Mathematical Enrichment

Sat Jan 26th 10 AM

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Sat Feb 23rd

Selection Test

www.ucd.ie/mathsciences/

1988 6

Let a and b be positive integers such that $ab+1$ divides a^2+b^2.

Show that $\frac{a^2+b^2}{ab+1}$ is the square of an integer.

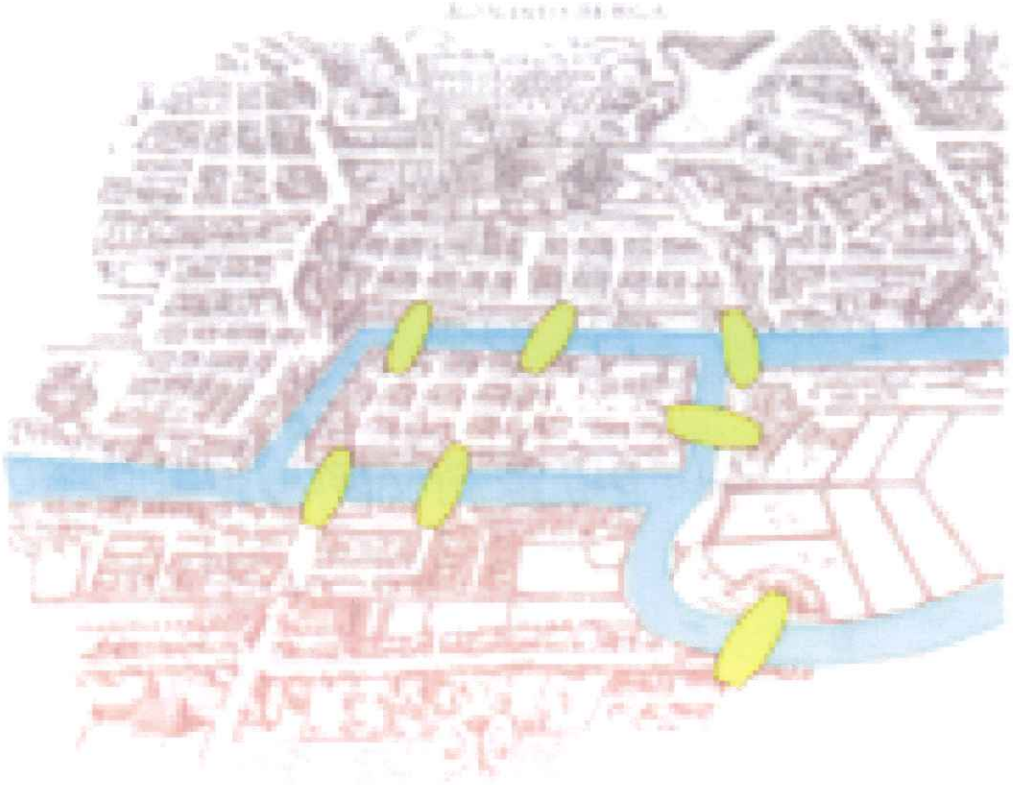
Problem-solving

insight, creativity, abstraction

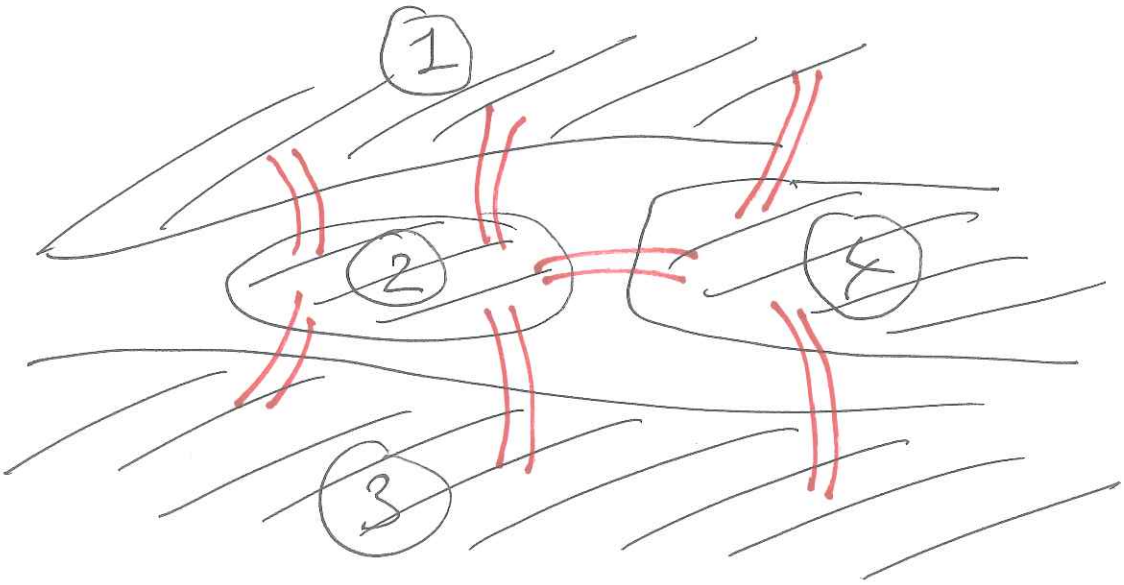
↓

"boiling a problem down to its essential elements"

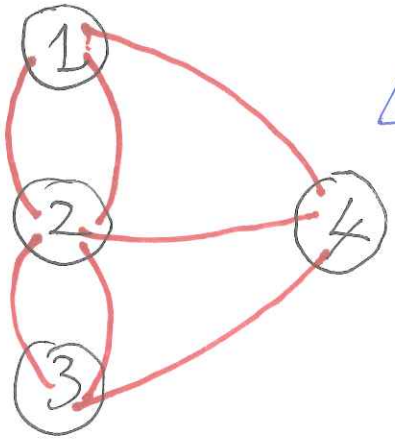
The Bridges of Konigsberg



Find a circuit beginning and starting at the same place and crossing each bridge exactly once.



Simplify/abstract further.



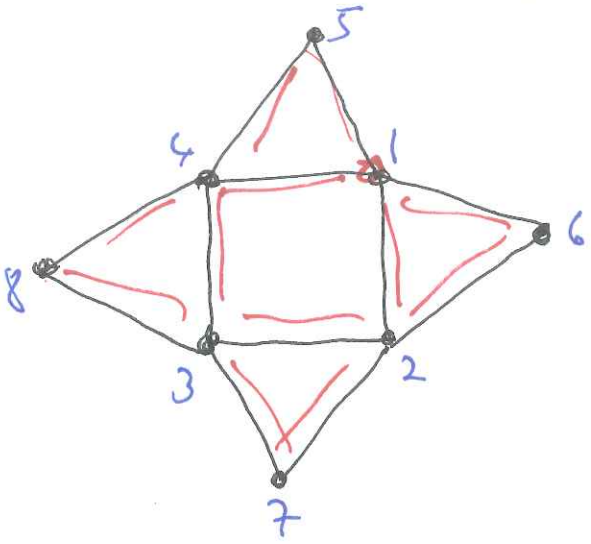
Graph with 4 vertices and 7 edges

Find an "Eulerian circuit"

"graph" (or 'network')

└ points (called 'vertices') joined by arcs (called 'edges')

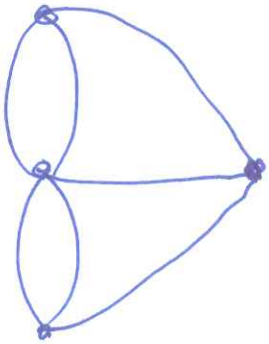
Each edge joins one vertex to another



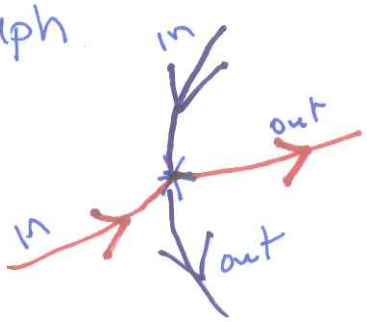
V = # of vertices = 8

E = # of edges = 12

Here, there is an Eulerian circuit.



Suppose a graph has an Eulerian circuit in a graph



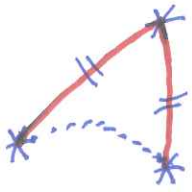
then the number of edges at each vertex has to be even.

The converse statement is also true.

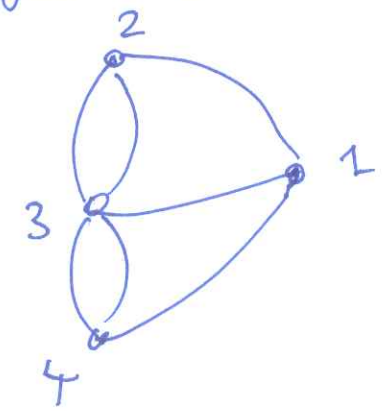
Theorem A connected graph has an Eulerian circuit if and only if there are an even number of edges through each vertex.

Idea of proof: Use "induction on number of edges"

and



The number of ~~vertices~~ edges through a vertex in a graph is called the degree of the vertex.



1 has degree 3
2 has degree 3
3 has degree 5 . etc.

Observe: Let $V_1, V_2, \ldots, V_n$ be the vertices of a graph. Then

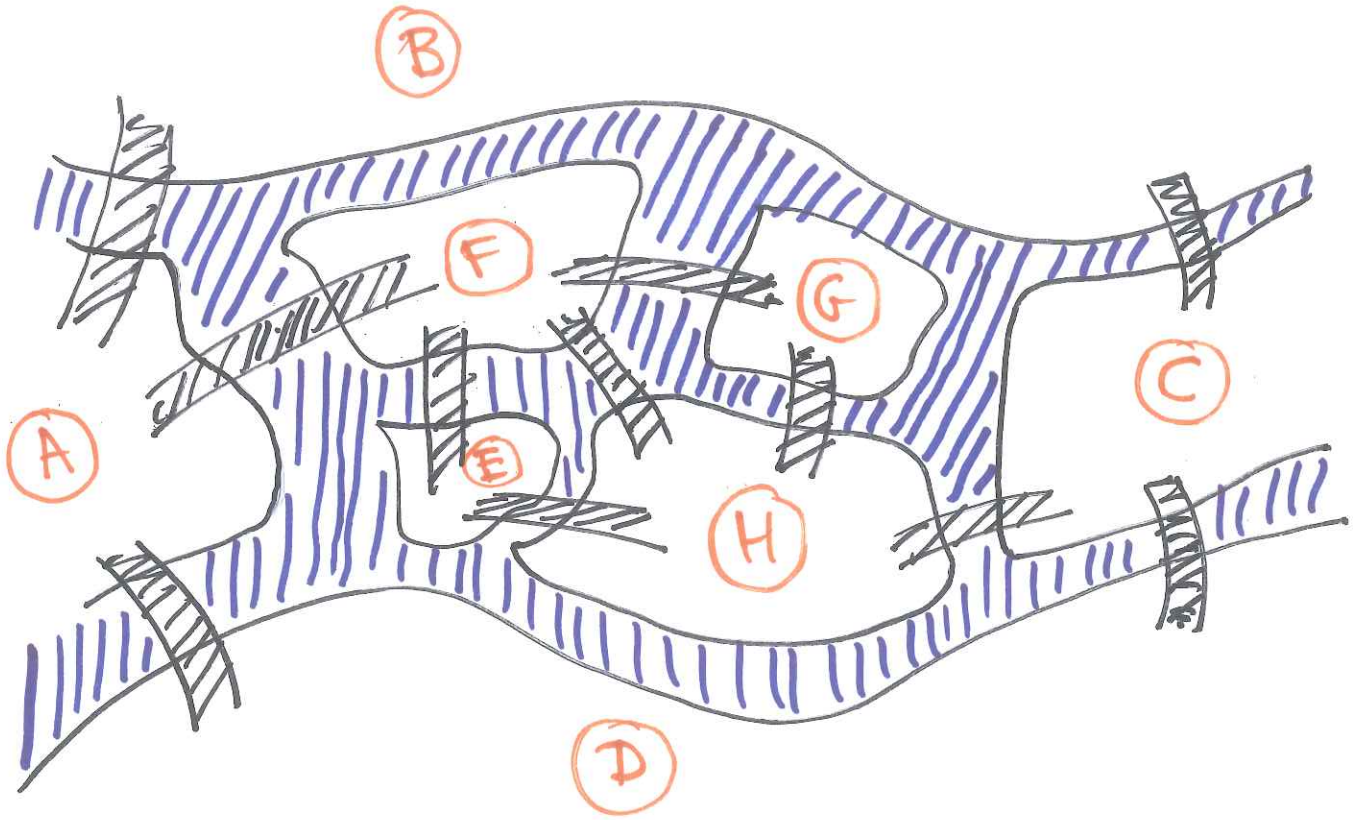
$$\deg(V_1) + \deg(V_2) + \cdots + \deg(V_n) = 2E$$

(E = # of edges.).

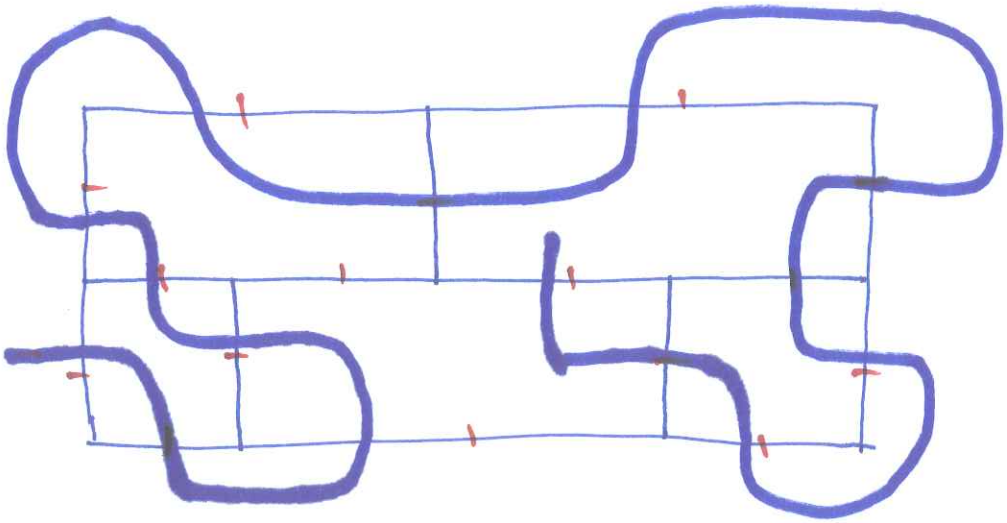
In particular, the sum of all the degrees is an even number

Example At a party, a number of people shake hands. Explain why the number of people who shake hands an odd number of times is even.

Exercise 11 people exchange Christmas cards among themselves (each sending at least one). Each card sent is reciprocated. Only Joe & and Mary send exactly the same number of cards as each other. Is this number even or odd?



The bridges of Göttingen



?

In this picture there are 16 bars.

Can you draw a continuous unbroken curve which crosses each bar exactly once?