PhD-Project title: Fischer Operators for Analytic Partial Differential Equations

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The search for the singularities of a solution of a partial differential equation with respect to analytic boundary data is an interesting but difficult problem, and even in the case of the Laplacian only partial results are known. Classical methods from complex analysis in one variable, like the Schwarz function, have been partially generalized by D. Khavinson and H.S. Shapiro to the context of several variables in the 80's and 90's, but many interesting questions are still open. In this project it is proposed to use new results of the principal investigator based on Fischer operator methods to analyse the behaviour of singularities. These methods have been successfully applied for solving the Khavinson-Shapiro conjecture for a large class of domains.

Key words: Cauchy-Kovaleskaya theorem, Fischer decomposition, Schwarz function, Schwarz potential, Polyharmonic function, Quadrature domain.

For first reading we refer to

1. D. Khavinson, E. Lundberg, *Linear holomorphic partial differential equations and classical potential theory,* Amer. Math. Soc., Providence, RI, 2018

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4. D. Khavinson, E. Lundberg, H. Render, *Dirichlet's problem with entire data posed on an ellipsoidal cylinder.* Potential Anal. 46 (2017), no. 1, 55–62.

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