly different. When $T \in [T_{1B}, T_{2B}]$, the seller chooses private treaty for low values of α, eventhough it is quite inefficient. When α is low, the market share from auction is not large enough to outweight the efficiency gain. In other words, auction becomes a best reply to private treaty for $T \in [T_{1B}, T_{2B}]$, provided the market it generates is wide enough.

5 Market equilibria

Under assumption 1 we have $T_{2A} = T_{1B}$ (see Appendix 4). Let $T^* = T_{2A} = T_{1B}$. Putting together all best replies leads us to the following result.

Proposition 4: The market equilibria depend on α and T. They are such that:

-Auction is a dominant strategy equilibrium either for $T > T_{2B}$, or for $T \in [T^*, T_{2B}]$ provided $\alpha > \alpha_P$.

-Private treaty is a dominant strategy equilibrium either for $T < T_{1A}$, or for $T \in [T_{1A}, T^*]$ provided $\alpha > \alpha_A$.

-The symmetric equilibria (A, A) and (P, P) form a Nash equilibrium either for $T \in [T_{1A}, T^*]$ provided $\alpha \leq \alpha_A$, or for $T \in [T^*, T_{2B}]$ provided $\alpha \leq \alpha_P$. (The

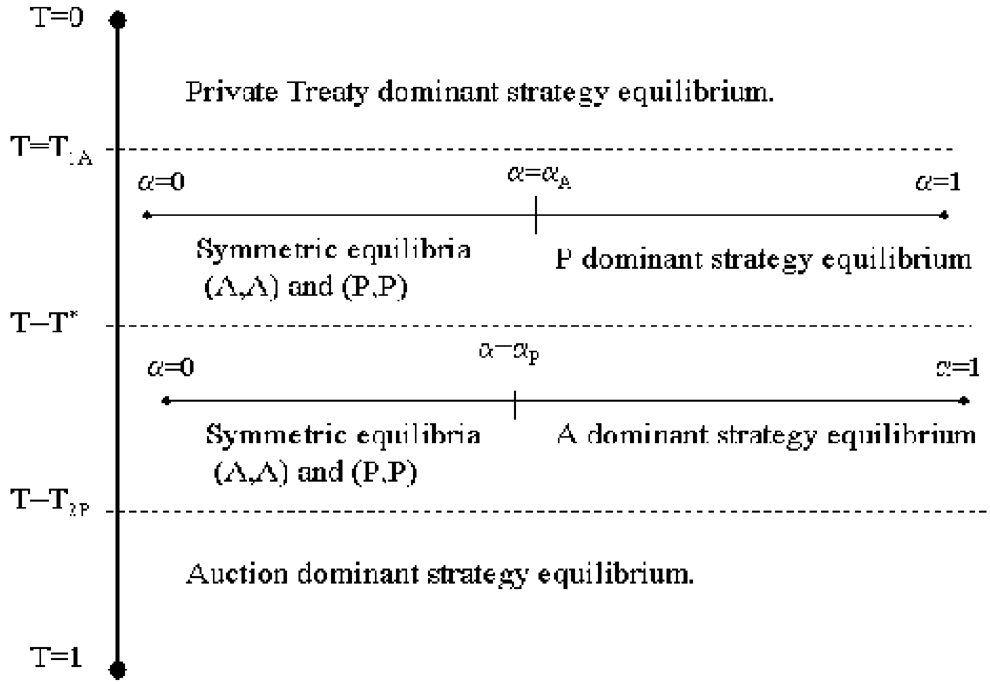

Figure 3: Market equilibria.

proof is trivial.)

The following graph gives a clearer idea of the equilibria.

Those results can be interpreted as follows. For $T < T^*$, that is when products are rather homogeneous, there are two possible configurations of equilibria. Either private treaty forms a dominant strategy equilibrium or, the two symmetric equilibria co-exist. In any case, auction is never a best reply to private treaty. In other words, when $T < T^*$, the seller is never willing to give up market share. This aversion to lose market share can drive both sellers to resort to inefficient allocations. In such a case, they would share the market in halve just as they would under (A, A), and both lose. This is summarized in the following lemma.

Lemma 6: The equilibrium payoff at (P, P) is dominated by the equilibrium payoff at (A, A).

Proof: By construction we have $\pi^S(P, P) \leq \pi^S(A, A)$ for all α, with equality at $\alpha = 1$ only.

Whenever (P, P) forms a dominant strategy equilibrium, the sellers reach a so-called prisoner's dilemma. Competition leads them to use private treaty no matter what their opponent does while the sellers would be better-off under joint-profit maximization.

When $T > T^*$, that is when the degree of product differentiation is large enough, a symmetric situation arises. In that case, there are two possible configurations of equilibria. The 2 symmetric equilibria or auction as a dominant strategy equilibrium. In any case, the best reply to auction is never private treaty. The seller is no longer interested in extending his market share at the expense of efficiency. Because products are more different, competition is somehow relaxed, and auction can arise in equilibrium.

Lemma 7: The equilibrium payoff at (A, A) is Pareto dominant. (The proof is trivial.)

Indeed, whenever (A, A) is played in equilibrium, the sellers reach a situation where they maximize their revenue.

6 Conclusion

This papers formalizes the following simple idea: as sellers become more concerned about guaranteeing a sale, they are willing to give up on efficiency to potentially gain market share. This may lead them to an inefficient outcome. As intuition suggests, whether they are willing to do so depend on how differentiated the products they sell are. It also depends on the degree of inefficiency they will have to tolerate to increase their market share.

From a theoretical point of view, it would be interesting to consider a model where a competing sellers could chose between auctioning to n interested buyers, versus auctioning to a subset of interested buyers that need not include the ones with the highest valuations.

7 Appendix

- **Appendix 1**: Proof of lemma 1.

We need to show that if a buyer acts as depicted, it is in his opponent's interest to adopt the same strategy. Assume the value θ_S^* exists and consider all possible choices of selling mechanisms.

Let $S=(A,A)$. Consider a buyer with type $\theta\in[0,1]$. Let $\pi_{1A}^{B}(\theta)$ denote this buyer's surplus upon attending seller 1's auction. Given his opponent's strategy, we have:

$$\pi_{1A}^{B}(\theta)=\begin{cases}\int_{\theta}^{\theta_{AA}^{*}}t\left(\widetilde{\theta}-\theta\right)f\left(\widetilde{\theta}\right)d\widetilde{\theta}+\int_{\theta_{AA}^{*}}^{1}t\left(1-\theta\right)f\left(\widetilde{\theta}\right)d\widetilde{\theta}\text{ if }\theta\leq\theta_{AA}^{*}\\ \int_{\theta_{AA}^{*}}^{1}t\left(1-\theta\right)f\left(\widetilde{\theta}\right)d\widetilde{\theta}\text{ if }\theta\geq\theta_{AA}^{*}\end{cases}\tag{4}$$

where $\widetilde{\theta}$ denotes his opponent's valuation. Let $\pi_{2A}^{B}(\theta)$ denote this buyer's surplus upon attending seller 2's auction:

$$\pi_{2A}^{B}(\theta)=\begin{cases}\int_{0}^{\theta_{AA}^{*}}t\theta f\left(\widetilde{\theta}\right)d\widetilde{\theta}\text{ if }\theta\leq\theta_{AA}^{*}\\ \int_{0}^{\theta_{AA}^{*}}t\theta f\left(\widetilde{\theta}\right)d\widetilde{\theta}+\int_{\theta_{AA}^{*}}^{\theta}t\left(\theta-\widetilde{\theta}\right)f\left(\widetilde{\theta}\right)d\widetilde{\theta}\text{ if }\theta\geq\theta_{AA}^{*}\end{cases}\tag{5}$$

Assume that $\theta_{AA}^{*}\in\,]0,1[$. Under this assumption, we have $\dfrac{d\pi_{1A}^{B}}{d\theta}<0$ and $\dfrac{d\pi_{2A}^{B}}{d\theta}>0$. Thus, the function $\pi_{1A}^{B}(\theta)-\pi_{2A}^{B}(\theta)$ is decreasing.

$$\begin{aligned}\text{At }\theta&=0:\pi_{1A}^{B}(0)-\pi_{2A}^{B}(0)=\pi_{1A}^{B}(0)>0\\ \text{At }\theta&=1:\pi_{1A}^{B}(1)-\pi_{2A}^{B}(1)=-\pi_{2A}^{B}(1)<0.\end{aligned}$$

There is therefore a single value θ_{AA}^{*} such that $\pi_{1A}^{B}(\theta_{AA}^{*})=\pi_{2A}^{B}(\theta_{AA}^{*})$. Solving, we get:

$$\theta_{AA}^{*}=1-F\left(\theta_{AA}^{*}\right).\tag{6}$$

From (6) we have obviously $\theta_{AA}^{*}\in\,]0,1[$. Finally, all buyers such that $\theta<\theta_{AA}^{*}$ (resp. such that $\theta>\theta_{AA}^{*}$) are such that $\pi_{1A}^{B}(\theta)>\pi_{2A}^{B}(\theta)$ (resp. $\pi_{1A}^{B}(\theta)<\pi_{2A}^{B}(\theta)$) and will therefore attend seller 1 (resp. seller 2).

Let $S=(P,A)$. Consider a buyer with type $\theta\in[0,1]$. Let $\pi_{1P}^{B}(\theta)$ denote this buyer's surplus upon attending seller 1's private treaty process. Given his opponent's strategy, we have:

$$\begin{aligned}\pi_{1P}^{B}(\theta)&=\alpha\int_{\theta}^{\theta_{PA}^{*}}t\left(\widetilde{\theta}-\theta\right)f\left(\widetilde{\theta}\right)d\widetilde{\theta}+\\&\quad t(1-\theta)\left[1-F\left(\theta_{PA}^{*}\right)+(1-\alpha)\left(\beta(F(\theta_{PA}^{*})-F(\theta))+(1-\beta)F(\theta)\right)\right]\end{aligned}$$

if $\theta\leq\theta_{PA}^{*}$,

$$\pi_{1P}^{B}(\theta)=t\left(1-\theta\right)\left[1-F(\theta_{PA}^{*})+F(\theta_{PA}^{*})(1-\alpha)(1-\beta)\right]$$

if $\theta \geq \theta^*_{PA}$.

If this buyer attends seller 2's auction his surplus will be given by expression (5) where we should replace θ^*_{AA} by θ^*_{PA}.

Assume that θ^*_{PA} is an interior solution. Under this assumption we have $\frac{d\pi^B_{1P}(\theta)}{d\theta} < 0$ given that $\beta \geq 1/2$. Thus, the function $\pi^B_{1P}(\theta) - \pi^B_{2A}(\theta)$ is decreasing, with $\pi^B_{1P}(0) - \pi^B_{2A}(0) > 0$ and $\pi^B_{1P}(1) - \pi^B_{2A}(1) < 0$. There is therefore a single value θ^*_{PA} such that $\pi^B_{1P}(\theta^*_{PA}) = \pi^B_{2A}(\theta^*_{PA})$. Solving, we have

$$\theta^*_{PA}F(\theta^*_{PA}) = (1-\theta^*_{PA})\left[1 - F(\theta^*_{PA}) + (1-\alpha)(1-\beta)F(\theta^*_{PA})\right]. \tag{7}$$

From the above, one can easily check that θ^*_{PA} is indeed an interior solution. Finally, it is true that all buyers such that $\theta < \theta^*_{PA}$ (respectively such that $\theta < \theta^*_{PA}$) will attend seller 1 (resp. seller 2).

Let $S = (A, P)$. The surplus to a buyer attending seller 1 is given by expression (4) where we should replace θ^*_{AA} by θ^*_{AP}. Let $\pi^B_{2P}(\theta)$ denote the surplus to a type θ buyer when attending seller 2's private treaty. We have:

$$\pi^B_{2P}(\theta) = t\theta\left[F(\theta^*_{AP}) + (1-\alpha)(1-F(\theta^*_{AP}))(1-\beta)\right]$$

if $\theta \leq \theta^*_{AP}$,

$$\begin{aligned}\pi^B_{2P}(\theta) &= \alpha\int_{\theta^*_{AP}}^{\theta} t\left(\theta - \widetilde{\theta}\right) f\left(\widetilde{\theta}\right) d\widetilde{\theta} \\ &\quad + (t\theta)\left[F(\theta^*_{AP}) + (1-\alpha)\left(\beta\left(F(\theta) - F(\theta^*_{AP})\right) + (1-\beta)(1-F(\theta))\right)\right]\end{aligned}$$

if $\theta \geq \theta^*_{AP}$.

Once again, if we assume that θ^*_{AP} is an interior solution then $\frac{d\pi^B_{2P}(\theta)}{d\theta} > 0$. Thus, $\pi^B_{1A}(\theta) - \pi^B_{2P}(\theta)$ is decreasing in θ. The indifferent type solves:

$$(1-\theta^*_{AP})(1-F(\theta^*_{AP})) = \theta^*_{AP}\left[F(\theta^*_{AP}) + (1-\alpha)(1-\beta)(1-F(\theta^*_{AP}))\right]. \tag{8}$$

One can easily check that there exists a single interior solution to the above equation. And the way buyers select which seller to go to follows the same logical argument as above.

Let $S = (P, P)$. Expressions (**??**) and (**??**) where we replace θ^*_{PA} and θ^*_{AP} by

θ^*_{PP}, give the surplus to a type θ buyer when attending seller 1 and 2 as both use private treaty. Once again if we assume that θ^*_{PP} is an interior solution, the function $\pi^B_{1P}(\theta) - \pi^B_{2P}(\theta)$ is decreasing. We have $\pi^B_{1P}(0) - \pi^B_{2P}(0) > 0$ and $\pi^B_{1P}(1) - \pi^B_{2P}(1) < 0$. The indifferent consumer is uniquely defined and such that

$$1 = F(\theta^*_{PP}))\left[1 - (1-\beta)(1-\alpha)\right] + \theta^*_{PP}\left[1 + (1-\beta)(1-\alpha)\right]$$

From the above one can verify that θ^*_{PP} is indeed an interior solution.

- **Appendix 2**: proof of proposition 2.

Let $\theta_\alpha \equiv \theta_{PA}$. Under assumption 1, we have $\theta_{AP} = 1 - \theta_\alpha$. Moreover, we know that θ_α is decreasing in α and $\theta_1 = \theta_{AA} = 1/2$. Assume that $\pi^S(P, A)$ is concave in α. Note that $\pi^S(A, A)$ is constant with respect to α.

Let T_{1A} be such that $\pi^S(P, A)\Big|_{\alpha=0} = \pi^S(A, A)$.

We have

$$\pi^S(P, A)\Big|_{\alpha=0} = a(\theta_0)(1-T).$$

Thus, T_{1A} solves

$$a(\theta_0)(1-T_{1A}) = a(\theta_1)(1-T_{1A}) + T_{1A}b(\theta_1)$$

leading to:

$$T_{1A} = \frac{a(\theta_0) - a(\theta_1)}{a(\theta_0) - a(\theta_1) + b(\theta_1)}. \tag{9}$$

Since $a(.)$ is an increasing function (see (**??**) in Appendix 2) and $\theta_0 > \theta_1 : a(\theta_0) - a(\theta_1) > 0$. Moreover, $b(\theta_1) > 0$. Thus the expression on the right side of (9) is always strictly within the interval $[0, 1]$. Since $\pi^S(P, A)\Big|_{\alpha=0}$ is decreasing in T, we have that $\pi^S(P, A)\Big|_{\alpha=0} > \pi^S(A, A)$ for $T < T_{1A}$ and $\pi^S(P, A)\Big|_{\alpha=0} < \pi^S(A, A)$ for $T > T_{1A}$.

Let T_{2A} be such that $\left.\frac{d\pi^S(P, A)}{d\alpha}\right|_{\alpha=1} = 0$. We have:

$$\left.\frac{d\pi^S(P, A)}{d\alpha}\right|_{\alpha=1} = \left[a'(\theta_1)(1-T) + Tb'(\theta_1)\right]\left.\frac{d\theta_\alpha}{d\alpha}\right|_{\alpha=1} + Tb(\theta_1) \tag{T2A}$$

where $a'(.) = \frac{da(\theta_\alpha)}{d\theta_\alpha}$ is given by expression (**??**) in Appendix 2 and $b'(.) = \frac{db(\theta_\alpha)}{d\theta_\alpha}$

is given by expression (**??**) in Appendix 2.

$$\text{At } T=0: \left.\frac{d\pi^S(P,A)}{d\alpha}\right|_{\alpha=1} = a'(\theta_1)\left.\frac{d\theta_\alpha}{d\alpha}\right|_{\alpha=1} < 0.$$

$$\text{At } T=1: \left.\frac{d\pi^S(P,A)}{d\alpha}\right|_{\alpha=1} = b'(\theta_1)\left.\frac{d\theta_\alpha}{d\alpha}\right|_{\alpha=1} + b(\theta_1) > \frac{1+\beta f(\theta_1)}{8(1+f(\theta_1))} > 0.$$

Moreover, $\left.\frac{d\pi^S(P,A)}{d\alpha}\right|_{\alpha=1}$ is increasing in T. Thus there exists a unique $T_{2A} \in]0,1[$ such that $\left.\frac{d\pi^S(P,A)}{d\alpha}\right|_{\alpha=1} = 0$. For all $T < T_{2A}$ $\left.\frac{d\pi^S(P,A)}{d\alpha}\right|_{\alpha=1} < 0$. and for $T > T_{2A}$ $\left.\frac{d\pi^S(P,A)}{d\alpha}\right|_{\alpha=1} > 0$.

Given the assumption 2 we have $T_{2A} > T_{1A}$. Indeed, by construction we have $T_{1A} = g_A(0)$. Moreover, $T_{2A} = \lim_{\alpha\to 1} g_A(\alpha)$. Since $g_A(\alpha)$ is increasing in α under assumption 2, we have $T_{1A} < T_{2A}$.

We can now summarize our findings in a table:

$T \in [0, T_{1A}]$	$T \in [T_{1A}, T_{2A}]$
$\left.\pi^S(P,A)\right\|_{\alpha=0} \geq \pi^S(A,A)$	$\left.\pi^S(P,A)\right\|_{\alpha=0} \leq \pi^S(A,A)$
$\left.\frac{d\pi^S(P,A)}{d\alpha}\right\|_{\alpha=1} < 0$	$\left.\frac{d\pi^S(P,A)}{d\alpha}\right\|_{\alpha=1} < 0$

$T \in [T_{2A}, 1]$
$\left.\pi^S(P,A)\right\|_{\alpha=0} < \pi^S(A,A)$
$\left.\frac{d\pi^S(P,A)}{d\alpha}\right\|_{\alpha=1} > 0$

- **Appendix 3**: Proof of proposition 3.

Let $\widehat{\theta}_\alpha \equiv \theta_{PA}$. By construction $\widehat{\theta}_\alpha$ is increasing in α and $\widehat{\theta}_1 = \theta_{PP} = 1/2$. Let T_{1P} be defined such that $\left.\frac{d\pi^S(A,P)}{d\alpha}\right|_{\alpha=1} = \left.\frac{d\pi^S(P,P)}{d\alpha}\right|_{\alpha=1}$. We have:

$$\left[a'\left(\widehat{\theta}_1\right)(1-T_{1P}) + Tb'\left(\widehat{\theta}_1\right)\right]\left.\frac{d\widehat{\theta}_\alpha}{d\alpha}\right|_{\alpha=1} = T_{1P}b\left(\widehat{\theta}_1\right).$$

Considering, (**??**) in Appendix 2, since it is true that $\widehat{\theta}_1 = \theta_1$ and $\frac{d\widehat{\theta}_\alpha}{d\alpha} = -\frac{d\theta_\alpha}{d\alpha}$, we obviously have $T_{2A} = T_{1P}$. Thus, there exist $T_{1P} \in]0,1[$ such that $\left.\frac{d\pi^S(A,P)}{d\alpha}\right|_{\alpha=1} =$

$\left.\frac{d\pi^S(P,P)}{d\alpha}\right|_{\alpha=1}$ at T_{1P}. For all $T < T_{1P}$, $\left.\frac{d\pi^S(A,P)}{d\alpha}\right|_{\alpha=1} < \left.\frac{d\pi^S(P,P)}{d\alpha}\right|_{\alpha=1}$. For all $T > T_{1P}$, $\left.\frac{d\pi^S(A,P)}{d\alpha}\right|_{\alpha=1} > \left.\frac{d\pi^S(P,P)}{d\alpha}\right|_{\alpha=1}$.

Let T_{2P} be such that $\left.\pi^S(A,P)\right|_{\alpha=0} = \left.\pi^S(P,P)\right|_{\alpha=0}$. Thus, T_{2P} solves:

$$a(\widehat{\theta}_0)(1-T_{2P}) + T_{2P}b(\widehat{\theta}_0) - a(\widehat{\theta}_1)(1-T_{2P}) = 0.$$

Let

$$F(T) = a(\widehat{\theta}_0)(1-T) + Tb(\widehat{\theta}_0) - a(\widehat{\theta}_1)(1-T).$$

We have $F(0) = a(\widehat{\theta}_0) - a(\widehat{\theta}_1) < 0$, and $F(1) = b(\widehat{\theta}_0) > 0$. Moreover, $\frac{dF(T)}{dT} = b(\widehat{\theta}_0) - \left[a(\widehat{\theta}_0) - a(\widehat{\theta}_1)\right] > 0$. Thus, there exists a unique $T = T_{2P}$ such that $F(T_{2P}) = 0$. For all $T < T_{2P}$, $\left.\pi^S(A,P)\right|_{\alpha=0} < \left.\pi^S(P,P)\right|_{\alpha=0}$. For all $T > T_{2P}$, $\left.\pi^S(A,P)\right|_{\alpha=0} > \left.\pi^S(P,P)\right|_{\alpha=0}$.

Under assumption 2, we necessarily have $T_{2P} > T_{1P}$. This is so because $T_{2P} = g_P(0)$ and $T_{1P} = \lim_{\alpha\to 1} g_P(\alpha)$. Since $g_P(.)$ is decreasing in α under assumption 2, we have $T_{2P} > T_{1P}$.

We can now summarize our findings in a table:

$T \in [0, T_{1P}]$	$T \in [T_{1P}, T_{2P}]$
$\left.\pi^S(A,P)\right\|_{\alpha=0} < \left.\pi^S(P,P)\right\|_{\alpha=0}$	$\left.\pi^S(A,P)\right\|_{\alpha=0} < \left.\pi^S(P,P)\right\|_{\alpha=0}$
$\left.\frac{d\pi^S(A,P)}{d\alpha}\right\|_{\alpha=1} < \left.\frac{d\pi^S(P,P)}{d\alpha}\right\|_{\alpha=1}$	$\left.\frac{d\pi^S(A,P)}{d\alpha}\right\|_{\alpha=1} > \left.\frac{d\pi^S(P,P)}{d\alpha}\right\|_{\alpha=1}$

$T \in [T_{2P}, 1]$
$\left.\pi^S(A,P)\right\|_{\alpha=0} > \left.\pi^S(P,P)\right\|_{\alpha=0}$
$\left.\frac{d\pi^S(A,P)}{d\alpha}\right\|_{\alpha=1} > \left.\frac{d\pi^S(P,P)}{d\alpha}\right\|_{\alpha=1}$

Under assumption 3, the only possible outcome from the above table is given in figure 2.

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