Random walks on random graphs

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Many random walk problems exhibit interesting and counter-intuitive behaviour when a small amount of disorder is introduced. One such example is the random conductance model (see [1]) which we will study in this project. This model naturally embeds itself into the fields of electrical networks, harmonic analysis and gradient fields. Natural questions for this model concern its long term behaviour, its hitting probabilities and the differences between its quenched and annealed behaviour.

One specific area of interest concerns the random conductance model on the complete graph in which we construct a sequence of weighted graphs in the following way. Let μ be a law on $\mathbb{R}_+$ and let $(c_{i,j})_{i,j=1}^{\infty}$ be a family of independent and identically distributed random variables with law μ. Let G_n be the complete graph on vertices $1, \dots, n$ with edges between vertices i and j weighted by the conductance $c_{i,j}$. Let $(X_k)_{k=0}^{\infty}$ be a random walk on G_n with $\mathbb{P}(X_0 = 1) = 1$ and

$$\mathbb{P}(X_{k+1} = z | X_k = x) = \frac{c_{x,z}}{\sum_y c_{x,y}}.$$

We can then ask questions such as: Is it possible for the walk to visit any vertex in the graph? Given fixed sets of vertices A and B, does the probability that the walk visits A before B converge as $n \to \infty$? What if the sets A and B expand as $n \to \infty$? How sensitive is the behaviour with respect to the starting point of the walk and the specific values of the conductances? Do the same results hold under weaker conditions than independence?

References

[1] Marek Biskup. Recent progress on the random conductance model. *Probab. Surv.*, 8:294–373, 2011.