

EXPERIMENT 8

To Determine the Wavelength of Sodium Light using Newton's Rings

Please read additional instructions on the bench for setting up the PC and camera for this experiment

Introduction

Newton's rings are interference fringes of equal thickness which are produced in the air film between a convex surface and an optical flat. It is interesting to note that these interference fringes, which demonstrate the wave nature of light, should be credited to Newton who was the chief proponent of the corpuscular theory.

The apparatus is set up as shown (Fig. 8.1). L is a convex lens placed on an optically flat plate of glass P, forming an air film of varying thickness. Light from the sodium lamp S, strikes a sheet of glass G, set at an angle such that the light is reflected downwards towards the lens and plate, P. Some of the amplitude is reflected at the lower convex lens surface and some at the glass plate. These two reflected rays travel upwards and enter the microscope and since they are coherent, they interfere in a way which depends on the phase difference introduced by the air film. Since the air film is symmetric about the point of contact, the fringes, which follow lines of equal thickness, will be concentric rings with their centre at this point. They are called fringes of equal thickness. This is an example of interference fringes produced by division of amplitude.

Background

Consider a ray of light incident on the airfilm at a point where its thickness is t . The optical path difference between the two reflected rays will be $2t$. Taking into account the phase change of π for reflection at the rare to dense surface, the conditions for constructive and destructive interference are

$$2t = \left(m + \frac{1}{2}\right) \lambda \quad (\text{constructive interference or bright rings})$$

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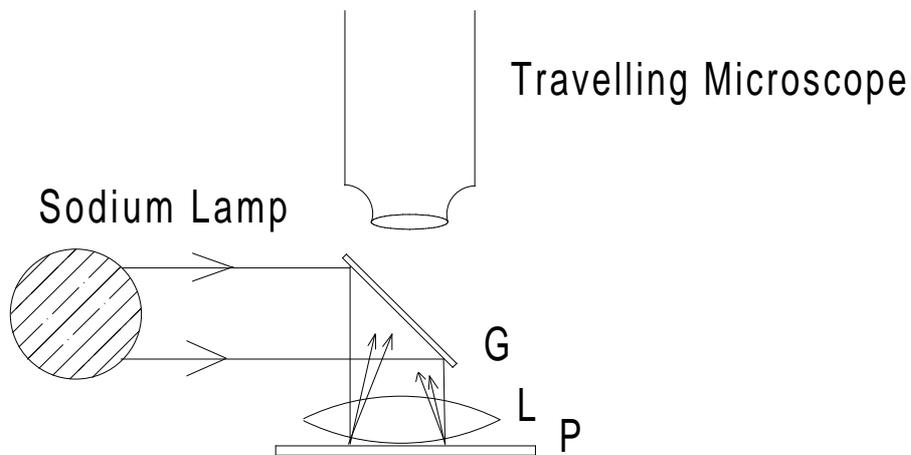


Figure 8.1: *Apparatus for observing Newton's rings*

(8.1)

$$2t = m\lambda \quad (\text{destructive interference or dark rings})$$

where m is the order of the ring and can take the values $m = 0, 1, 2, 3 \dots$

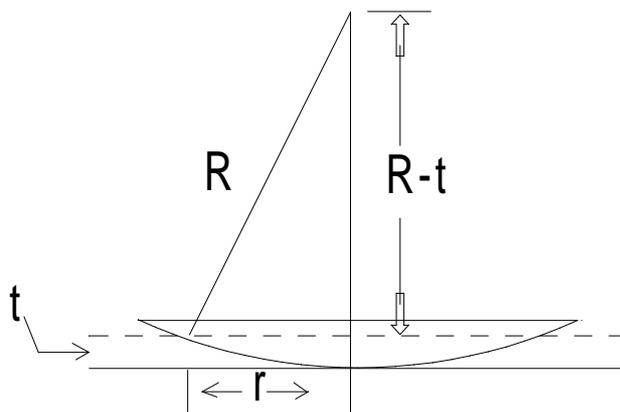


Figure 8.2: *Geometry of Newton's rings arrangement*

If R is the radius of curvature of the lens and r , the distance of the point under consideration to the point of contact of the lens and glass plate (see Fig. 8.2) then

$$R^2 = (R - t)^2 + r^2$$

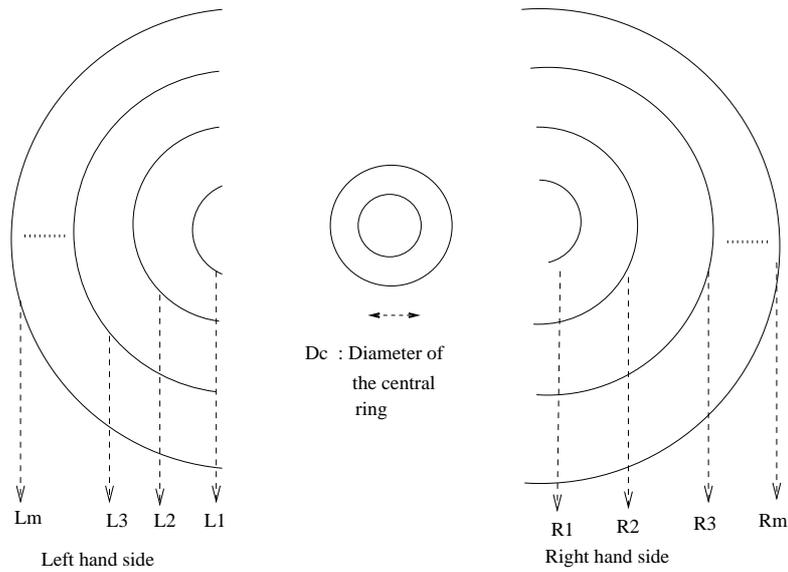


Figure 8.3: Measure the diameter of the central ring (D_c) and the positions of the rings on the left hand (L_1 to L_M) and right hand sides (R_1 to R_M).

$$\begin{aligned}
 &= R^2 - 2tR + t^2 + r^2 \\
 2t &= \frac{r^2}{R} = \frac{D^2}{4R} \quad (8.2)
 \end{aligned}$$

since $t^2 \ll r^2$ and $D = 2r$ diameter of a ring.

Combining this with the condition for, say the m^{th} dark ring, Eq. 8.1, one gets for the diameter of that ring:

$$D_m^2 = 4Rm\lambda \quad (8.3)$$

hence λ can be determined.

This is the equation used to determine λ . The same equation would be obtained if the bright rings had been taken.

Experimental Procedure

Measure the diameter of the central ring (D_C) and the positions of the rings on the left hand (L_1 to L_M) and right hand sides (R_1 to R_M) (Fig. 8.3).

The diameter of the m^{th} ring is given by:

$$D_m = |L_m - L_1| + D_c + |R_m - R_1|$$

Using Eq. 8.3, calculate the mean λ and the standard error on the mean.

Table 8.1: Sample data table

Ring Number	L_m	$ L_m - L_1 $	R_m	$ R_m - R_1 $	D_m
24					
23					
22					
..					
1					

Spherometer

A spherometer is used to find R , the radius of curvature of the lens. The lens is a segment of a sphere of radius R . The three legs of the spherometer form an equilateral triangle of side C and lie on a circle of radius a (Fig. 8.3).

The zero of the spherometer is checked using the plane surface of the optical flat and the spherometer is placed on the lens and h is found. The dashed horizontal line (Fig. 8.3) is a side view of the circle on which the legs of the spherometer lie. Then by reasoning similar to that used for Eq. 8.2.

$$\begin{aligned}
 2Rh &= a^2 \\
 &= \frac{c^2}{3} \quad \text{i.e.} \quad a = \frac{c}{\sqrt{3}} \\
 R &= \frac{c^2}{6h}
 \end{aligned} \tag{8.4}$$

Substitute for R into Eq. 8.3 and hence determine λ .

The Use of Interference Fringes to Test the Quality of Optical Components

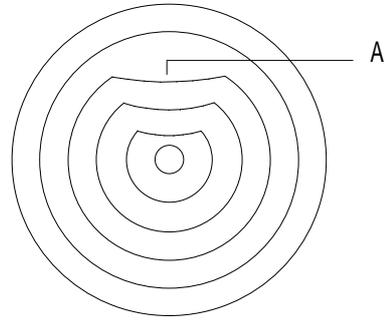
The fringes of equal thickness observed in this experiment obviously correspond to contours of the air film formed between the optical flat and the lens. The change in thickness of the air film corresponding to the distance between adjacent fringes is $\frac{\lambda}{2}$ which is of the order of 0.3 microns.

To Check that the Lens Surface is Spherical

It has been assumed that the lens surface is spherical. This can be checked using Eq. 8.3. This implies that if the lens surface is spherical

$$D_m^2 \propto m$$

Therefore imperfections of this order on the surface of the lens or the optical flat will be apparent since they will give rise to observable distortions of the fringe system. The fringe pattern shown would correspond to a hill on the optical flat or the lens surface at A of height $\frac{\lambda}{2}$. This forms the basis of interferometers used to test the quality of optical components.



The appropriate information is already available in Table 8.1. Plot D_m^2 versus m and using a linear least squares fit, determine the slope and the error on the slope. Using your value for λ calculate R and determine the error on R .

To Measure the Thickness of a Metal Foil

Interferometry can be used to accurately measure small distances or displacements. In the present experiment the thickness of a thin metal foil is measured. The arrangement used is as shown in Fig. 8.4. The metal foil is used as a spacer between the two optical flats to form a thin wedge shaped film of air.

The glass plate G is positioned so that the light from the sodium lamp is reflected downwards toward the air wedge. Some of the amplitude is reflected at the bottom surface of the first optical flat and some at the top surface of the second optical flat. These two reflected beams travel upwards and since they are coherent they will interfere depending on the path difference introduced by the air film. Since the light of interest is incident normally on the air film, to a very good approximation the fringes will follow lines of constant thickness which, in this case, are straight lines parallel to the apex of the air wedge.

For simplicity the air wedge alone is drawn in Fig. 8.5. Consider a ray of light incident where the thickness of the wedge is t . The optical path difference between the two reflected rays is $2t$ since the refractive index of air is 1.00029 and can be put equal to 1. Taking into account the phase change of π for reflection at the rare to dense surface this will be the position of the n^{th} order dark fringe if

$$2t = n\lambda \quad (8.5)$$

The thickness t can be written in terms of the position of the fringe with respect to the apex of the wedge, r_n , and the angle of the wedge α .

$$t = \alpha r_n \quad (8.6)$$

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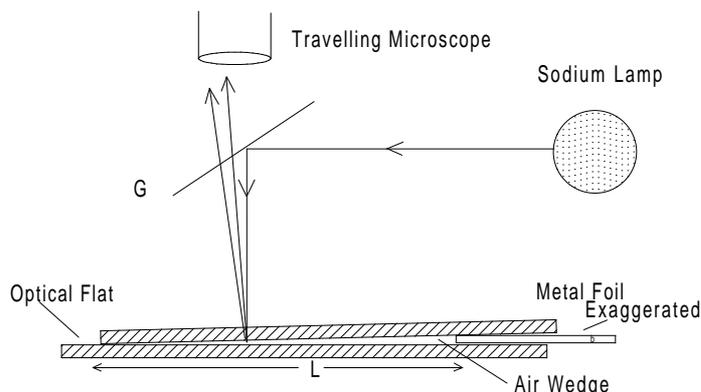


Figure 8.4: Set-up to measure the thickness of a metal foil

Substituting this in Eq. 8.5

$$2\alpha r_n = n\lambda \quad (8.7)$$

From this it follows that the distance between adjacent fringes, d , is

$$d = \frac{\lambda}{2\alpha} \quad (8.8)$$

Therefore the equation

$$\alpha = \frac{\lambda}{2d}$$

can be used to find the angle of the wedge since d can be measured. The thickness of the foil can be found from the equation (see Fig. 8.4).

$$b = L\alpha = \frac{L\lambda}{2d} \quad (8.9)$$

In setting up the apparatus (see Fig. 8.4) the upper optical flat should rest on the polished edge of the lower optical flat and the thin foil should be placed so that it is parallel to the apex of the wedge as judged using the travelling microscope. The fringes as in Newton's rings are localized fringes and to observe them the microscope should be focussed on the air wedge. One crosswire should be set perpendicular to the line of travel of the travelling microscope and if necessary the two optical flats rotated as a unit so that the fringes are aligned with the crosswire. Starting near the apex of the wedge the position of every fourth fringe is recorded and tabulated as in Table 8.2.

In Eqs. 8.6 and 8.9 it is assumed that the thickness of the wedge increases linearly with the distance from the apex. This can be checked using column 3. A value of d can also be obtained as indicated. The distance L (Fig. 8.4) should be measured a number of times and the best estimate obtained.

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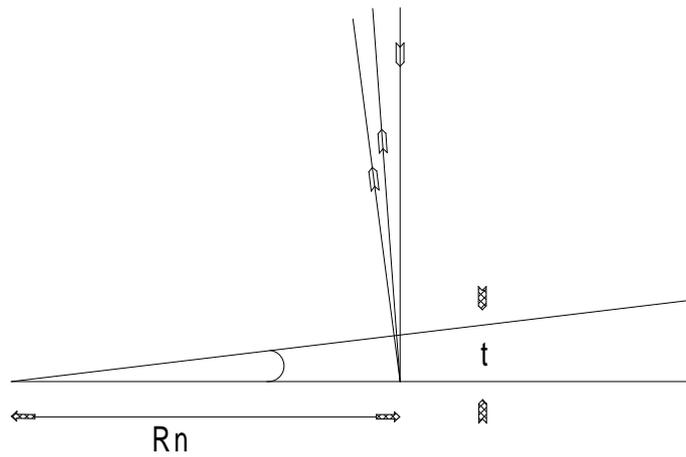


Figure 8.5: Geometry of the air wedge

Table 8.2: $d = \left(\frac{r_{m+4} - r_m}{4} \right) \pm \Delta d$

Fringe No.	Position r_m	$r_{m+4} - r_m$
0		
4		
8		
12		
16		

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The value of λ used is that determined by Newton's rings. This value will have a definite standard error. The useful precision to which d and L should be determined depends on the precision of λ . From Eq. 8.9 the fractional errors are related by

$$\left(\frac{\Delta b}{b}\right)^2 = \left(\frac{\Delta \lambda}{\lambda}\right)^2 + \left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 \quad (8.10)$$

From this equation it is seen that the quantity with the largest fractional error will make the biggest contribution to the fractional error in b . If a particular fractional error is less than $\frac{1}{3}$ that of the largest one it can be neglected since this will only make a 5% error in $\frac{\Delta b}{b}$. **To determine d or L such that their fractional errors are smaller than $\frac{1}{3} \frac{\Delta \lambda}{\lambda}$ is labour in vain. This should be kept in mind when measuring d and L**

To make a rough estimate of the separation of the Sodium D lines

You will notice that the quality of the fringes deteriorates as you move away from the apex of the wedge, and that they soon disappear. The visibility, V , is a measure of the quality of the fringes and is defined as

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad (8.11)$$

where I_{max} is the intensity of a bright fringe and I_{min} the intensity of the adjacent dark fringes. $V = 0$ corresponds to uniform intensity i.e. no fringes. When $I_{min} = 0$, $V = 1$ corresponding to the sharpest fringes. The reason why the visibility of the fringes decreases as you move away from the apex is that the yellow sodium line is not monochromatic but is a doublet i.e. it consists of two lines with a small wavelength separation. Each line will produce its own set of interference fringes and the eye will see the resultant intensity distribution. According to Eq. 8.7 the position of the zero order fringe ($n = 0$) is independent of wavelength and is situated at the apex of the wedge. The positions of the first order fringes will be at $\frac{\lambda_1}{2\alpha}$ and $\frac{\lambda_2}{2\alpha}$ where λ_1 and λ_2 are the wavelengths of the lines in the doublet. The first order fringes are therefore separated by

$$\frac{\lambda_1 - \lambda_2}{2\alpha}$$

The ratio of this separation to the distance between fringes (Eq. 8.8) is

$$\frac{(\lambda_1 - \lambda_2)/2\alpha}{\bar{\lambda}/2\alpha} = \frac{\lambda_1 - \lambda_2}{\bar{\lambda}} \quad \text{where } \bar{\lambda} = \frac{\lambda_1 + \lambda_2}{2}$$

and can be very small, typically of the order of 10^{-3} . $\bar{\lambda}$ is the wavelength determined in the previous experiment. The separation of the n^{th} order fringes due to λ_1 and λ_2 will be

$$\frac{n(\lambda_1 - \lambda_2)}{2\alpha}$$

It follows that near the apex of the wedge (i.e. small n) the fringes due to λ_1 and λ_2 will be more or less in step and the visibility of the resultant fringe system will be ~ 1 (Fig. 8.6a). As you move

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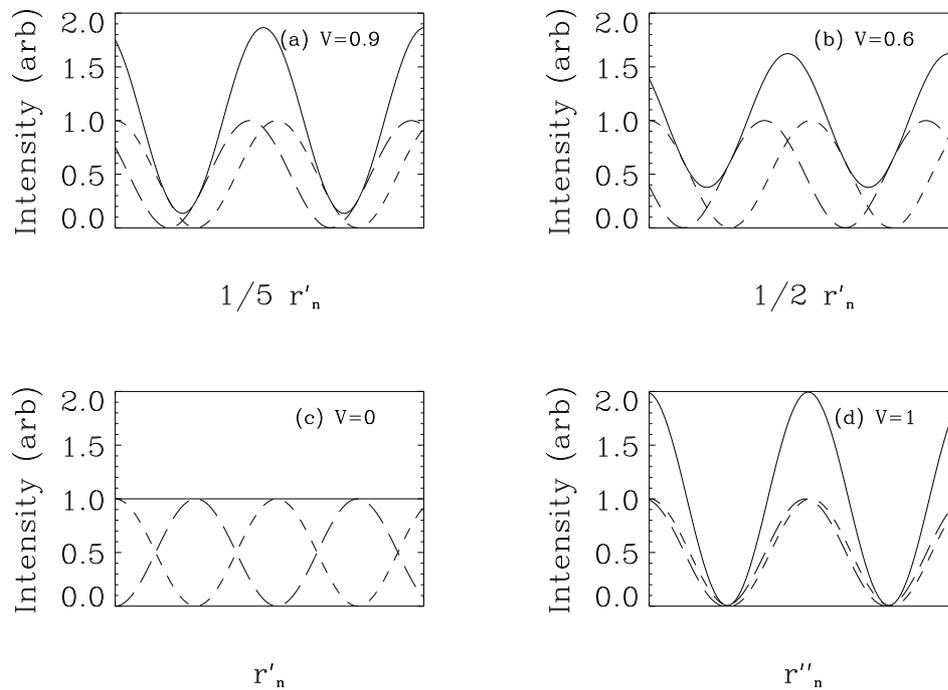


Figure 8.6: Visibility of the fringes due to the Na doublet lines

away from the apex of the wedge (i.e. as n increases) the fringes due to λ_1 and λ_2 will get more and more out of step with a corresponding decrease in the visibility of the resultant fringe system (Fig. 8.6b). As you move further out the fringes will eventually get out of step by half the distance between fringes, at which stage the order of the fringe n' is given by

$$\frac{n'(\lambda_1 - \lambda_2)}{2\alpha} = \frac{1}{2} \frac{\bar{\lambda}}{2\alpha} \quad (8.12)$$

and if the intensity of the two lines is the same, the fringes will combine to give uniform intensity i.e. $V = 0$ (Fig. 8.6c). As you move further out the fringes should appear again and their visibility increase until it again becomes equal to 1, when the separation between the fringe system due to λ_1 and λ_2 is equal to the distance between fringes (Fig. 8.6d). and the order n'' is given by

$$\frac{n''(\lambda_1 - \lambda_2)}{2\alpha} = \frac{\bar{\lambda}}{2\alpha} \quad (8.13)$$

In practice you will find that although the fringes reappear their visibility is considerably less than what it was near the apex of the wedge. The reason for this will be explained in the next section.

Eqs. 8.12 and 8.13 can be written in terms of the distance from the apex of the wedge at which these situations occur by replacing n' and n'' using Eq. 8.7.

$$\text{Eq. 8.12} \quad (\lambda_1 - \lambda_2) = \frac{\bar{\lambda}^2}{4\alpha r n'}$$

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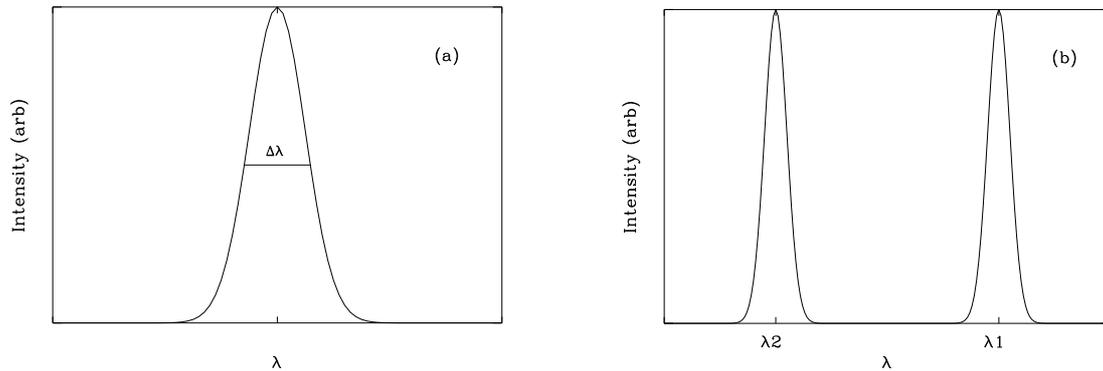


Figure 8.7: Finite line width of spectral lines

$$\text{Eq. 8.13 } (\lambda_1 - \lambda_2) = \frac{\bar{\lambda}^2}{2\alpha r_n''}$$

Only a rough estimate can be made of $r_{n'}$ and $r_{n''}$ but these can be used to get a rough value for $\lambda_1 - \lambda_2$.

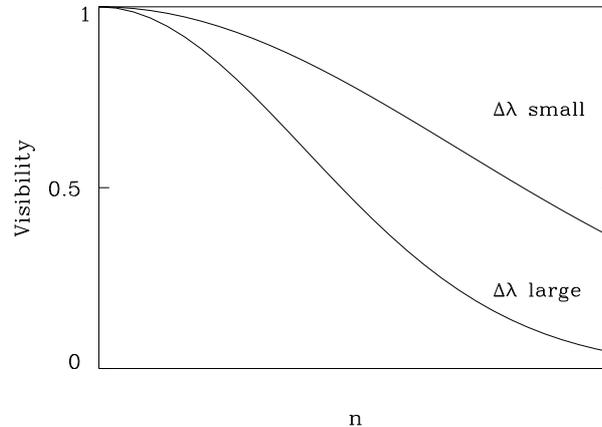


Figure 8.8: Dependence of the visibility as a function of n for different line widths

Width of spectral lines

So far we have assumed that we were dealing with monochromatic light, that is light of a definite wavelength. This is an idealization that cannot be achieved in practice. In practice all spectral lines have a finite width which depends on the transition involved and on the conditions of excitation. The energy distribution in a spectral line will typically be as shown in Fig. 8.7a.

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It was noted in the last section that when the fringes reappeared their visibility was much reduced from what it was near the apex of the wedge. This is due to the finite width of the two lines of the yellow sodium doublet (Fig. 8.7b). Consider one of those lines for simplicity. It contains a small range of wavelengths. Each wavelength could be regarded as producing a set of interference fringes and the eye will see the resultant intensity distribution. The zero order fringe corresponding to all the wavelengths will coincide since its position is independent of wavelength (Eq. 8.7). For small values of n the fringe systems will be in step for all practical purposes and the resultant fringe system will have unit visibility. As n increases, the fringe systems, due to the different wavelengths in the line, will get out of step with a corresponding decrease to zero. It follows from Eq. 8.12 that the rate of change of the visibility with order $\frac{dV}{dn}$ will depend on the width of the spectral line, the smaller the width the smaller $\frac{dV}{dn}$ as shown in Fig. 8.8.

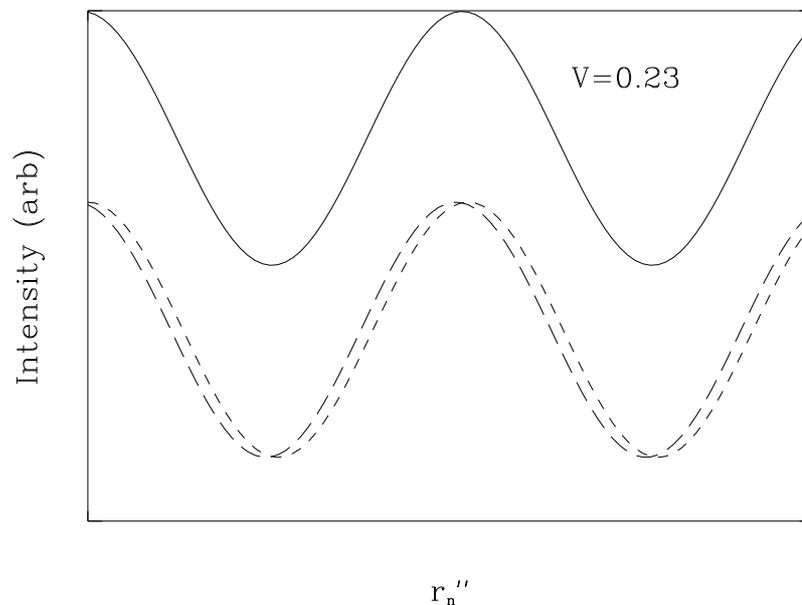


Figure 8.9: Modified fringe pattern, taking finite line widths into account

It follows that the width of a spectral line could be investigated by determining how the visibility of its fringes varies with n . The Michelson interferometer can be used for this purpose (see write-up in this book).

Taking this effect into account Fig. 8.6 would have to be modified and in particular the modified form of Fig. 8.6d would be as shown in Fig. 8.9 hence accounting qualitatively for what is observed.

Question

- 1) Why is a sodium lamp, rather than an incandescent or fluorescent lamp, used in the experiments?

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References

1. 'Physics', D. Giancoli
2. 'Optics', Hecht & Zajac