

# EXPERIMENT 18

## Millikan Oil-Drop

### Introduction

With the apparatus supplied it is possible to repeat Millikan's very important experiment of 1909 in which he established the discreteness of the electronic charge and determined its magnitude. The charge carried by an electron is a fundamental physical constant. Techniques employed prior to R.A. Millikan's work in 1909 permitted users to deduce only the "average" electron charge. Millikan was the first to show that this quantity was discrete, or single valued. For this work, and that on the photoelectric effect, he was awarded the Nobel prize for Physics in 1923.

In the experiment a small, charged drop of oil is observed in a closed chamber between two horizontal, parallel plates. By measuring the velocity of fall of the drop under gravity and its velocity of rise when the plates are at a high electrical potential difference, data is obtained from which the charge on the drop may be computed.

### Description of the apparatus

The apparatus consists of a cylindrical viewing chamber containing an ionizing source, with a deep spray chamber above it, mounted at the centre of a support carrying a measuring microscope and focusing light source. The latter are directed at the viewing chamber. This assembly is mounted, with 10 cm of elevation adjustment, in a base incorporating a polarity-reversal and shorting switch, binding posts for voltage input, outlet for a cable providing a voltage to the plates of the viewing chamber, and levelling screws. A scale for microscope calibration, a non-flooding aspirator and a bottle of high-quality, light oil are also included.

The viewing chamber (Fig. 18.1) comprises two parallel, cylindrical, metal plates 5 cm in diameter, separated by a plastic ring of uniform thickness. It can be disassembled by removing the two thumb nuts beneath the spray chamber. A hole in the upper plate facilitates entry of oil drops from the spray chamber above. An ionizing source for use in changing the charge on the oil drop under observation is located in a cavity within the plastic separator ring. It is a small rod plated with Polonium 210, an alpha emitter. Its shield is actuated by a spring-return lever located on the side of the chamber.

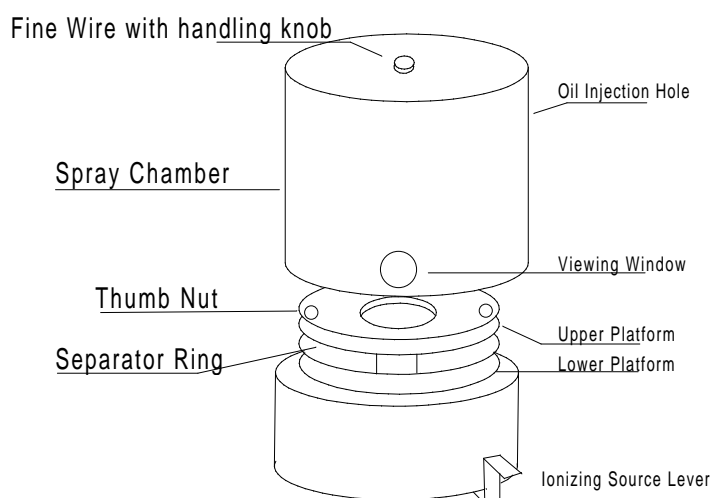


Figure 18.1: *The viewing chamber and spray chamber*

The spray chamber slips over the viewing chamber and is held in place by a bracket. When in position it effectively shields the viewing chamber from drafts and ambient light and protects the user from electrical shock. There are two windows for illuminating and viewing the oil drops. A fine wire with handling knob is stored in the top of the spray chamber. It is used in the upper plate of the viewing chamber for initial focusing of the light source and microscope. Oil is sprayed into the spray chamber by inserting the aspirator nozzle into the oil-injection hole near the chamber top.

The light source contains an incandescent bulb requiring 6.3 volts A.C. or D.C. at 0.5 amps. The bulb assembly slides within the outer sleeve to allow the light to be focused at the centre of the viewing chamber. An infrared-absorbing filter is mounted in front of the bulb to reduce heating of the viewing area by the light source.

The measuring microscope has a fixed magnification and is provided with rack-and-pinion focusing. Measurements are made by means of the graticule in the microscope which is to be calibrated using the scale supplied. The depth of focus is sufficiently short that the calibration will remain unchanged provided that the image remains in sharp focus.

The calibration scale, 10 mm long and graduated in 0.1 mm steps, is mounted on a glass disk in a metal frame. When calibrating the microscope, the scale can be supported in a vertical position at the centre of either the upper or lower plate of the viewing chamber.

Binding posts on the base provide electrical connections, through a high resistance to minimize the chance of electrical shock, to the plates. The three-position switch reverses the polarity of the plates and when in the centre position, disconnects the voltage source and short-circuits the plates, permitting free fall of the oil drops. When the switch is moved to its upper position, the upper plate is positive and, when in the lower position, the upper plate is negative (Fig. 18.2). The voltage-input cable to the plates plugs into the outlet in one of the legs. Each leg is provided with a levelling

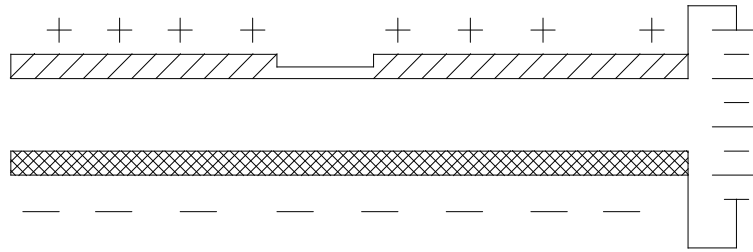


Figure 18.2: Detail of the plates with a potential difference applied

screw. A knurled-head screw near the top of the base permits the elevation of the apparatus to a convenient height.

## Theory

The oil drops in this experiment are subject to three different forces: viscous, gravitational and electrical. By analyzing these various forces an expression can be derived which will enable measurement of the charge on the oil drop and determination of the unit charge on the electron.

When there is no electric field present, the drop under observation falls slowly, subject to the downward pull of gravity and the upward force due to the viscous resistance of the air to the motion of the oil drop. The resistance of a viscous fluid to the steady motion of a sphere is obtainable from Stokes's law, from which the retarding force acting on the sphere is given by

$$F_r = 6\pi a\eta v \quad (18.1)$$

where  $a$  is the radius of the sphere,  $\eta$  the coefficient of viscosity of the fluid, and  $v$  the velocity of the sphere.

For an oil drop with mass,  $m$ , which has reached constant or terminal velocity,  $v_g$ , the upward retarding force equals the downward force taking buoyancy into account.

$$mg - \frac{4}{3}\pi a^3 \alpha_1 g = 6\pi a\eta v_g \quad (18.2)$$

where  $\alpha_1 =$  density of air. Now let an electric field,  $E$ , be applied between the plates in such a direction as to make the drop move upward with a constant velocity,  $v_E$ . The viscous force again opposes its motion but acts downward in this case. An oil drop with an electrical charge  $q$ , when it reaches constant velocity is again in a state of equilibrium and

$$Eq - mg + \frac{4}{3}\pi a^3 \alpha_1 g = 6\pi a\eta v_E \quad (18.3)$$

Solving Eq. 18.3 for  $mg - \frac{4}{3}\pi a^3 \alpha_1 g$  and equating this to Eq. 18.2 gives

$$q = \frac{6\pi a\eta}{E}(v_E + v_g) \quad (18.4)$$

*Experiment 18. Millikan Oil-Drop*

---

The electric field,  $E$ , is obtained by applying a voltage,  $V$ , to the parallel plates which are separated by a distance,  $d$ . Therefore,

$$q = \frac{6\pi a\eta d}{V}(v_E + v_g) \quad (18.5)$$

From Eq. 18.5 it is seen that for the same drop and with a constant applied voltage, a change in  $q$  results only in a change in  $v_E$  and

$$\Delta q = c\Delta v_E \quad (18.6)$$

where  $c$  is the constant  $\frac{6\pi a\eta d}{V}$ .

When many values of  $\Delta v_E$  are obtained, it is found that they are always integral multiples of a certain small value. Since this is true for  $\Delta v_E$  the same must be true for  $\Delta q$ ; that is, the charge gained or lost is the exact multiple of a unit charge. Thus the discreteness of the electronic charge may be demonstrated without actually obtaining a numerical value of the charge.

In Eq. 18.5 all quantities are known or measurable except  $a$ , the radius of the drop. To obtain the value of  $a$ , Eq. 18.2 may be written

$$\frac{4}{3}\pi a^3 \alpha g - \frac{4}{3}\pi a^3 \alpha_1 g = 6\pi \eta a v_g$$

therefore

$$v_g = \frac{2ga^2(\alpha - \alpha_1)}{9\eta} \quad (18.7)$$

where  $\alpha$  is the density of the oil,  $\alpha_1$  the density of the air, and  $\eta$ , as stated previously, is the viscosity of the air. Since the density of the air is very much smaller than the density of the oil,  $\alpha_1$ , is negligible and Eq. 18.7 is solved to give

$$a = \sqrt{\frac{9\eta v_g}{2g\alpha}} \quad (18.8)$$

Substitution of this value of  $a$  in Eq. 18.5 gives an expression for  $q$  in which all the quantities are known or measurable:

$$q = \frac{6\pi d}{V} \sqrt{\frac{9\eta^3}{2\alpha g}} (v_E + v_g) \sqrt{v_g} \quad (18.9)$$

Millikan, in his experiments, found that the charge resulting from the measurements seemed to depend somewhat on the size of the particular oil drop used and on the air pressure. He suspected that the difficulty was inherent in Stokes' law which he found not to hold for very small drops. It is necessary to make a correction, multiplying the viscosity by the factor  $\left(1 + \frac{b}{pa}\right)^{-1}$  where  $p$  is the barometric pressure (in centimeters of mercury),  $b$  is a constant of numerical value  $6.17 \times 10^{-6}$ , and  $a$  is the radius of the drop in meters. This correction is sufficiently small that the rough value of  $a$  obtained from Eq. 18.8 may be used in calculating it.

The corrected charge on the drop is, therefore, given by

$$q = \frac{6\pi d}{V} \sqrt{\frac{9\eta^3}{2\alpha g}} \left(1 + \frac{b}{pa}\right)^{-\frac{3}{2}} (v_E + v_g) \sqrt{v_g} \quad (18.10)$$

Using the MKS system of units, the electrical charge  $q$  is expressed in *Coulombs* when

b	= $6.17 \times 10^{-6}$
p	is barometric pressure in <i>cm of mercury</i>
a	is the radius of drop in <i>meters</i>
$\eta$	is the viscosity of air = $1.83 \times 10^{-5}$ <i>ns/m<sup>2</sup></i> at 23°C
$\alpha$	is the density of oil = $890 \text{kg/m}^3$ for Nye's watch oil
g	is the acceleration due to gravity = $9.8 \text{m/sec}^2$
$v_E$	is the velocity of a rising drop in <i>m/sec</i>
$v_g$	is the velocity of a falling drop in <i>m/sec</i>
d	is the plate separation in <i>meters</i>
V	is the potential difference between plates in <i>volts</i> .

## Experimental Procedure

### (1) Aligning and focusing the system

Assemble the apparatus as shown in Fig. 18.1. The chamber should be rotated so that the bracket and cable are toward the rear, in line with the microscope. Plug the cable into the outlet in the rear support leg.

Connect the light source to a 6.3 volt supply. Align the source parallel to its support arm so that light falls on the viewing chamber. Rotate the bulb assembly within the sleeve so that the image of the filament is vertical. Centre the image over the chamber window by raising or lowering the light source.

Loosen the thumb nut at the rear of the spray chamber. Remove the chamber and upper plate. Slide the bulb assembly within the sleeve to focus the vertical image of the filament at the centre of the plate. To centre the image it may be necessary to rotate the light source slightly, being careful not to raise or lower it. It is helpful to place a small paper screen at the centre of the plate upon which to focus the image.

Replace the upper plate. Unscrew the fine wire with handling knob from its storage place in the top of the spray chamber and insert it through the hole in the upper plate. Elevate the support-arm assembly to place the microscope at a convenient viewing height.

Focus the microscope eyepiece on the graticule, then focus the microscope on the vertical line of light reflected from the side of the wire. Position the microscope so that the wire is centered in the field of view. Both the top and bottom plates should be visible. They appear as horizontal streaks of light, due to reflections from their surfaces. The wire does not appear to terminate at the plate surfaces, but instead seems to be continuous across each surface, again due to reflection. If it is not clear whether the wire or a reflection is being viewed, slowly withdraw the wire while observing through the microscope. Remember, the microscope inverts the image so that the wire will appear to be pulled down through a hole in the lower plate. Finally refocus the eyepiece and microscope until no parallax is noticeable. The position of the microscope should not be changed during the remainder of the experiment.

Remove the wire and screw it back into its storage hole in the top of the spray chamber. Before replacing the spray chamber, check that the plates are horizontal, preferably by laying a small level on the upper plate. If necessary level the apparatus by means of the three supporting screws. Replace the spray chamber. The chamber may now need to be rotated slightly to centre the illuminating light on the window. Be careful not to raise or lower it in the process.

Set the polarity-reversal switch to the "plates shorted" position. Connect the D.C. voltage source to the binding posts (red post should be positive). Also connect a high-resistance D.C. voltmeter (1000 ohms/volt or greater) across the binding posts. Keep the meter in the circuit while data is being taken.

## **(2) Observing the oil drops**

Place oil in the aspirator and insert the nozzle into the hole near the top of the spray chamber. One or two squeezes of the bulb are usually sufficient to introduce fine oil drops into the chamber. The atomizing process also serves to electrically charge most of the drops.

Now look through the microscope. At first many drops, appearing as bright spots of light, should be seen. The drops fall under the force of gravity, with the larger ones moving more rapidly than the smaller ones. Since the image is inverted, they actually appear to rise. The large drops quickly fall out of the field of view. Select one of the slowest moving ones and follow its movements as the polarity of the field is changed. Practice applying the field to bring the drop up near the upper plate, then allowing it to free fall through the graticule divisions. Follow the drop up and down to develop the technique of observing it and controlling its motion before starting to record data.

Large, rapidly falling drops (rising in the microscope) are not used because they require a large electrical force to overcome the oppositely directed gravitational force. In order that the electric field be able to produce this force, the drop must be highly charged. When a highly charged drop changes charge by gaining or losing a single ion, only a small percentage change in the total charge results. It may be so small as to be comparable to the uncertainty in the experimental data. This is undesirable since the value of the charge on the electron is obtained directly from these changes of charge.

Occasionally a drop changes charge during a timing of its rise in the applied field. This trial should, of course, be disregarded. However the corresponding percentage change in the velocity of a highly charged drop is small and may go unnoticed, resulting in experimental error. It is advisable to choose a drop that rises slowly when the electric field is applied. This indicates that the charge on the drop is small, therefore probably only a small multiple of that on the electron. If too low a potential difference is applied to the plates, it may not be possible to overcome the gravitational force on drops carrying only a few electron charges.

The best data is obtained from drops that have, say, five or fewer electron charges on them. With a potential difference of 200 volts, a drop that free falls through the three graticule divisions in the microscope in about 20 seconds will, in the electric field, rise the same distance in about 20 or 30 seconds if it carries 5 or 4 charges, respectively. A drop that free falls the distance in about 30 seconds carries 5 or 4 charges if it rises in about 10 or 15 seconds, respectively. A drop that falls through the three spaces in about 50 seconds and rises in about 20 seconds carries 2 electron charges. Of course a drop moving this slowly would probably be timed through only one or two graticule divisions. Practice selecting the best drops.

Some drops are too small to use. Not only are they very difficult to see but also very small drops do not travel up and down in a straight line but waver back and forth with Brownian motion. If drops tend to drift out of focus or to one side it may be that the plates are not horizontal. It would then be necessary to re-level the apparatus.

Drops with large charges can be removed by applying the electric field. Then switch off the field to eliminate the heavy drops. The remaining drops may be suitable for use in the experiment.

Finally practice changing the charge on the drop under observation. To do this, bring the drop near the upper plate (lower in the microscope), making a mental note of its velocity in the applied field. Set the polarity-reversal switch to the "plates shorted" position and hold the ionizing-source lever in the "in" position. Allow the drop to free fall for several seconds. Now apply the field. If there has been a change in the velocity, then the drop has changed charge. If there has been no change, short the plates again for several seconds, repeating until a velocity change results. Release the spring-return lever. Should the change in charge be such that the drop moves rapidly downward when the field is applied, reverse the plate polarity to make the drop rise again.

### **(3) Making Measurements**

**(Please also read the procedures for using the Computer Programs in the Appendix)**

1. Record the barometric pressure in centimeters of mercury at the beginning and end of the experiment. Use the average value.
2. Either before or after recording the data on the oil drop motion, the microscope must be calibrated. To do this remove the spray chamber and place the calibration scale at the centre of the upper plate. Elevate the microscope and focus on the 0.1-mm scale. Determine the distance represented by the divisions on the microscope reticule.  
After calibrating the microscope, remove the upper plate and the plastic separator ring. Use a metric micrometer caliper to measure the thickness of the ring. This is the plate separation,  $d$ .
3. Switch on the PC and printer. Set up the computer as described in the Appendix so it is ready to begin taking readings. Select a satisfactory drop and position it below a line of the graticule. With 'Field-off', time the drop passing between 2 lines as it (apparently) rises under gravity. Wait until the drop has passed the upper line before switching on the E field. Time the 'fall' of the drop between the same 2 lines. At least 15 measurements on each drop are required for good statistical accuracy. Be sure to record the times as the drop disappears behind the lines - try not to anticipate this as it will result in systematic errors on your timing measurements.

Measure and record the time of rise of the drop for the same charge  $\sim 15$  times, recording also the number of divisions travelled. Use the average. (A sudden change in the velocity indicates that the charge on the drop or the mass of the drop has changed, invalidating that trial.) If it is found that the voltage varies too much (say, more than the volt meter accuracy) it will be necessary to record its value after each timing. Then, rather than using the average voltage and time in the calculations, a separate calculation must be made for each trial.

If, after several timings, the drop has not changed charge, use the ionizing source to induce a change. Again time its rise several times. Continue in this manner for as long as possible using the same drop. After the drop is finally “lost”, select a new one and continue the procedure.

## Analysis of Data

1. Calculate the velocities of rise,  $v_E$  and fall,  $v_g$ , for the drop.
2. Compute the approximate radius,  $a$ , of the drop using Eq. 18.8.
3. Compute the electrical charge,  $q$ , on the drop from Eq. 18.10. Note that many of the quantities in the equation remain constant and need be calculated only once. Others are invariant for a given drop and must be computed only once for that drop. The remaining quantities change each time the charge on the drop varies.
4. It will be found that the charge,  $q$ , is always some small positive integer,  $n$ , times the value of the electron charge,  $e$ ; that is  $q = ne$ . By a rough inspection of the data, determine the number,  $n$ , of electron charges on the drop for each charge calculated. This will be easy if the drop carried relatively few electron charges, which is another reason for selecting a drop that moves upward in the electric field at a slow rate.
5. Divide the charge on the drop by the number of electron charges,  $n$  which it carried to obtain a value for the charge on the electron. These values may vary but their average should be in close agreement with the accepted value of  $e$  if the experiment has been carefully performed.
6. An alternative method is to subtract the various values of  $ne$  ( $q$ ) from each other and then continue subtracting these differences from each other until a common difference (which is the value of  $e$ ) is found.

The use of the computer allows the rapid and easy calculation of the drop radius, the charge carried by the drop and the measured charge multiplicity.

## Supplementary Comments

In your report summarise the computer printout in a Table (see example below).

Table 1: Summary of data recorded with sample data shown

Drop #	Number Readings	Rise Time [s]	Fall Time [s]	Drop radius [ $\mu\text{m}$ ]	Charge [ $10^{-19}$ C]	$e^{-1}$ charges
1	30	11.23 $\pm$ 0.15	13.92 $\pm$ 0.01	0.56 $\pm$ 0.02	1.723 $\pm$ 0.06	1.01 $\pm$ 0.03
2	15	15.53 $\pm$ 0.66	32.01 $\pm$ 1.03	0.49 $\pm$ 0.04	3.3 $\pm$ 0.5	2.13 $\pm$ 0.21

1. The accepted value of the charge on the electron is  $1.6022 \times 10^{-19}$  Coulomb.



2. Read through the entire experiment first and then familiarize yourself thoroughly with the apparatus and its adjustments. It may take considerable practice to get just the right size of oil drop and the proper charge. Only after several trial runs should actual data be taken.
3. An interesting variation of Millikan's Oil-Drop Experiment consists of suspending a charged oil drop in the electric field. A continuously adjustable voltage source is used to adjust the potential difference to the exact value necessary to suspend the oil drop. The velocity of fall of the drop is determined as previously described.

If we neglect the buoyant force of the atmosphere, the force of gravity equals the electrical force when the particle is suspended, or

$$\frac{4}{3}\pi a^3 \rho g = q \frac{V}{d}$$

where  $V$  is the potential difference required to suspend the oil drop. Using the value of  $a$  given by Eq. 18.8 the value of  $q$  can be determined.