Enumerative Combinatorics:
Patterns, Posets, and Models in Statistical Physics

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Enumerative combinatorics concerns the study of discrete structures and their representations that have proven useful in an enumerative context. Some people working in this area consider it a tool or a way of thinking rather than an area due to its broad applicability. A contemporary topic in this area is the theme of patterns in structures, such as permutations and partially ordered sets, and structures that avoid a given sub-structure. One famous example of this is Wagner's theorem (1937), a classical result in graph theory which tells us that a graph can be drawn in the plane with no overlapping edges if and only if it contains no minor that is isomorphic to the two graphs K_5 and $K_{3,3}$. My recent research in the area of pattern avoidance has shown that a wealth of connections between very different structures can be proven if one is imaginative about what precisely a pattern is or can be.

In addition to this and within the realm of enumerative combinatorics, but motivated by statistical physics, another theme in my research has been combinatorial aspects of the Abelian sandpile model. The gist of this research has been exploring combinatorial characterizations of recurrent (or critical) states of the sandpile model on a variety of classical graphs. The correspondences we have uncovered have formed the basis for a more algebraic consideration of the objects and given rise to new classes of polynomials that have some surprising properties.

I have a range of topics and problems in the areas of pattern avoidance (in permutations, posets, and different types of sequences) and the combinatorics of the sandpile model that would be ideal for a PhD student to explore and build their research profile upon.