

# Matrices with Prescribed Spectral and Structural Constraints

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What lists of real numbers can be the eigenvalues of a symmetric matrix with nonnegative entries? This open question [1] is an example of the type of questions considered in the project.

Joint consideration of spectral and structural conditions on a matrix lead to challenging questions that call upon methods not only from linear algebra, but also combinatorics, number theory, and analysis. The spectral data involved may consist of complete or only partial information on the rank, the eigenvalues or eigenvectors, and the structural constraints may include zero-nonzero pattern, entry-wise nonnegativity, symmetry and sparsity. We will explore classes of structured matrices and matrix constructions.

The project has the scope to cater to students with diverse interests: a combinatorially minded student may explore structured matrices whose pattern is determined by a graph [2], while a student with a bent more towards analysis may choose to investigate sign patterns of coefficients of polynomials (or even power series) and connect this investigation with companion-type matrices [3]. Matrices with integer entries connect this project with number theory [4]. Potential applications can be explored [5].

## References

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