

CENTRE FOR ECONOMIC RESEARCH

WORKING PAPER SERIES

2005

The Invention of Invention

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WP05/15

September 2005

**DEPARTMENT OF ECONOMICS
UNIVERSITY COLLEGE DUBLIN
BELFIELD DUBLIN 4**

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Abstract

This paper models an industrial revolution as a qualitative transition from a world where innovation is infrequent and haphazard to one where it is continuous and systematic. Pre-industrial innovation is treated as a social process where an individual's effectiveness as an innovator depends on the skills of other individuals in his social network. As technology improves, individuals invest more time in learning through social contact. This gradual increase in linkage formation leads to a sudden change in the size of knowledge networks from small, isolated clusters, to a large connected cluster spanning most of the economy, causing a sudden increase in the effectiveness of innovation: an industrial revolution. The predicted sequence of typical innovators—from gifted amateurs, to lucky amateurs, to professionals—is consistent with empirical evidence.

JEL: O40. Keywords: Industrial revolution, social networks, innovation.

1 Introduction.

Innovation is a social process that consists in bringing together the knowledge and skills of different individuals to generate new knowledge and skills. For a contemporary firm undertaking R&D, these skills can be brought together by hiring researchers with appropriate training and experience. In a pre-industrial economy, with limited technological knowledge and education, this combining of skills has to occur primarily through personal contact, so that an individual's effectiveness as an innovator will depend on the skills of those in his social network. This paper shows how the gradual expansion of these networks of personal contact leads to a sudden rise in the knowledge and effectiveness of innovators, turning innovation from an occasional, haphazard activity into a continuous, systematic one.

*This paper is part of the International Trade and Investment Programme of the Geary Institute at UCD.

We model an economy where kitchen table inventors devote time to forming networks of social contacts in the hope of gaining useful knowledge, and show that, if individual effectiveness at innovating is supermodular in skills over the network, the most able individuals devote the most time to forming social contacts. Successful innovations increase the skill of innovators and cause the number of linkages between agents to grow slowly through time.

Our central result is that these knowledge networks undergo a sudden, qualitative change as the number of linkages between agents reaches a critical density. Below the critical density, the network is split into small, isolated clusters of individuals. As the critical density is reached these clusters coalesce into a large, connected cluster that spans most of the agents in the economy. Within a few generations, innovators go from having a small number of intellectual contacts, with a limited probability that their collective knowledge can generate some useful innovation; to having many contacts and a far greater probability of meeting someone whose knowledge will enable them to make a significant breakthrough.

Recent models of industrial revolutions, including [Galor and Weil \(1999\)](#), and [Jones \(2001\)](#), start from the result that that the growth rate in models where firms allocate labour to R&D is determined by the rate of population growth ([Kremer 1993b](#), [Jones 1995](#)), and model the acceleration of innovation through a demographic transition in a Malthusian model. In these models, pre-industrial innovation is qualitatively the same as contemporary innovation; there are simply fewer researchers engaged in R&D. [Hansen and Prescott \(1998\)](#) model an industrial revolution without innovation as a shift from a land intensive to a capital intensive technology. [Weitzman \(1998\)](#) develops a model of innovation as the combination of ideas but without the social component of linkages between individuals that it central here. Our approach to innovation as a social activity is most closely related to the literature on social interactions in economics surveyed by [Brock and Durlauf \(2001\)](#) and [Glaeser and Scheinkman \(2000\)](#). The network formation game here differs from existing models of network formation ([Jackson and Wolinsky, 1996](#); [Bala and Goyal, 2000](#)) in that agents have incomplete knowledge of the characteristics of their potential partners.

The rest of the paper is as follows. Section 2 discusses why social networks aid innovation. Sufficient conditions for the number of personal linkages between agents to grow through time are derived for a network formation game in Section 3, while Section 4 shows that gradual growth in linkages causes the sudden emergence of a large, connected knowledge network. Section 5 examines how

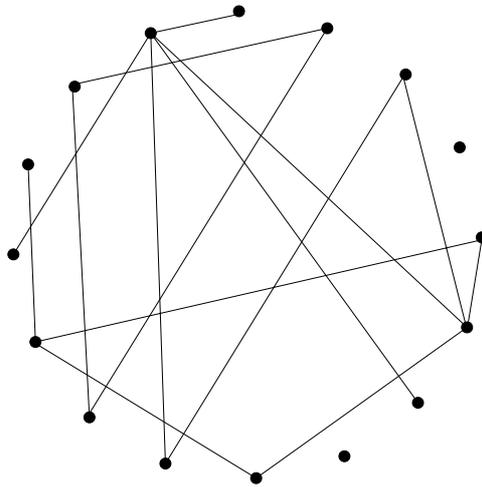


Figure 1: Social network: individuals are points, social contacts are lines.

the characteristics of innovators are predicted to change through time, and shows that these match the characteristics of American inventors during the nineteenth century.

2 Social Cognition.

Innovation is a social process; pre-industrial innovation particularly so. In a world with little technical knowledge and formal education, an individual's success as an inventor depends in large part on his access to the knowledge, skills, and ideas of other aspiring innovators. We model the social network of innovators as a random graph: potential innovators are points (vertices), and personal contacts between them are lines (edges). Edges form at random as individuals meet and choose to pool their knowledge.

The social networks here are undirected graphs: the transfer of knowledge goes both ways. This means that the inventors that compose the network are individuals of roughly similar ability, neither geniuses or botchers, who can benefit mutually from intellectual contact.

A social network has chains of acquaintance: individuals can know each other directly, or be indirectly connected through other acquaintances. Being indirectly connected to someone does not imply that he necessarily knows anything about you but it does give a chance for individuals with complementary interests and skills to

learn of each other and come together. In particular the higher the ability of an individual in a connected chain of acquaintance, the greater the probability that news of his skills will diffuse through the network and attract linked individuals with complementary interests and skills. Inventors that belong to separate networks, by contrast, have no possibility of learning of each other and do not benefit from each other's knowledge.

The importance of social networks can be seen in many accounts of historical innovations. For example, in examining why the mechanization of the cotton industry took place in Manchester rather than any of the other equally successful textile centres in eighteenth century England, [Hall \(1998, 310–347\)](#) concludes that the decisive factors were social: the presence of a relatively large middle class of small capitalist entrepreneurs engaged in textile production who, as Dissenters excluded from mainstream Anglican society, had close and relatively egalitarian social networks based on common churches, schools and the Manchester Scientific Society which, like other scientific societies served as a forum for intellectual networking, and a marketplace where knowledge was traded for patronage ([Mokyr, 2002, 44](#)). In this society that combined strong group ties with personal competitiveness, news of successful innovations could spread rapidly and serve as a foundation for further successful innovation.

2.1 What makes two heads better than one?

There are four benefits to be derived from intellectual contact with others: complementary knowledge, interaction of ideas, division of labour, and overcoming cognitive barriers; and one possible cost: conformity. We discuss each in turn.

No individual can know and do everything, and innovation will occur most readily where complementary skills are brought together. The mechanization of the cotton industry, for instance, required combining a knowledge of textile production with a knowledge of mechanical engineering which, in the case of Manchester, was present in the form of clock makers who were able to design and build parts for textile machines.

When individuals share similar knowledge and skills—the participants at an academic seminar, for instance—the interaction of ideas through challenge, justification and revision acts to clarify and improve thoughts ([Levine, Resnick and Higgins, 1993](#)). In his study of intellectual and personal networks among philosophers from ancient Greece until the mid-twentieth century, [Collins \(1998, 37–40\)](#) shows how important is group contact, both for new participants who can acquire the knowl-

edge of existing members, and for established members who can recharge their enthusiasm for their individual research.

When tasks require specific skills, there is an incentive to specialize in one task and pool one's knowledge with other specialists. In [Becker and Murphy \(1992\)](#), for example, output is the product of skill and time devoted to production, where skill is proportional to time invested in human capital formation. Output in each activity is therefore quadratic in time, giving an incentive to specialize in a few activities as part of a team of similar specialists.

Innovation is not merely about coming up with new ideas, but also about rejecting existing ideas that others accept unquestioningly. In overcoming personal cognitive barriers, contact with the ideas of others is vital. For example, [Margolis \(1987, 224–249\)](#) shows that the decisive turning point in Copernicus's thought occurred after seeing Waldseemüller's 1507 map of the world. This was the first map to show America as a separate continent, something irreconcilable with the Aristotelian view of Europe, Africa and Asia alone on a sphere surrounded by water, and surrounded in turn by the celestial spheres. Copernicus's successful rejection of geocentric cosmology led his followers in turn to reject other aspects of the deductive Aristotelian physics in favour of the empirical scientific method whose importance for subsequent technological development is shown by [Mokyr \(2002\)](#).

While individual knowledge and creativity can be stimulated by contact with others, they can also be stunted by Groupthink: pressure to conform with the existing ideas and goals of a group. Conformity is a problem for groups where the costs of exit are high, but in the informal networks that interest us here, anyone who does not like the ideas of the group can leave and work alone or move to another group.

Given that an individual innovator can increase his effectiveness by expanding his network of contacts with other innovators, we now derive sufficient conditions for social connections between innovators to grow endogenously through time.

3 Network Formation Game.

Any innovative process involves learning followed by doing. Here we consider kitchen table inventors who devote some of their leisure time to innovation. In the first period of their lives they devote time to learning to increase their technical skills; in the second period they apply these skills to inventing.

An inventor i who has acquired a level of technical skill or effectiveness e_i in the first period of his life, allocates time w_i in the second period to working at inventing to maximize his expected payoff

$$R(w_i, e_i) - c(w_i) \quad (1)$$

where c is the cost of leisure time devoted to invention and R is the expected return from inventive activity. We assume that R is increasing in and has strictly increasing differences in its arguments so that the most effective inventors devote most time to inventing. An example of a such a function, used by [Becker and Murphy \(1992\)](#), is $R = w_i e_i$.

With expected return in the second period determined by effectiveness e_i , the problem in the first period is to allocate time l_i to developing human capital to maximize

$$V(e_i) - c(l_i) \quad (2)$$

where V , the present value of the second period payoff, is an increasing function of e_i . We assume moreover that it is convex: in generating new knowledge there are increasing returns to ability. In modern economies a person can increase his effectiveness through formal study and technical training, but in pre-industrial economies he is forced to learn mainly through personal contact with people who possess the skills he wishes to acquire.

There is a fixed population of N part-time inventors: population growth will play no role in any takeoff that occurs. There are M types of skill. Agent i has a single skill of type j , $\tau_i^j \geq 0$ that is exogenously given by his day job and the current state of technology in all fields. If no innovation occurs, each generation has the same vector of skills $\boldsymbol{\tau}$ as its predecessor. Any successful innovation that occurs will increase the skill of one or more inventors in the next generation.

We denote the types of agents other than i as $\boldsymbol{\tau}_{-i}$, and types in i 's network as $\boldsymbol{\tau}^{(i)}$. We expand $\boldsymbol{\tau}^{(i)}$ to include all N agents by setting the types of agents outside i 's network to zero. The effectiveness of agent i

$$e_i \left(\tau_i^j, \boldsymbol{\tau}^{(i)} \right) \quad (3)$$

depends on his own skill and the skill of his contacts. Distance between agents in a network can be allowed for by allowing the effective skill of other agents $\boldsymbol{\tau}_{-i}$ to

diminish with their distance from i , although nothing essential changes if distance is ignored.

By increasing the time l_i devoted to forming linkages with other innovators, inventor i can improve the composition of his network $\tau^{(i)}$ by increasing the number of individuals to whom he is linked; reducing their distance from him; and, if matching is assortative, their individual skill.

The network formation game therefore has N agents who invest time l_i in linkage formation to increase their skill e_i and consequently their expected payoff from innovation (2). We wish to see how actions l_i change as successful innovation increases the skill of agents τ_i^j .

3.1 Comparative Statics.

While comparative static results in non-cooperative games customarily rely on strategic complementarity in actions (Milgrom and Roberts, 1990), in many linkage formation games actions are strategic substitutes. If many linkages have already formed then it is unlikely that your adding an extra linkage will bring someone really useful into your network who was not there already.¹ We require instead that individual effectiveness e_i is supermodular in types.

Assumption. e_i is increasing and supermodular in $(\tau_i, \tau^{(i)})$.

For the reasons discussed in Section 2.1 there are positive externalities in learning. Supermodularity requires further that the value of these spillovers increase with individual type: the smarter you are, the greater the benefit you are able to derive as your contacts become smarter. An example of a skill function where knowledge is complementary across players is $e_i = \tau_i^j + \tau_i^j \prod_{m \neq j} \max_{k \in \tau^{(i)}} \tau_k^m$.

Supermodularity will not hold where learning is uninfluenced by the ability of others: the innovator is an intellectually self-sufficient genius with nothing to gain from contact with his contemporaries; or retarded by it. The skill of others can retard your learning if learning takes place in formal teams of limited size into which agents are sorted by ability (such as the O-ring technology of Kremer, 1993a and the Leontief technology of Becker and Murphy, 1992) so that a rise in the skill of an outsider can displace you onto a weaker team. However in the pre-industrial

¹Suppose for example, that your payoff depends only on the highest type of player in your network $e_i = \tau_i + \max \tau^{(i)}$. The expected number of highest types that have joined the network by the time it has reached size m is approximately $\log m$ (Arnold, Balakrishnan and Nagaraja, 1998, 24) so that the expected payoff to adding new agents falls rapidly with m .

economies that concern us here, opportunities for learning in formally structured teams are negligible.

Theorem 1. *The network formation game has a pure strategy Nash equilibrium where each player's equilibrium strategy is non-decreasing in his type.*

Proof. Agent i allocates effort l_i from a compact set $\mathcal{L}_i \in \mathbb{R}$ to forming contacts with other agents. Expected payoff V is an increasing, convex function of e_i and is therefore supermodular in $(\tau_i, \tau^{(i)})$.

Given l_i and the set of strategies of other players $\lambda_{-i}(\tau_{-i})$, the distribution function of types in i 's network is $F_i(\tau^{(i)}; l_i, \lambda_{-i}(\tau_{-i}), \tau)$ which is ordered by first order stochastic dominance in l_i : the probability that agents in i 's network exceed a given quality τ increases as he spends more time on network formation. Given τ_{-i} , the agent's expected payoff is $U(l_i, \lambda_{-i}(\tau_{-i}), \tau) = \int V_i(\tau_i, \tau^{(i)}) dF_i(\tau^{(i)}; l_i, \lambda_{-i}(\tau_{-i}), \tau)$.

Given the first order stochastic dominance of F in l_i and the supermodularity of $V_i(\tau_i, \tau^{(i)})$ in $(\tau_i, \tau^{(i)})$, $U_i(l_i, \lambda_{-i}(\tau_{-i}), \tau)$ satisfies single crossing in incremental returns in (l_i, τ_i^j) : [Athey \(1998\)](#). The set of types of other agent τ_{-i} has conditional distribution function $G(\tau_{-i}|\tau_i)$ so the agent has the objective function $\int U_i(l_i, \lambda_{-i}(\tau_{-i}), \tau) dG(\tau_{-i}|\tau_i)$. The result follows directly from [Athey \(2001\)](#). \square

The economy starts with a low value τ of individual skills, so agents have little incentive to learn from each other. Through time, however, a small number of inventive efforts will be successful. Each successful innovation improves the quality τ_i^j of a subset of innovators in the next period and causes the intensity of their network formation efforts to increase.²

Innovation and network formation are treated here as a closed cycle: successful innovation increases the skill of innovators τ , who invest more time in learning by forming social networks, generating new inventions that increase τ further. In practice other, exogenous factors increased skills and reinforced this process: improved literacy, increased division of labour associated with market expansion ([Becker and Murphy, 1992](#)), the spread of the experimental method from the sciences to industry ([Mokyr, 2002](#)), and greater parental investment in the education of children ([Galor and Moav, 2002](#)).

²The increasing time that researchers devote to learning from each other can be seen in the length of bibliographies in economics. Ricardo and Smith cite almost nobody, Marshall and Keynes cite a few more, whereas contemporary researchers typically cite large numbers of other works.

4 Evolution of Social Networks.

We now consider how a network with a gradually increasing number of connections between individuals evolves through time and how this gradual process of network expansion gives rise to a sudden takeoff in innovation.

4.1 Random Matching.

We start with a network where each individual i forms linkages with k_i other individuals at random, where k_i is an increasing function of l_i , the time devoted to forming linkages with other players. Let p_k be the proportion of agents in the network with $k = 0, 1, 2, \dots$ linkages. The behaviour of such random graphs has been analysed by [Newman, Strogatz and Watts \(2001\)](#).

The generating function for p_k is $G_0(x) = \sum_{k=0}^{\infty} p_k x^k$ so the average number of nearest neighbours of any agent is $z_1 = \sum_k k p_k = G_0'(1)$, while it can be shown that the average number of second nearest neighbours is $z_2 = G_0'(1)G_1'(1) = G_0''(1)$ where $G_1(x) = G_0'(x)/z_1$ is the generating function for the number of vertices reached by following an edge.

For N large, the average size of a connected cluster of agents is

$$\begin{aligned}\bar{s} &= 1 + \frac{G_0'(1)}{1 - G_1'(1)} \\ &= 1 + \frac{z_1^2}{z_1 - z_2}.\end{aligned}\tag{4}$$

This expression diverges when $G_1'(1) = 1$ or, more transparently, when the average agent has as many second nearest neighbours as nearest neighbours $z_2 = z_1$. The system suddenly goes from small, isolated clusters of agents to a large connected cluster. When linkages are added, their effect at first is to join isolated agents into connected pairs. As the critical density is approached, adding new linkages causes these many isolated pairs to coalesce rapidly into a large connected cluster.

4.2 Assortative and Localized Matching.

While networks where people form connections with each other at random are tractable and a useful benchmark, it is more realistic to allow for individuals to sort themselves by ability so that more able individuals attempt to seek each other out, and reject linkages from less able ones. The behaviour of such assortative

networks has been analysed by [Newman \(2002\)](#) who generalizes condition (4) for the emergence of the large connected component, and shows that the tendency of the best connected individuals to link together causes the large connected cluster to emerge earlier than in random matching models.

In examining the social networks of job seekers, [Granovetter \(1973\)](#) found the prevalence of strong ties: people tended to be linked directly to people to whom they were also linked indirectly, breaking the network into tight, local clusters. The way that this local clumping delays the emergence of a large connected cluster can be seen by comparing a network where anyone can link to anyone else with probability p , with one where agents are placed as points on a square grid and can only link to their 4 nearest neighbours.

For the economy where anyone can link, for N large the average number of linkages of each agent is $z_1 = Np$, and the distribution of linkages across agents is Poisson with parameter z_1 , giving the generating function $G_1(x) = e^{z_1(x-1)}$. From (4), the large connected cluster appears when the average number of linkages per agent $z_1 = 1$. For the nearest neighbour model, the large cluster appears when $p = 0.5$ ([Grimmett, 1989](#), 192) so that each agent needs an average of $z_1 = 2$ linkages, twice the number in the random matching economy.

4.3 Example: Exponential Distribution of Contacts.

We demonstrate the sudden transition from small isolated clusters of agents to a large connected cluster for the case where the distribution of number of contacts across agents is exponential

$$p_k = Ce^{-k/\kappa} \tag{5}$$

where the normalizing constant $C = 1 - e^{-1/\kappa}$. In the terms of the linkage formation game in Section 3, the parameter κ reflects the average effort that agents put into forming contacts, and rises through time as average ability increases as a result of successful innovation.

For random matching, the generating function is $G_0(x) = (1 - e^{-1/\kappa})/(1 - xe^{-1/\kappa})$. In this case $G_1(x) = G_0(x)^2$ so that condition (4) for the emergence of a giant cluster implies that the critical value for the connectivity parameter κ is $\kappa^* = 1/\log 3 \approx 0.9$.

For assortative matching, we allow each agent to meet ck prospective partners, where $c \geq 1$ is an integer, and to choose the best k . The higher is c the greater is the degree of sorting, with $c = 1$ corresponding to random matching. To preclude

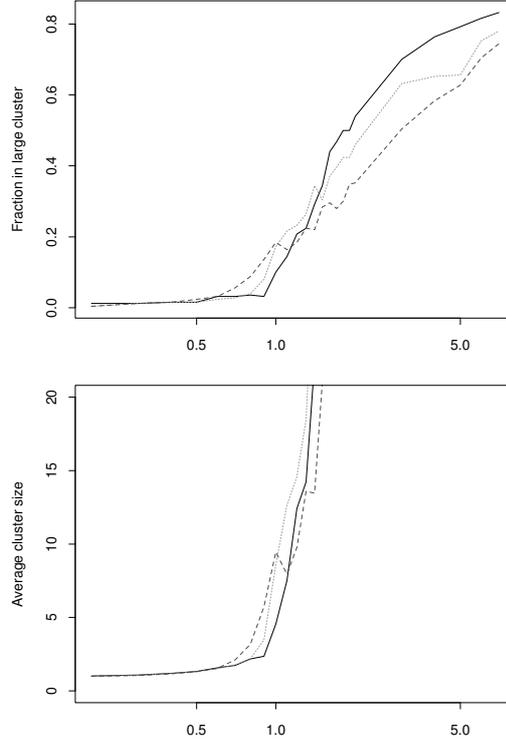


Figure 2: Average cluster size and fraction of agents in large cluster versus connectivity parameter κ for a network with 250 agents, with assortative matching parameters $c = 1$ (solid line), $c = 2$ (dotted line), and $c = 4$ (dashed line).

marriage-problem complications, where agents refuse a match in the hope of making a better one later, agents get to choose partners in descending order of k so no later offer will be more attractive than the one being currently made.

Figure 2 shows how the average cluster size and largest cluster size change as κ varies in a simulated economy with 250 agents, with values of the assortative matching parameter c of 1, 2, and 4. For random matching, the takeoff around $\kappa = 0.9$ is apparent, while assortative matching is seen to cause the takeoff to start earlier but to slow down as the more highly connected agents continue to link with each other, leaving lower ranking agents out of the large cluster.

4.4 General Matching.

Finally, we show that critical behaviour is a general feature of social networks that holds for almost any pattern of linkage formation. Let I be the set of agents and

R the set of possible linkages with associated σ -algebra $\mathcal{F}(R)$ and cardinality n : $|R| = n$. Let $S \subset \mathcal{F}(R)$ be the set of linkages that connect $O(N)$ agents with the properties that $\emptyset \notin S$, $R \in S$, and if $A \in S$ and $A \subset B$, then $B \in S$.

Theorem 2. *There exists a critical number of linkages m^* for the network (I, R) . For n large the probability of a giant connected cluster tends to zero below m^* and to one above it.*

Proof. Let $R^{|k|}$ be the set of subsets of R with k elements, $0 \leq k \leq n$ and define $S_k = S \cap R^{|k|}$. Given k connections, define the measure of large connected components as $P_k(S) = |S_k| / \binom{n}{k}$. $P_0(S) = 0$, $P_n(S) = 1$, and $P_k(S)$ is increasing in k .

Define $m^* = \max_k \{k : P_k(S) \leq \frac{1}{2}\}$ and let $\omega(n) \geq 1$ be increasing in n . By Theorem 4 of [Bollobás and Thomason \(1986\)](#), for $m \leq m^*/\omega(n)$, $P_m(S) \leq 1 - 2^{-1/\omega}$; while for $m \geq (m^* + 1)\omega(n)$, $P_m(S) \geq 1 - 2^{-\omega}$. For n large the result follows. \square

We examined an economy where kitchen table inventors form social networks in the hope of learning from each other, and showed that an arbitrarily slow expansion in the number of social connections leads to a relatively sudden change in the economy as a critical density of connections is reached. Below the critical density, agents are split into small isolated networks that offer agents with complementary skills little chance of meeting, and their effectiveness as innovators is retarded as a consequence. Above the threshold these isolated networks coalesce into a network that spans most of the economy, giving each innovator access to a very much larger pool of skills and ideas. The industrial revolution: a qualitative transition from a world where invention is haphazard to one where it is routine; is a real process.

5 Empirical Predictions: Characteristics of Inventors.

How do the characteristics of inventors change as networks evolve? We look at the three stages of the network: before, during, and after the emergence of a large connected cluster.

In the pre-threshold stage with small knowledge networks, an individual's effectiveness as an innovator e_i depends largely on his own ability τ_i so that the inventions that do occur will usually be made by more talented individuals, who may make several inventions in their careers. As the economy reaches the threshold and knowledge networks start to grow rapidly, some individuals of low personal ability will, though serendipitous contacts, make significant inventions, but are unlikely to be lucky twice.

The time that an individual devotes to inventing w_i is increasing in his effectiveness e_i . As individual skill rises through successful innovation and the knowledge network comes to span the entire economy, some individuals will become sufficiently effective as innovators that they can expect to earn more at full time inventing than in their day jobs, and turn professional. These professional innovators can increase their effectiveness by grouping into teams with a formal division of labour (Becker and Murphy, 1992) which compete against other teams in familiar patent races. However, while Rosen (1981) superstars account for an increasing share of inventions, the inherent unpredictability of innovation means that talented garage entrepreneurs are never entirely displaced.

This predicted sequence of innovators—first talented amateurs, then an influx of lucky amateurs, and finally an increasing number of professionals—matches the sequence found among early-nineteenth century American inventors by Sokoloff and Khan (1990). Analysing a large sample of patentees from 1790 to 1846, they find at the start of the period that patenting is dominated by merchants, professionals and skilled artisans, and that many individuals have several patents in their careers. As the nineteenth century progresses, an increasing share of patents go to ordinary artisans, and to individuals with only a single patent in their lifetime. By the middle of the nineteenth century, when their sample ends, professional inventors start to account for a significant share of patents.

6 Summary.

Just as energy consumption is higher now than in the eighteenth century, not because cars back then had very small engines but because internal combustion engines have fundamentally changed transportation technology; so innovation now is not merely higher than in the pre-industrial world, it is a qualitatively different activity. The industrial revolution marks the transition from a world where innovation is intermittent and haphazard to one where it is systematic and continuous: a transition that Whitehead termed “the invention of the method of invention”.

To model industrial revolutions as a qualitative change in the effectiveness of innovation, we modelled pre-industrial innovation as a social process where an individual’s effectiveness as an innovator depends on the skills of other individuals in his social network. Assuming that an individual’s effectiveness as an individual is supermodular in the skills in his network, as technology improves individuals invest more time in learning through social contact (Theorem 1). This gradual

increase in linkage formation leads to a sudden change in the size of social networks, from networks spanning a few individuals, to networks spanning most of the economy (Theorem 2), leading to a jump in the effectiveness of innovation: an industrial revolution.

The story here is intended to explain the industrial revolution, but can equally apply to the scientific revolution. In the fourteen hundred years that separate Ptolemy and Copernicus no major scientific discovery occurred, but around 1600 there was a burst of discovery, most notably by Galileo, Kepler, Stevin, and Gilbert (Margolis, 1992). While the threshold effect in knowledge networks is the same for both revolutions, the form that the linkages take is different. Face to face contact is the most important way of forming connections between agents in the industrial revolution, while letter writing and printing are the decisive conduits of information in the scientific revolution.

References

- Arnold, Barry C., N. Balakrishnan and H. N. Nagaraja. 1998. *Records*. New York: Wiley. 7
- Athey, Susan. 1998. Characterizing Properties of Stochastic Objective Functions. Working Paper 96-1R Department of Economics, MIT. 8
- Athey, Susan. 2001. "Single Crossing Properties and the Existence of Pure Strategy Equilibria in Games of Incomplete Information." *Econometrica* 69:861–889. 8
- Bala, Venkatesh and Sanjeev Goyal. 2000. "A Noncooperative Model of Network Formation." *Econometrica* 68:1181–1229. 2
- Becker, Gary S. and Kevin M. Murphy. 1992. "The Division of Labor, Coordination Costs, and Knowledge." *Quarterly Journal of Economics* 107:1137–1160. 5, 6, 7, 8, 13
- Bollobás, Bela and A. Thomason. 1986. "Threshold Functions." *Combinatorica* 7:35–38. 12
- Brock, William and Steven N. Durlauf. 2001. Interactions-Based Models. In *Handbook of Econometrics, Volume 5*, ed. James J. Heckman and Edward Leamer. Amsterdam: North Holland. 2

- Collins, Randall. 1998. *The Sociology of Philosophies: A Global Theory of Intellectual Change*. Cambridge: Harvard University Press. 4
- Galor, Oded and David N. Weil. 1999. "From Malthusian Stagnation to Modern Growth." *American Economic Review, Papers and Proceedings* 89:150–154. 2
- Galor, Oded and Omer Moav. 2002. "Natural Selection and the Origin of Economic Growth." *Quarterly Journal of Economics* 117:1133–1191. 8
- Glaeser, Edward and José A. Scheinkman. 2000. Non-Market Interactions. Working Paper 8053 National Bureau of Economic Research. 2
- Granovetter, Mark. 1973. "The Strength of Weak Ties." *American Journal of Sociology* 78:1360–1380. 10
- Grimmett, Geoffrey. 1989. *Percolation*. New York: Springer-Verlag. 10
- Hall, Peter. 1998. *Cities in Civilization: Culture, Innovation, and Urban Order*. London: Weidenfeld and Nicholson. 4
- Hansen, Gary D. and Edward C. Prescott. 1998. Malthus to Solow. Working Paper 6858 National Bureau of Economic Research. 2
- Jackson, Matthew O. and Asher Wolinsky. 1996. "A Strategic Model of Social and Economic Networks." *Journal of Economic Theory* 71:44–74. 2
- Jones, Charles. 2001. "Was an Industrial Revolution Inevitable? Economic Growth over the Very Long Run." *Advances in Macroeconomics* 1(2):Article 1. <http://www.bepress.com/bejm/advances/vol1/iss2/art1>. 2
- Jones, Charles I. 1995. "Time Series Tests of Endogenous Growth Models." *Quarterly Journal of Economics* 110:495–525. 2
- Kremer, Michael. 1993a. "The O-Ring Theory of Economic Development." *Quarterly Journal of Economics* 108:551–576. 7
- Kremer, Michael. 1993b. "Population Growth and Technological Change. One Million B.C. to 1990." *Quarterly Journal of Economics* 108:681–716. 2
- Levine, John M., Lauren B. Resnick and E. Tory Higgins. 1993. "Social Foundations of Cognition." *Annual Review of Psychology* 44:585–612. 4

- Margolis, Howard. 1987. *Patterns, Thinking, and Cognition*. Chicago: University of Chicago Press. 5
- Margolis, Howard. 1992. *Paradigms and Barriers*. Chicago: University of Chicago Press. 14
- Milgrom, Paul and John Roberts. 1990. “Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities.” *Econometrica* 58:1255–1277. 7
- Mokyr, Joel. 2002. *The Gifts of Athena: Historical Origins of the Knowledge Economy*. Princeton: Princeton University Press. 4, 5, 8
- Newman, M. E. J. 2002. “Assortative Mixing in Networks.” *Physical Review Letters* 89:208701. 10
- Newman, M. E. J, S. H. Strogatz and D. J. Watts. 2001. “Random Graphs with Arbitrary Degree Distributions and their Applications.” *Physical Review E* 64:026118. 9
- Rosen, Sherwin. 1981. “The Economics of Superstars.” *American Economic Review* 71:845–858. 13
- Sokoloff, Kenneth L. and B. Zorina Khan. 1990. “The Democratization of Invention during Early Industrialization: Evidence from the United States, 1790–1846.” *Journal of Economic History* 50:362–378. 13
- Weitzman, Martin. 1998. “Recombinant Growth.” *Quarterly Journal of Economics* 113:331–360. 2