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A joint model of turnout and vote choice

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A joint model of turnout and vote choice

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Abstract: Most existing studies of referendums examine voter turnout and their vote choice separately. Our theoretical model suggests that this can lead to biased results. The model we propose links participation to the informational power of political campaigns in a setting where risk averse voters are uncertain about the options' precise political locations. It predicts that voters who generally prefer one of the two possible referendum outcomes, but who are relatively uncertain about the consequences of their preferred option, tend to abstain from voting. Greater uncertainty about a referendum option not only reduces its value, but also, for more "distant" voters, the value of participating. Uncertainty, thus, has a double effect: potential supporters of one referendum option are less likely to vote; and citizens who vote are less likely to support this option. We use data from the 'Brexit' vote to show how individual assessments of uncertainty about the two-options affect turnout and the vote. Our empirical analyses provide support for our theoretical model.

Keywords: referendums, turnout, uncertainty, European integration, electoral behavior and Brexit.

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I. Introduction

Recent referendums in the European Union (EU) indicate that voter turnout has a substantial effect on the outcome of the vote. One example is the 'Brexit' referendum in the United Kingdom (UK) in June 2016 over the question whether Britain should leave the EU. Several analysts blamed low turnout among younger voters for lack of support for staying within the EU.[1] An even more obvious example is the repeated referendums in Ireland on the Nice Treaty in 2001/02. The Irish electorate first rejected the treaty and then approved it in a second referendum because supporters of the treaty who previously abstained decided to participate (Sinnott, 2003). We see similar patterns in other popular votes such as national elections. For example, the presidential elections of France in 2017 were likely influenced by the low turnout of voters from the political left.[2]

Although existing research has long suggested that turnout matters for referendum outcomes, the general mechanisms through which this happens remain unclear. The anecdotes above support the view that a critical channel through which the outcome of a vote can be influenced is by affecting turnout (Zhang, 2018). In the context of EU referendums, the link between turnout and referendum outcomes has been highlighted repeatedly (Sinnott, 2003; Hobolt, 2009). But, despite this consensus, most analyses still examine the decisions whether to vote and how to vote as separable choice problems (e.g., Hug and Schulz, 2007; Hobolt, 2009; Dür and Konstantinidis, 2013; Elkink and Sinnott, 2015). Only very few theoretical models explicitly model the connection between turnout and vote choice (e.g., Matsusaka, 1995; Feddersen and Pesendorfer, 1996).

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[1] *The Guardian*, "EU referendum: youth turnout almost twice as high as first thought", 10 July 2016, https://www.theguardian.com/politics/2016/jul/09/young-people-referendum-turnout-brexit-twice-as-high.

[2] *BBC News*, "French election: Turnout sharply down in Le Pen-Macron battle", 7 May 2017, http://www.bbc.com/news/world-europe-39833831; *Independent*, "France presidential elections: As French go to polls, could abstentions prove decisive in Macron-Le Pen contest", 7 May 2017, http://www.independent.co.uk/news/world/europe/france-presidential-elections-le-penmacron-french-polls-vote-abstentions-odds-who-will-win-a7722356.html; see Galam (2017) for an analysis of the interaction between turnout and vote choice in the French elections.

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Our theoretical model shows that an important mechanism linking turnout to vote choice is through uncertainty about the consequences of the referendum options. First, and contrary to the previous literature, greater uncertainty about the consequences of one option biases turnout in favor of the less uncertain option. Specifically, we show that the citizens who are highly uncertain about their preferred choice decide to stay home. When a voter's preferred option is subject to much uncertainty, the distinction between the two options blurs, and voting loses its appeal. Second, and consistent with the previous literature, increased uncertainty about one option reduces its support from participating voters. Voters are critical of options for which the consequences are uncertain and hence tend to vote for the less uncertain option. Uncertainty, thus, has a double impact: first, voters on one side stay home; and second, some of the participating voters change their vote.

We test the predictions of our model using survey data on the 'Brexit' referendum in the UK (2016). This survey is unique because it directly measures uncertainty of the two referendum options. Specifically, respondents were asked how certain they are about the consequences of remaining in the EU and of leaving the EU prior to the referendum taking place. Furthermore, questions related to the expected impact of 'Brexit' on the UK allow us to include a proxy for the difference between the status quo and the proposed position in our spatial model of voting. We are therefore able to derive an empirical model specification closely aligned with our theoretical model (cf. Signorino, 1999).

The empirical results confirm our hypotheses. They show that voters whose preferred position is closer to the more uncertain option are less likely to vote and less likely to vote for that option. As an additional implication of our theoretical model, we also find that as the two options are closer to each other, turnout declines. Not only do we provide support for our theoretical model, but we also demonstrate how the overall estimated outcome of the referendum can be substantially mis-estimated when only vote choice is taken into account.

The mechanism that we describe is not unique to referendums, but can be applied to elections more generally, at least where it concerns two possible outcomes such as the second round of the French Presidential Elections. In such a context similar variation in uncertainty about the relative policy positions, or the implications of someone being elected president, can lead to similar voter dynamics and thus a similar suboptimal election outcome.

The following section provides an intuitive understanding of our theoretical argument, prior to the formal model specification in the third section. The fourth section compares our theoretical model to existing formal models that link turnout and vote choice to the aggregate referendum outcome. The fifth section tests the empirical implications of the model on the 'Brexit' referendum data. It also evaluates how much ignoring turnout leads to an underestimation of the impact of uncertainty on the vote in this referendum. Finally, section six provides our conclusions.

II. Uncertainty, turnout, and vote choice

Recent referendums on European integration have been associated with substantial uncertainty due to, among other factors, the general complexity of the issue being voted upon, the lack of predictability of international reactions (Farrell and Schmitt-Beck, 2003; Nicholson, 2003), and the unclear impact each option has on national sovereignty which affects the government's ability to implement domestic policies (Sattler and Urpelainen, 2012). The literature shows that uncertainty impacts both, turnout, and vote choice, via education (Nie et al., 1996), cognitive ability (Denny and Doyle, 2008), political awareness (Zaller, 1992) and political knowledge (Elkink and Sinnott, 2015).[3] Specifically, informed citizens participate more often in elections and EU referendums (Matsusaka 1995; Hobolt 2005) and they are more likely to support European integration proposals (Hobolt 2009; Elkink and Sinnott 2015).

[3] See, as well, Campbell et al., 1960; Nie et al., 1979; Zaller, 1992; Carpini and Keeter, 1996.

To explore the link between uncertainty, participation, and vote choice we consider a setting with individuals guided by their intrinsic ideology and an aversion to uncertainty. In a European context, we consider voters facing two options, who are aware that one option is clearly more supportive of integration. However, there is uncertainty as to how much integration this option will lead to, as well as uncertainty as to how little integration the other option will lead to (Walter et al., 2018).[4] With voters critical of options for which consequences are uncertain, and in line with the literature, our model predicts that greater uncertainty around one option leads to a reduced probability of a vote in favor of that option.

The link we establish between uncertainty and participation has not yet been captured in the literature. Participation is shown to depend on the voters' reaction to information released during a campaign held after the announcement of an upcoming referendum. Such information could take the form of pamphlets highlighting the main political, economic, and social implications associated with each outcome thereby giving a more precise idea about the true consequences of the two options. Feddersen and Pesendorfer (1996, 1999) show that information impacts participation in situations where one option is objectively *superior*, for economic or social reasons, but voters are unsure as to which is so that uninformed voters prefer to abstain fearing that their ballot could lead to the selection of the *inferior* option. According to Larcinese (2009) this outcome also arises when voters have some intrinsic preferences and the fear of being pivotal leads them to acquire costly information.[5]

In this paper we show that uncertainty impacts participation in a context where none of the two options is objectively superior and where pivotality is not a concern. Specifically, our approach provides a rationale for political disengagement on behalf of some citizens. It shows that additional

[4] We adopt a general model departing from the assumption that only the new treaty is associated with uncertainty.

[5] Several other formal models of voting behaviour focus on the relationship between pivotality and information (Matsusaka, 1995; Ghirardato and Katz, 2002; Larcinese, 2009; Degan and Merlo, 2011; Oliveros, 2013).

information is not always associated with a positive value-added even though it reduces the uncertainty. Indeed, some voters may realize, during the campaign, that the option closer to them is not as close as what they originally thought. From the point of view of *extreme* voters, this implies that the distinction between the two options blurs, which dampens their willingness to participate. Thus, and unlike predictions from previous models, we show that a specific group—those favoring the more uncertain option—prefer to disengage from the referendum when the uncertainty increases. More specifically, we show that what matters is not the level of uncertainty, but the relative uncertainty levels between the two referendum options. Consequently, our model is applicable in a wide range of referendums, with cooperative or non-cooperative reversion points (Hobolt, 2009) and different levels and types of uncertainty (Walter et al., 2018).

III. Theoretical model

Model setup

Consider a policy space representing the set of policy options available in a particular policy field. Without loss of generality, we assume that the policy space varies between 0 and 1. Applied to EU referendums, the policy space reflects the depth of European integration. The value 0 represents a situation in which European integration ends, and all member states continue as individual states. The value 1 represents a situation where Europe fully integrates into a federal state. Voters are distributed across the policy space according to a probability distribution $F(.)$. Each voter has an ideal point $i \in [0,1]$, which reflects the depth of European integration that the voter favors most. A greater i represent more pro-integrationist EU attitudes.

In the referendum, citizens can choose between two options with different *expected* positions in the policy space: $t_0 = \alpha t$ and $t_1 = t$ with $0 < t < 1$ and $0 < \alpha < 1$. In the 'Brexit' referendum, for instance, option t_1 represents the status quo, which is the current level of European integration; and option t_0 represents the proposal to leave the European Union, which reduces the current integration level. In past referendums, such as those on

the Maastricht, Nice and Lisbon Treaties, option t_0 represents the status quo, and option t_1 represents the proposed treaty.

The citizen's decision problem is complicated by uncertainty about the consequences of the two options. Uncertainty about an option is captured by taking t_0 and t_1 as random variables. Their realization captures the level of integration that will be implemented and is not perfectly predictable. We assume that there are two possible true positions for each option. Specifically, t_1 is either at a low (L) position, $t - \Delta_1$, in which case the proposed option leads to a moderate deepening of EU integration, or at a high (H) position, $t + \Delta_1$, in which case the proposed option leads to significantly more EU integration. Similarly, t_0 is either at $\alpha t - \Delta_0$ or $\alpha t + \Delta_0$. We assume that the joint distribution is such that there are only two possible outcomes that can be implemented: the states of the world *LL* = $(\alpha t - \Delta_0, t - \Delta_1)$ and *HH* = $(\alpha t + \Delta_0, t + \Delta_1)$.[6] The variables $t, \alpha t, \Delta_0$ and Δ_1 are all common knowledge and such that $\alpha t + \Delta_0 < t - \Delta_1$ and $0 < \alpha t - \Delta_0 < t + \Delta_1 < 1$. Allowing some of these terms to be equal would not alter our results.

To guarantee that the expectations of t_0 and t_1 are $E(t_0) = \alpha t$ and $E(t_1) = t$, we consider that, *a priori*, each state of the world is equally likely: $\Pr(LL) = \Pr(HH) = \frac{1}{2}$. It follows that, *a priori*, their variances are $V(t_w) = \Delta_w^2$, where $w \in \{0,1\}$ for the two referendum options. Hence, the uncertainty about an option is reflected by the size of Δ_w. When Δ_w increases, citizens are more uncertain about the true consequences of option t_w. This allows us to explore how changes in Δ_0 or Δ_1 impact the voters' decisions. We should mention that contrary to what is commonly assumed, we consider that both options can have uncertain consequences. In the case of 'Brexit', the greater uncertainty is likely to be around the UK leaving the EU, not the status quo. In Ireland, for the Lisbon II referendum, a significant amount of rhetoric revolved around the uncertain outcome if the treaty was not supported given the dependency of the country on financial aid from European partners. Thus,

[6] See the extension provided in Appendix which shows how the analysis is impacted when HL and LH are considered as possible states of the world.

in Ireland, the uncertainty around the status quo might have been greater than the uncertainty around the treaty.

We consider risk averse citizens who dislike uncertainty. To do so, we follow Hobolt (2009) and assume that individuals have a concave, quadratic utility function. This is the conventional approach to modelling risk aversion in utility functions.[7] Specifically, the utility function of a voter located at $i \in [0,1]$ is given by:

$$U_i(t_w) = -(t_w - i)^2. \quad (1)$$

Given this specification, the expected utility associated with outcome t_w is given by

$$E[U_i(t_w)] = -(E(t_w) - i)^2 - V(t_w). \quad (2)$$

The variance has a clear negative impact on the utility—the more uncertain a voter is about an option, the less she values that option.

Figure 1, below, illustrates our setup graphically. Euroskeptic citizens i_1 and i_2 are closer to option t_0 and, hence, should reject option t_1. Eurofriendly citizens i_4 and i_5 are closer to option t_1 and, hence, should support this option.

[7] See Hobolt (2009), ch. 2, esp. p. 49 and fn. 12.

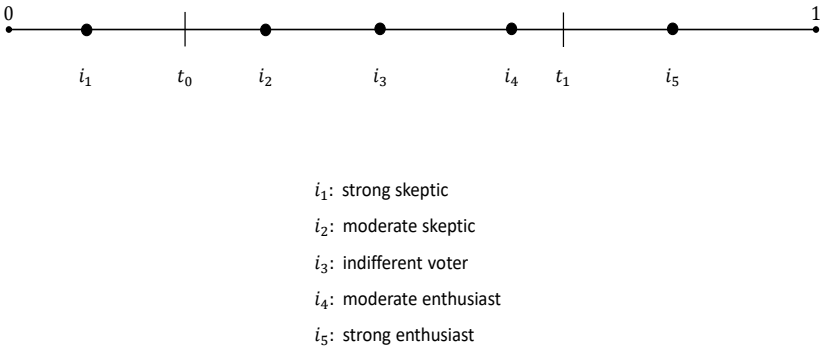

Figure 1: Possible attitudes towards European integration

Following the announcement that a referendum will be held, a campaign starts during which citizens receive a signal $s \in \{L, H\}$ which is non-negatively correlated with the true position of the options. This assumption is in line with existing strategic literature on information transmission and learning (Schneider and Cederman 1994; Schneider and Weitsman 1996; Hug 2002, ch.4; Dür and Konstantinidis 2013).[8] We consider that, *a priori*, each signal is equally likely, so that $\Pr(s = L) = \Pr(s = H) = \frac{1}{2}$ and that the joint probabilities are defined such that:

$$\Pr(HH \cap s = H) = \Pr(LL \cap s = L) = p,$$

$$\Pr(HH \cap s = L) = \Pr(LL \cap s = H) = \frac{1}{2} - p, \text{ where } p \in \left[\frac{1}{4}, \frac{1}{2}\right].$$

We can compute the conditional probabilities associated with each possible state of the world using the information in the previous paragraph. This conditional probability represents the probability that the true position of

[8] In our setup, all citizens receive the same signal. It could be argued, based on "echo chamber" effects of communication, that citizens are more likely to receive signals that are more comforting given their own position (see, for instance, Bessi et al., 2015; Del Vicario et al., 2016). In the Appendix we explore how our results would change in such an alternative setting.

a referendum option is either high (HH) or low (LL), given that the citizen receives a high or low signal. Following Bayes' Theorem and given that either signal is equally likely, we have

$$\Pr(HH|s = H) = \Pr(LL|s = L) = 2p,$$

$$\Pr(HH|s = L) = \Pr(LL|s = H) = 1 - 2p.$$

Notice that, within this setting, we also have

$$\Pr(t_w = H|s = H) = \Pr(t_w = L|s = L) = 2p.$$

When $p = \frac{1}{2}$ the signal is fully informative about the true position options t_0 and t_1. Once the signal is received, citizens have perfect knowledge of the position of both referendum options. When $p = \frac{1}{4}$ the signal is not informative, meaning that the campaign has no informational power. This means that, depending on p, during the campaign, the variance may or may not decrease and reduce the uncertainty about the absolute and relative positions of the status quo and the treaty proposal.

The timing of events is as follows. Initially, at time *T* = 0, the government announces a referendum. Citizens can vote for either option. Citizens are free to vote. At *T* = 1, a campaign starts, and all citizens receive some information (the signal). Finally, at *T* = 2, citizens who participate cast their vote. As is customary in such sequential settings, we assume that citizens can perfectly anticipate the outcome of the subsequent period and solve accordingly.

Voting in favor or against

The voter would vote in favor of option t_1 if and only if

$$E(U_i(t_1)|s) \geq E(U_i(t_0)|s), \tag{3}$$

where $s \in \{H, L\}$. By plugging (2) into (3) and solving for i, we can derive the threshold values, i_s, that separates voters into supporters and opponents of the two options after receiving signal s:

$$i_s = \frac{V(t_1|s) - V(t_0|s)}{2[E(t_1|s) - E(t_0|s)]} + \frac{1}{2}[E(t_1|s) + E(t_0|s)]. \quad (4)$$

If a voter is to the right of this position ($i > i_s$), she votes in favor of the option proposing more integration. Otherwise, she votes in favor of the option proposing less integration. The expression for i_s implies that if there is greater uncertainty around the implications of one option than there is around the other, voters are less likely to vote in favor of the former.

H$_1$: *Voters who participate in the referendum are more likely to vote against the more uncertain option.*

To test H_1 we can simplify matters and eliminate the noise added by the signal and its precision.[9] Using our setup, the indifferent types for *s* = *L* and *s* = *H* are given by

$$i_s = \frac{t^2(1-\alpha^2) + (\Delta_1^2 - \Delta_0^2) + 2\phi t(4p-1)(\Delta_1 - \alpha\Delta_0)}{2t(1-\alpha) + 2\phi(4p-1)(\Delta_1 - \Delta_0)}, \quad (5)$$

where $\phi = -1$ when $s = L$ and $\phi = +1$ when $s = H$. Clearly, the division of voters is impacted by the signal received (via ϕ), the precision of the signal (via p) and the variance associated with each option. Considering that signals are uninformative allows us to focus on the impact of uncertainty. When $p = \frac{1}{4}$, which includes the situation prior to the campaign, a citizen who decides to vote supports the treaty provided her position is such that

$$2ti(1-\alpha) \geq t^2(1-\alpha^2) + (\Delta_1^2 - \Delta_0^2). \quad (6)$$

Participation

[9] To capture the impact of the signal's precision, note that $i_s = \frac{t}{2}(1+\alpha) + \Delta\phi(4p-1)$ when $\Delta_0 = \Delta_1 = \Delta$. Thus, an increase in the precision of the signal widens the gap between i_L and i_H.

Motives for participation diverge across citizens and political campaigns can have an impact on participation via the provision of information (Farrell and Schmitt-Beck, 2003). Kenski and Stroud (2006) and Dimitrova et al. (2014) provide evidence of the positive impact of *digital* information on participation.

In our setting, the signal that is sent affects a citizen's assessment of the referendum's outcomes through three different channels. Firstly, it unambiguously increases a voter's expected utility by reducing the uncertainty (or variance) associated with options t_0 and t_1. Secondly, it affects the expected location of t_0 and t_1. This may have positive or negative impact on the overall utility. For voters located on the extreme left, $s = H$ informs them that both options are further away from them than what they originally thought. The opposite holds for voters located at the extreme right when they receive $s = L$. For voters in the middle, the campaign has a more balanced impact. They learn that one of the two options is closer to their ideal outcome while the other is further away. Lastly, the signal impacts participation by influencing the location of the indifferent voter which, depending on the distribution of voters, can sway the outcome of the referendum.

Effectively, the decision to participate can change during the campaign as information is released. As we want to understand the role played by uncertainty, as opposed to role played by specific signals, we focus on a subset of abstaining voters: those who decide to disengage from the referendum before the campaign.[10] We link their decision to the uncertainty that surrounds the two options. We postulate that a citizen's decision to disengage reflects a perfect and accurate anticipation that the value attributed to the referendum's outcomes will not rise during the campaign. Said differently, we consider that staying at home reflects a form of pessimism: Those who do not participate are the ones who expect that nothing good will come from the

[10] In some countries, such as the U.S., voters must register to vote. Our approach focuses on citizens who decide not to register and thereby forgo the possibility to participate prior to the campaign. We understand that, ultimately, more voters may decide not to vote but their decision is more likely to be related to the information they gathered.

campaign.[11] This approach enables us to establish a clear link between participation and uncertainty as it is not dependent on the type of information that is released during the campaign. We also believe that this approach is reasonable in a framework where citizens are sophisticated. They are fully aware of the signals that may be sent during the campaign and how these will impact the value they attribute to the potential outcomes. In other words, all that can possibly be learnt during the campaign is perfectly and accurately anticipated by all citizens.

Within this framework we identify all perfect Bayesian Nash equilibria. In equilibrium, the following conditions must hold:

- Given her political preferences (i) and beliefs about the referendum's outcome, and the behaviour of all other citizens, each citizen participates if she anticipates that the expected value she associates with the referendum's outcome will rise during the campaign.
- Given the participation strategies, the beliefs are consistent, meaning that each citizen perfectly anticipate the outcome of the referendum.

Let $\rho_s \in \{0,1\}$ capture a citizen's belief about the outcome of the referendum following a campaign where signal $s \in \{H, L\}$ was released. Specifically we adopt a convention whereby $\rho_s = 1$ when a citizen believes that t_1 will be adopted and $\rho_s = 0$ when the citizen believes that t_0 will be adopted. Since the referendum's outcome is the same for all, consistent beliefs must be identical across all citizens.

A citizen's decision to participate is based on the comparison of two expected values of the referendum's outcomes, given her beliefs and her political preferences captured via her location on the interval $[0,1]$. The

[11] Krizan and Sweeny (2013) rely on data on Cannabis legalization in California and find that residents who maintained their enthusiasm about the initiative's outcome over time were more likely to vote.

difference between these values measures the impact of the campaign as a mechanism that can either curb or foment her enthusiasm.

Let $U_{i,s}^{P}$ be the post-campaign value given to the referendum's outcome by a citizen located at $i \in [0,1]$ who has received signal $s \in \{H, L\}$ during the campaign and holds beliefs $\rho_s \in \{0,1\}$:

$$U_{i,s}^{P} = \rho_s E(U_i(t_1)|s) + (1-\rho_s)E(U_i(t_0)|s). \quad (7)$$

Let $E_s\left(U_{i,s}^{P}\right)$ be the expectation of $U_{i,s}^{P}$ which is assessed prior to receiving a signal

$$E_s\left(U_{i,s}^{P}\right) = \frac{1}{2}\sum_{s\in\{H,L\}} \rho_s E(U_i(t_1)|s) + (1-\rho_s)E(U_i(t_0)|s). \quad (8)$$

Let U_i^R refer to this citizen's *reservation* value. This reflects the expected value given to the referendum's outcome in the absence of any campaign:

$$U_i^R = \rho E[U_i(t_1)] + (1-\rho)E[U_i(t_0)]. \quad (9)$$

We characterize the set V of voters as

$$V = \left\{i: U_i^R < E_s\left(U_{i,s}^{P}\right)\right\}. \quad (10)$$

We thereby suggest that voters who anticipate their utility to rise strictly, on expectation, are the ones who participate. The strict inequality captures the fact that there is an infinitesimal cost associated with voting. Voters who expect that the campaign won't change anything, in relation to their utility, prefer to stay at home.[12]

In equilibrium ρ must be consistent with the fact that, *a priori*, the signal is equally likely to be high or low, so that:

[12] In the Appendix where we prove the Lemma and proposition characterizing the equilibrium behaviour, we have an addendum where we consider the added equilibria that arise when considering that these voters participate.

$$\rho = \frac{1}{2}(\rho_H + \rho_L).^{13} \quad (11)$$

Using (11) the set of voters is defined by all i such that

$$(\rho_H - \rho_L)[i(\Delta_1 - \Delta_0) - t(\Delta_1 - \alpha\Delta_0)] > 0. \quad (12)$$

Below we present our main results. Lemma 1 captures a situation where all citizens participate. The Proposition captures the equilibria we are more interested in as it reflects situations in which some voters stay home. A proof of all of these results can be found in Appendix.

Lemma: *When* $\Delta_1 \in [\alpha\Delta_0, \Delta_0]$ *and the individuals have political tastes distributed in such a way that* t_1 *is only adopted when the signal s=L, there exists an equilibrium with full participation.*

Consider a situation where the median citizen within the set $[0,1]$ is in the interval $[i_L, i_H]$.[14] In such a case, subject to full participation, the only consistent beliefs are such that $\rho_L = 1$ and $\rho_H = 0$. If, in addition, $\Delta_1 \in [\alpha\Delta_0, \Delta_0]$ meaning that the uncertainty surrounding t_1 is not much greater than the uncertainty surrounding t_0, then (12) holds for all i and we have $V = [0,1]$.

This equilibrium is such that full participation arises in a context where the options relatively equally uncertain and where citizens believe the signal will impact the outcome of the referendum.

[13] If the outcome of the referendum is believed to be the same regardless of the signal so that $\rho_H = \rho_L = 0$ or $\rho_H = \rho_L = 1$, then $\rho = 0$ or $\rho = 1$. If the citizen believes that the signal impactes the outcome and $\rho_H \neq \rho_L$, then $\rho = \frac{1}{2}$ because each signal is equally likely a priori.

[14] Note that $i_L \leq i_H \Leftrightarrow$[(t −Δ1) − (αt −Δ0)][(t +Δ1) − (αt +Δ0)] ≥ 0, which is always true.

Proposition: *There are two equilibria in which some of the voters stay home. In each of these the individuals have political tastes that are not skewed so that option 1 is only adopted when the signal is s = L. In equilibrium the voters who do not participate are those who prefer the most uncertain outcome:*

-Voters on the extreme left stay home when $\Delta_1 < \alpha\Delta_0$.

-Voters on the extreme right stay home when $\Delta_1 > \Delta_0$.

Figure 2, below, captures our findings in terms of participation on a graph.

Let us consider that citizens believe that the signal sent during campaign can sway the outcome of the referendum so that $\rho_L = 1$ and $\rho_H = 0$.

If option t_0 is subject to a greater uncertainty relative to its alternative so that $\Delta_1 < \alpha\Delta_0$ then (12) holds for all $i \in \left]\frac{t(\alpha\Delta_0 - \Delta_1)}{(\Delta_0 - \Delta_1)}, 1\right]$ so that some voters on the extreme left prefer not to participate. If option t_1 is subject to a greater uncertainty relative to its alternative so that $\Delta_1 > \frac{1-\alpha t}{1-t}\Delta_0$ then (12) holds for all $i \in \left[0, \frac{t(\Delta_1 - \alpha\Delta_0)}{(\Delta_1 - \Delta_0)}\right[$ so that some voters on the extreme right prefer not to participate.

The initial beliefs held by the citizens are consistent provided the distribution of political tastes are such that the median voter within the set of participating citizens is always located in the interval $[i_L, i_H]$.

These results show that freely available information that reduces the uncertainty individuals face in a referendum is not valued equally by all citizens, despite their aversion to risk. The value added depends on the *relative* variances associated with each outcome. When the option that is closest to one voter is associated with high uncertainty relative to her less favored option, this voter can lose interest in the referendum. While one option offers an outcome close to her political ideology, its associated uncertainty reduces its appeal.

H_2: Voters at the extreme end of the policy space on the side of the more uncertain option are more likely to abstain.

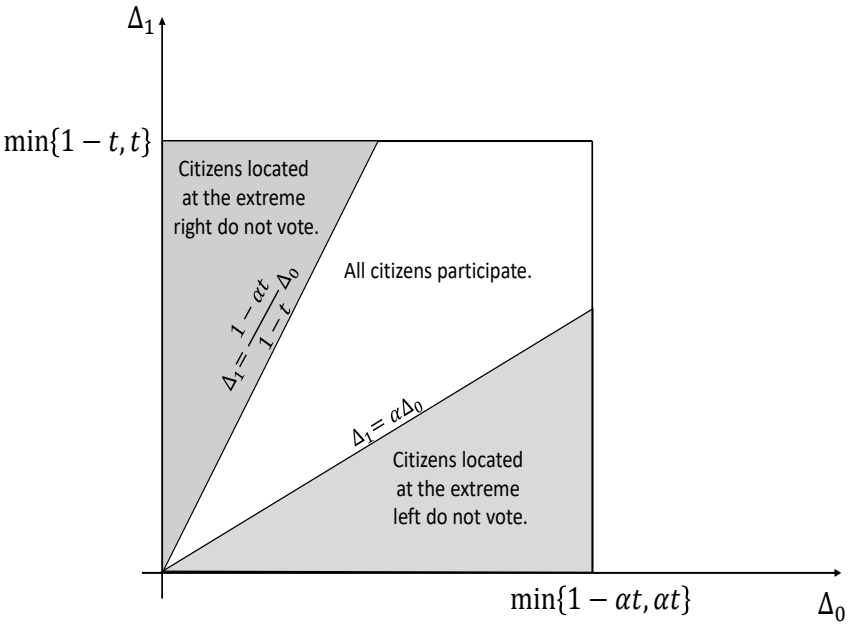

Figure 2: Graphical depiction of the equilibrium outcomes of voter turnout. The boundaries for Δ_0 and Δ_1 are such that $\alpha t - \Delta_0 > 0$, $\alpha t + \Delta_0 < 1$, $t - \Delta_1 > 0$, and $t + \Delta_1 < 1$. D

Discussion

Our modelling approach does not rely on the assumption that a voter fears the likelihood of being pivotal. Pivotality mostly matters in close elections (Bursztyn et al., 2018), especially when uncertainty about the election outcome (rather than the consequences of the election options) is large (Myatt, 2015). There is only weak evidence that voters assess the probability of being pivotal (Grant and Toma, 2008).

In addition, our model does not require a significant cost component that discourages citizens from participating, as is the case in the main alternatives to the pivotal voter models. In ethical turnout models, for instance, voters face a cost of abstaining, which arises from

social pressure to vote and the fear that like-minded voters learn about the voter's failure to vote for their preferred party (Della Vigna et al., 2017). In this class of models, greater turnout can affect the electoral advantage of large or small parties and produce a bias. But, unlike our model, this advantage is unrelated to the ideological position of the party or referendum option (Levine and Mattozzi, 2018).

We also deviate from the assumption that there is a superior option. In previous models, the link between turnout and choice arises when uninformed voters decide not to vote out of strategic considerations. As a result, informed voters determine the outcome which is optimal from the perspective of the uninformed voter (Feddersen and Pesendorfer, 1996, 1999). But in many instances, there is no objectively right or wrong answer to a referendum question. Instead, utilities differ between voters—either based on political ideology or personal circumstances—and the vote decision and the importance of additional information varies between voters.

Relatedly, empirical research evaluates the conditions under which elections deliver on information aggregation assessing whether the overall aggregate results match the underlying distribution of opinion in the public in a context of uninformed voters (Battaglini et al., 2008; Kim and Fey, 2007; Lau and Redlawsk, 1997, 2001). As far as information and uncertainty about the consequences of referendum options are related, our model suggests that the outcome of an election is not the same when voters are informed or uninformed. Especially when there is less information (and hence greater uncertainty) about one option, the referendum outcome will be biased and not reflect the underlying distribution of opinions.

Empirical analysis

The 'Brexit' referendum

We test our model using survey data collected before and after the 'Brexit' referendum in the United Kingdom in June 2016. This referendum is not only interesting because of its great political importance, but it also provides a hard test for our theoretical model. Due to the high political salience, turnout is this referendum was relatively high for reasons that are complementary to our analysis. Under these circumstances, the importance of uncertainty for turnout moves into the background because other factors, like political duty and interests, played a decisive role for the decision to vote.

The 'Brexit' referendum survey is also useful for our analysis because it differs in one important respect from most surveys after European integration referendums: the panel of referendum surveys before and after the 'Brexit' referendum asked explicitly about the uncertainty concerning the referendum proposal (leave) and the status quo (remain).[15] The question asked "How sure are you about what would happen to the UK if it left the EU or if it remained in the EU?", scoring each of the two questions on a four point scale from "very unsure" to "very sure".

Aside from the uncertainty variable, the key independent variable is the level of Euroscepticism or pro-European attitude i.[16] In the same pre-

[15] We make use of the 2014–2018 British Election Study Internet Panel (Fieldhouse et al., 2015), using variables from waves 8 and 9. In total 33,502 respondents took Wave 8, which was conducted by YouGov between 6th May 2016 and 22nd June 2016. In total 30,036 respondents took Wave 9—27,555 of these also took Wave 8, an overall wave on wave retention rate of 82.2%. Wave 9 was conducted by YouGov between 24th June 2016 and 4th July 2016, right after the referendum on 23rd June 2016..

[16] For brevity sake, we take this as a unidimensional variable, in line with the theoretical model. See Boomgaardenet al. (2011) and de Vreese et al. (2019) for a discussion on the multidimensional nature of EU support.

referendum survey, the question "Some say European unification should be pushed further. Others say it has already gone too far. What is your opinion?" was posed, with the answer captured on an eleven-point scale from "unification has already gone too far" to "unification should be pushed further". We rescale this to 0-1 to align with our theoretical model specification of the continuum of European integration.

We proxy $E(t_1) - E(t_0)$ using the question "How much impact do you think that Britain leaving the EU would have on the country as a whole?" This variable is measured on a five-point scale from "no impact at all" to "a very large impact", which we rescale the 0-1, the theoretically maximum distance between t_0 and t_1. The combination of these variables allows us to directly translate the theoretical model into a set of regression equations that can be statistically tested.

Our analysis complements the studies that examine the main drivers of the 'Brexit' referendum. Consistent findings in this literature align with what we capture in our model with pro-integration attitude. Whether the analysis is performed at aggregate, regional or individual level, the 'leave' vote is primarily driven by demographic factors such as age, education, and income (Hobolt, 2016; Vasilopoulou, 2016; Clarke et al., 2017; Goodwin and Heath, 2016; Goodwin and Milazzo, 2017; Matti and Zhou, 2017; Arnorsson and Zoega, 2018; Zhang, 2018). These factors largely correspond to theories of European integration related to the idea of "winners" and "losers" of European integration (Gabel, 1998; Tucker et al., 2002). Indeed, regions that are most strongly affected economically by globalization in recent times were most likely to vote for 'Brexit' (Becker et al., 2017; Colantone and Stanig, 2018). Attitudes towards immigration and European integration consistently come up as important factors (Hobolt, 2016; Vasilopoulou, 2016; Clarke et al., 2017; Goodwin and Milazzo, 2017). Our analysis takes these studies as a starting point and examines how uncertainty about the referendum options factors into the political considerations of voters.

Based on this literature, as well as earlier literature on referendum voting behavior, we control for education, income, age, and gender. Table 1, below, provides summary statistics for all relevant variables.[17]

	Minimum	Maximum	Mean	Standard deviation	N obs.
Turnout	0	1	94%		29,936
Remain vote	0	1	50%		28,069
Pro-Integration	0	1	0.32	0.31	25,847
Uncertainty (Δ1)	0	1	0.37	0.24	23,100
Uncertainty (Δ0)	0	1	0.44	0.25	22,960
Impact "Brexit" (t1 – t0)	0	1	0.66	0.26	25,544
Political knowledge	0	6	4.25	1.83	25,749
Efficacy	0	5	2.19	1.44	27,960
Education	0	1	48%		29,476
Income	1	15	6.70	3.52	18,615
Female	0	1	51%		30,036
Age	15	94	50.8	15.8	30,036

Table 1: Summary statistics for all variables. Mean with standard deviation for continuous variables and percentage of ones for dummy variables. Note that Education, in addition to the 48% ones, 12% were coded as ½. For "Brexit" referendum, $t = 0$ is the position of leaving the EU, $t = 1$ the status quo.

While there are many variables included in studies on voter turnout (Smets and van Ham, 2015), these are the main variables that can be expected to affect both levels of uncertainty and turnout and vote choice. While controlling for second-order effects is common in the referendum

[17] *Pro integration* is a 0-1 scale of pro integration attitudes; the *Uncertainty* variables 0-1 scales of subjective assessments of uncertainty regarding the two possible outcomes, remain and leave, respectively; *Education* is a dummy variable whether the respondent has a university degree; the *Female* variable is a dummy variable; *Age* represents age in years; *Income* is an ordinal scale of income at fifteen levels; *Impact 'Brexit'* is a subjective assessment of the impact of the 'Brexit' referendum on the country as a whole.

literature (Reif and Schmitt, 1980), none of the main parties was clearly supportive of Brexit and all parties were internally split—we therefore have no reason to expect significant differences when controlling for party support.[18] In her more general explanation of vote choice in the 'Brexit' referendum, Hobolt (2016) includes a range of other variables in the empirical analysis, including identity, trust, affiliation, and attitude variables, but these are unlikely to be confounders for the relationship between uncertainty and voting behavior.

Analysis and results

A full test of our model requires multiple regression analysis to include appropriate controls and to incorporate the interactions between the components of our model as implied by the theoretical model. Because of the two-step nature of the decision to vote—first whether to participate and then how to vote—we use a Heckman selection model. For both choice problems, we use a probit Heckman model to estimate

$$\Pr(y_k = 1) = \Pr[y_k^* \geq 0],$$

with $k = 1$ for the turnout model and $k = 2$ for the vote choice model, and where y_k^* is a latent utility that follows from the theoretical model above. In the selection component of the Heckman model, the dependent variable y_1 takes the value 1 if a voter turns out to vote and 0 if not. In the outcome component, the dependent variable y_2 takes the value 1 if a voter supports the integrationist position, i.e. she votes 'remain', and 0 if she rejects integration, i.e. she votes 'leave'.[19]

[18] Including a dummy variable for support for an opposition party does not affect the estimates significantly. Estimates available upon request.

[19] No survey weighting was applied in the analysis. Survey weighting is primarily important when focusing on descriptive statistics or when correcting for explicit sampling schemes. Here we use multiple regression models, while the respondents are not purposely sampled based on the relevant outcome variables in the model. Nevertheless, there is potentially a correlation between participating in the referendum and participating in the survey, and we do indeed oversample voters. Endogenous sampling is a motivation to apply survey weights (Solon et al., 2015). Upon request we can provide results when applying weights based on turnout, using a linear probability model for the vote choice. See table A3 in Appendix for results when applying weights based on turnout, using a linear probability model for the vote choice.

Our model specification is directly derived from the theoretical model (see Appendix for details). The latent regression for participation is[20]

$$y_1^* = \beta_0 + \beta_1 i(\Delta_0 - \Delta_1) + \beta_2 \Delta_0 + \beta_3 \Delta_1 + \beta_4 i + x_1' \beta_5 + \epsilon_1, \qquad (13)$$

where x_1 is a vector of control variables. This is equivalent to equation (13). In our empirical estimation, we leverage survey questions assessing i, Δ_0 and Δ_1 directly, while we take $E[t_0]$ and $E[t_1]$ to be incorporated in the regression coefficients for the Δ_0 and Δ_1 dependent variables, respectively. It should be noted that in our theoretical model, Δ_0 and Δ_1 do not vary by voter. We take the individual answers to the survey questions as individual assessments of this aggregate level of uncertainty. Note that since $0 < t_0 \leq t_1 < 1$we can assume $E[t_0]$ and $E[t_1]$ to be positive and therefore provide expectations with regards to the signs of the coefficient estimates. Indeed, our theoretical model suggests that $\beta_1 > 0, \beta_2 < 0$ and $\beta_3 > 0$.

The latent regression for vote choice is

$$\begin{aligned} y_2^* = \gamma_0 + \gamma_1(\Delta_0^2 - \Delta_1^2) + \gamma_2(E[t_1] - E[t_0])i + \gamma_3 i \\ + \gamma_4(E[t_1] - E[t_0]) + x_2' \gamma_5 + \epsilon_2, \end{aligned} \qquad (14)$$

where x_2 is a vector of control variables.

This empirical specification follows directly from equation (6). Since we assume that voters are aware that $t_1 > t_0$, we can take $E[t_1] - E[t_0]$ to be the equivalent of $|E[t_1] - E[t_0]|$ and therefore use the question on the impact of the referendum on the UK as a measure of this difference. Table 2 presents the main result from our model.[21]

[20] For the sake of readability, subscripts indicating the respondent have been omitted.

[21] We make use of the Toomet and Henningsen (2008) package in R for our estimations.

We first turn to the selection mechanism of the model—the turnout equation. The derived model specification suggests that there should be a positive impact of the interaction between ideal point i and the difference in uncertainty between the 'Brexit' option and remaining in the EU, respectively, $(\Delta_0 - \Delta_1)$. With or without control variables, we find indeed a strong, positive effect. This provides direct support for H_2, which summarizes this interaction, stating that for pro-integration voters (high i), greater uncertainty around remaining in the EU (Δ_1) should lead to lower turnout, and that for anti-integration voters (low i), greater uncertainty around leaving the EU (Δ_0) should lead to lower turnout. Looking separately at the impact of uncertainty, we find the expected negative effect for β_2, but not the positive sign on β_3.

Secondly, we investigate the outcome component of the model—the vote choice equation. Hypothesis H_1 states that the greater the uncertainty around 'Brexit', relative to the uncertainty around the status quo, the greater the probability of a vote in support of remaining in the EU, and vice versa. Here we look at the model explaining vote choice, in this case a probit regression explaining a vote in support of remaining. Following our derivation of the theoretical model, we should find that there is a positive interaction between ideal point i and the difference between the status quo t_1 and leaving the EU t_0 and we indeed find a strong, positive coefficient, with or without control variables.

Lastly, we expect that greater uncertainty around the 'Brexit' option should lead to more support for the option to remain in the EU, i.e. we should obtain a positive coefficient on $(\Delta_0^2 - \Delta_1^2)$, which we do. We therefore find strong support for our theoretical model, and H_1, based on the regression analysis.

Given the outcome of the referendum, one expects a model that shows how turnout and uncertainty generated the 'Brexit' outcome. Instead, our empirical analysis shows how uncertainty and turnout reduced support for 'Brexit'. The key insight from this analysis is that British voters are sufficiently skeptic of European integration that had they been less uncertain about the consequences of the 'Brexit' option, the result would have been even more strongly in the 'Brexit' direction. High levels of uncertainty around the 'Brexit'

option reduced turnout and made some moderate supporters reluctant to vote leave.

	(1)		(2)	
	turnout	remain	turnout	remain
Pro integration (I)	−0.459***	3.411***	−0.339***	3.445***
	(0.053)	(0.133)	(0.075)	(0.167)
Uncertainty Remain ($\Delta 1$)	−0.193*		−0.204	
	(0.106)		(0.158)	
Uncertainty Leave ($\Delta 0$)	−0.379***		−0.244	
	(0.108)		(0.162)	
$I \times (\Delta_0 - \Delta_1)$	0.542***		0.703**	
	(0.179)		(0.270)	
Impact Brexit ($t_1 - t0$)		1.338***		1.397***
		(0.076)		(0.110)
$(t_1 - t_0) \times I$		1.597***		1.415***
		(0.169)		(0.220)
($\Delta 2 - \Delta 2$) 0 1		1.287***		1.216***
		(0.058)		(0.081)
N	21,607		13,388	

*p < .1; **p < .05; ***p < .01

Table 2: Heckman selection models for voting behavior in the 'Brexit' referendum. Both the selection mechanism, turnout, and the outcome, voting for remain, are modelled using probit regression specifications. The turnout and vote choice equations are estimated jointly using Maximum Likelihood estimation. Model 2 includes controls for education, gender, age, and income, which are omitted from the table.

The interpretation of an interaction effect in a binary choice model is notoriously problematic (Ai and Norton, 2003; Berry et al., 2010), due to the inherent interactions between all exogenous variables in a non-linear model. Ai and Norton (2003) demonstrate how a positive and significant coefficient on an interaction term is neither a sufficient nor a necessary requirement for a positive interaction effect. Indeed, in our turnout model specification we find a positive and significant interaction effect when we use a non-linear logistic or probit regression, but we find a negative and significant interaction effect when we use a linear probability model.[22] Ai and Norton (2003) focus on the expected value of the outcome variable, thus the impact of the exogenous variables on the probability of observing a positive outcome. Our theoretical model, however, does not stipulate that the probability (y), but that the underlying utility function (y^*) is affected by the interaction term. In this case we can directly interpret the coefficient in the probit model (Berry et al., 2010, 261).

When one side stays home

We have demonstrated how uncertainty, partly through turnout, resulted in weaker support for 'Brexit' than might have been, given British attitudes towards integration. This also shows the importance of incorporating turnout in empirical models explaining referendum voting behavior. Here we

[22] These empirical results are available upon request.

will evaluate the extent to which ignoring turnout biases aggregate empirical predictions of the referendum vote.

To do so we estimate predicted vote probabilities for a hypothetical set of voters, who are at the median of all variables, but vary on pro-integration attitudes and (relative) levels of uncertainty. For these we calculate predicted probabilities to vote in favor of remaining within the EU based on only the outcome component of the Heckman model. We then calculate predicted participation probabilities based on the selection component multiplied by the probability to vote 'remain'. Figure 3, below, shows the difference in predicted probabilities when only looking at the vote choice model, or when including turnout probabilities—i.e. by calculating

$$P(\textit{remain vote}) = P(\textit{remain vote} \mid \textit{participation}) \cdot P(\textit{participation}).$$[23]

[23] Curves are smoothened curves using the generalized additive model implementation of the geom_smooth() function in Version 2.2.1 of the ggplot2 library in R (Wickham, 2015), using second order polynomials.

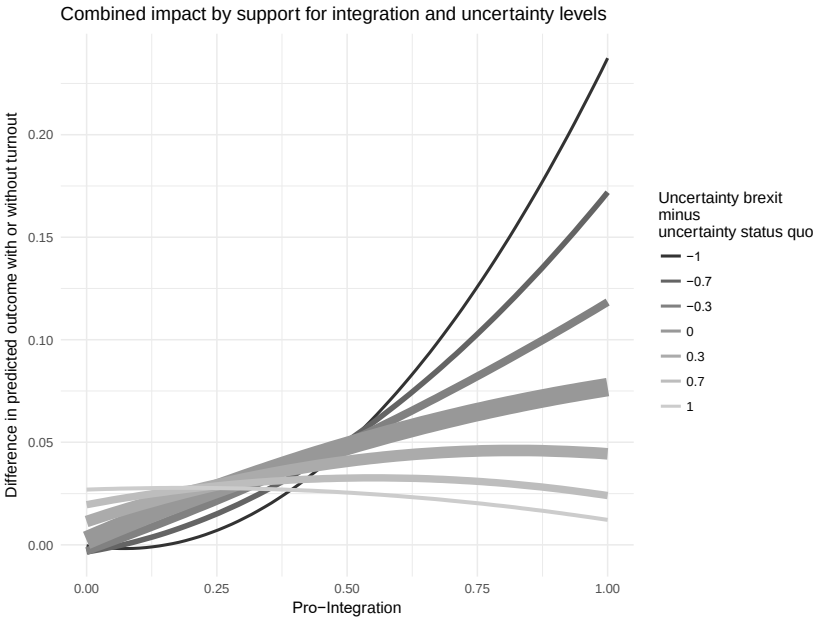

Figure 3: Overall overestimation in impact when ignoring turnout for the 'Brexit' referendum, using smoothened curves. The difference is calculated as the probability of a 'remain' vote calculated for voters only, minus the probability of a 'remain' vote multiplied by the probability to turn out. The predicted vote for 'remain' based on median values on all remaining variables. Thickness of lines is proportional to the frequency in the sample.

This can lead to an overestimate of the support for the 'remain' option by up to approximately 20%. When looking at the more common situation of slightly greater uncertainty around 'Brexit' than around the status quo, we still overestimate the 'remain' vote by 8%. Given that the 'Brexit' option won by 3.8 percentage points, a prediction error of 8 percentage points is substantial.

An alternative approach is not to take a hypothetical population keeping variables at their median, but to use the sample distribution of the survey to calculate predicted probabilities. As shown in Figure 4 below.

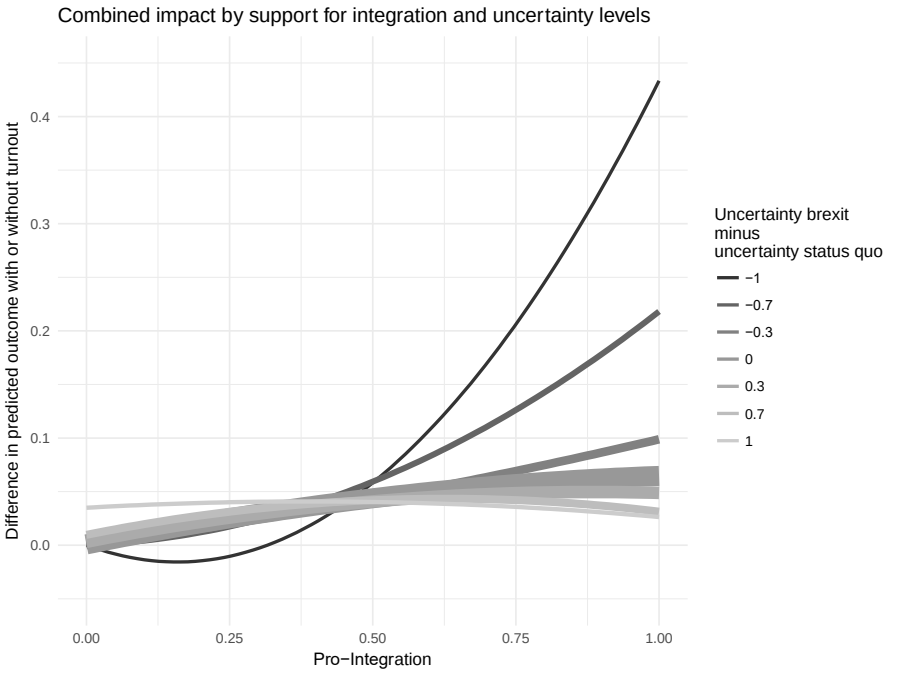

Figure 4: Captures the same overestimation in impact as figure 3 but bases the predicted vote for 'remain' on the empirical distribution of all variables in the sample. Thickness of lines is proportional to the frequency in the sample.

Here the misestimation is even larger. For those voters who are very uncertain about remaining within the EU, but quite confident about the situation in the UK after 'Brexit', we dramatically overestimate support for 'remain'. This is a rare combination—26% of the respondents are more uncertain about 'Brexit' than about the status quo, while the reverse is true for only 9%. Based on the empirical distribution of covariates, we therefore expect that the overall support for the 'remain' option in the referendum is overestimated by up to 7 percentages points when only vote choice and not turnout is considered. On average, across all voters in the sample, the overestimate is 2.7%.

IV. Conclusion

We present a theoretical model describing how uncertainty in a referendum impacts turnout and vote choice in a direction that exacerbates the impact on the overall outcome of the election. The model predicts that extreme voters who prefer one of the two options tend to abstain when the uncertainty about their preferred option is large. Greater uncertainty about its consequences not only reduces the value of an option, but the difference in value between the two options also vanishes for more *distant* voters. Uncertainty, thus, has a double effect: potential supporters of one option are less likely to vote; and citizens who vote are less likely to support that option.

Our theoretical model contributes significantly to the theoretical literature on turnout and vote choice in binary elections and referendums as it proposes an argument for participation that is not based on being pivotal and does not assume one option is superior. Instead, we capture participation as a reflecting increased interest in the referendum. This enables us to consider political campaigns as mechanism that have the potential to curb or foment the citizens' decision to contribute.

We test our theoretical predictions using a survey after the 'Brexit' referendum in the UK. The empirical analysis confirms our expectations and provides new insights into voting behavior. We find that turnout among 'Brexit' voters would have been higher and even greater numbers would have voted for 'Brexit', if uncertainty around the 'Brexit' option had been smaller. Our analysis therefore demonstrates the importance of taking turnout into account when explaining or predicting voting behavior. In the 'Brexit' referendum, ignoring turnout leads to an overestimation of support for the status quo by 2.7% on average across our sample—and much higher for some groups of voters.

While we apply the model to the 'Brexit' referendum, it can also be applied to any referendum or election with two candidates or two parties. The original inspiration for this paper were the referendums on the Nice Treaty in Ireland in 2001–02, where a change in turnout resulted in a radically different outcome when the same referendum was repeated a year later. We thus continue the endeavor by Sanders (1998) and Sattler and Urpelainen (2012) to

encourage empirical researchers to combine turnout and vote choice in single model specifications.

The findings do not only affect how we should model voting behavior in referendums and elections with two candidates, but it also has implications for democracy and legitimacy more generally. The magnified role of uncertainty in referendums suggests that creating confusion and providing misinformation—or "fake news"—can successfully undermine the deliberative quality of the democratic process. Secondly, the mechanism as outlined in our model leads to a misalignment between the referendum outcome and the preference distribution of the electorate, whereby a minority potentially manages to impose its preferred option although a majority would be better off with the losing alternative.

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APPENDIX: Proofs of the Lemma and Proposition

The set *V* contains all i such that equation (12), in the text, holds. Thus, the condition that must hold for a voter to participate is

$$(\rho_H - \rho_L)[i(\Delta_1 - \Delta_0) - t(\Delta_1 - \alpha\Delta_0)] > 0.$$

A Bayesian Nash equilibrium refers to a situation where the set of voters V and the beliefs $\rho_H \in \{0,1\}$ and $\rho_L \in \{0,1\}$ are consistent. This means that the above inequality holds given the beliefs held by each citizen and that the beliefs are accurate given the voters' behaviour.

Since $i_L \leq i_H$ (see footnote 13), the set of voters supporting option t_1 when the signal is H is a subset of the set of voters supporting option t_1 when the

signal is L, regardless of participation. A low signal will lead all voters $i \in [i_L, i_H]$ who prefer t_0 when $s = H$ to change their mind, while all other voters still support the same option. Therefore, as the signal changes from H to L, the support for option t_1 widens (or stays the same) regardless of

participation. Hence, any consistent beliefs must be such that: $\rho_H = 1 \Longrightarrow \rho_L = 1$.[24]

Let i_M^V denote the median voter within set $V \subset [0,1]$.[25]
In what follows we will show that potential abstaining voters are either on the extreme right or the extreme left but never in the middle. Therefore, under a majority rule, we have

$$\rho_S = 1 \Leftrightarrow i_S \leq i_M^V$$

Proof of the Lemma
Assume that voters believe that option t_1 is adopted only when the signal is low $\rho_H = 0$ while $\rho_L = 1$. The condition can be re-written as

$$[i(\Delta_1 - \Delta_0) - t(\Delta_1 - \alpha\Delta_0)] < 0.$$

For any $\Delta_1 \in [\alpha\Delta_0, \Delta_0]$ the above always holds for all i and $V = [0,1]$.
The individuals' initial beliefs $\rho_H = 0$ and $\rho_L = 1$ reflect the outcome of the referendum, provided $i_L \leq i_M^{[0,1]} \leq i_H$. In any other situation, the voters' beliefs would not be consistent.

Proof of the Proposition

Let us consider that citizens believe that the signal sent during campaign can sway the outcome of the referendum so that $\rho_L = 1$ and $\rho_H = 0$.

When $\Delta_1 < \alpha\Delta_0$, the condition given above can be re-written as

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[24] This enables us to discard beliefs such that $\rho_H = 1$ and $\rho_L = 0$ as these would be inconsistent, irrespective of the turnout decision.
[25] The exact location depends on the distribution of voters.

$$i > \frac{t(\alpha\Delta_0 - \Delta_1)}{(\Delta_0 - \Delta_1)}.$$

This means that some voters on the extreme left stay at home and

$$V = \left]\frac{t(\alpha\Delta_0 - \Delta_1)}{(\Delta_0 - \Delta_1)}, 1\right].$$

The voters' initial beliefs are consistent provided the distribution of voters is such that $i_L \leq i_M^V \leq i_H$ where $i_M^{[0,1]} \leq i_M^V$.

When $\Delta_1 > \Delta_0$, the condition given above can be re-written as

$$i < \frac{t(\Delta_1 - \alpha\Delta_0)}{(\Delta_1 - \Delta_0)}.$$

This means that some voters on the extreme right stay at home and

$$V = \left[0, \frac{t(\Delta_1 - \alpha\Delta_0)}{(\Delta_1 - \Delta_0)}\right[.$$

The voters' initial beliefs are consistent provided the distribution of voters is such that $i_L \leq i_M^V \leq i_H$ where $i_M^V \leq i_M^{[0,1]}$.

Addendum

Assume that we assume now that consider that $V = \{i: U_i^R \leq E_s(U_{i,s}^P)\}$. The participation condition becomes $(\rho_H - \rho_L)[i(\Delta_1 - \Delta_0) - t(\Delta_1 - \alpha\Delta_0)] \geq 0$. Assume that voters believe that the signal released during the campaign will not affect the outcome so that we have $\rho_H = \rho_L$. The inequality above now binds for all i so that $V = [0,1]$ and two possibilities arise.

Possibility 1: The population is sufficiently polarised and option 1 is either always selected or never selected: $i_L \leq i_H \leq i_M^{[0,1]}$ or $i_M^{[0,1]} \leq i_L \leq i_H$. In such a case believing that $\rho_H = \rho_L$ is consistent and we have an equilibrium as beliefs are consistent with the observed behaviour and the behaviour is consistent with the beliefs.

Possibility 2: the population is not polarised and option 1 is only selected when the signal is L: $i_L \leq i_M^{[0,1]} \leq i_H$. In this case believing that $\rho_H = \rho_L$ is not consistent.