MATH00030 Formula Sheet

Arithmetic and Algebra

Rules of Indices:

$$x^m \times x^n = x^{m+n}.$$
$$(x^m)^n = x^{mn}.$$
$$x^m \div x^n = x^{m-n}.$$
$$x^0 = 1.$$
$$x^1 = x.$$
$$x^{-n} = \frac{1}{x^n}.$$
$$(x \times y)^n = x^n \times y^n.$$
$$\sqrt[n]{x} = x^{\frac{1}{n}}.$$

Rules of Logarithms:

Definition: $\log_a x$, is the number y such that $x = a^y$.

$$\log_a(xy) = \log_a x + \log_a y.$$
$$\log_a (x^m) = m \log_a x.$$
$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y.$$
$$\log_a 1 = 0.$$
$$\log_a a = 1.$$
$$\log_a x = \frac{\log_b x}{\log_b a}.$$
$$a^{\log_a x} = x.$$
$$\log_a (a^x) = x.$$

The Binomial Theorem:

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i.$$
$$(x+y)^n = x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \cdots + y^n.$$

Binomial Coefficients:

$$\binom{n}{i} = \frac{n!}{i!(n-i)!} = \frac{n(n-1)(n-2)\cdots(n-i+1)}{i!}.$$
$$\binom{n}{i} = \binom{n}{n-i}.$$
$$\binom{n+1}{i} = \binom{n}{i-1} + \binom{n}{i}.$$

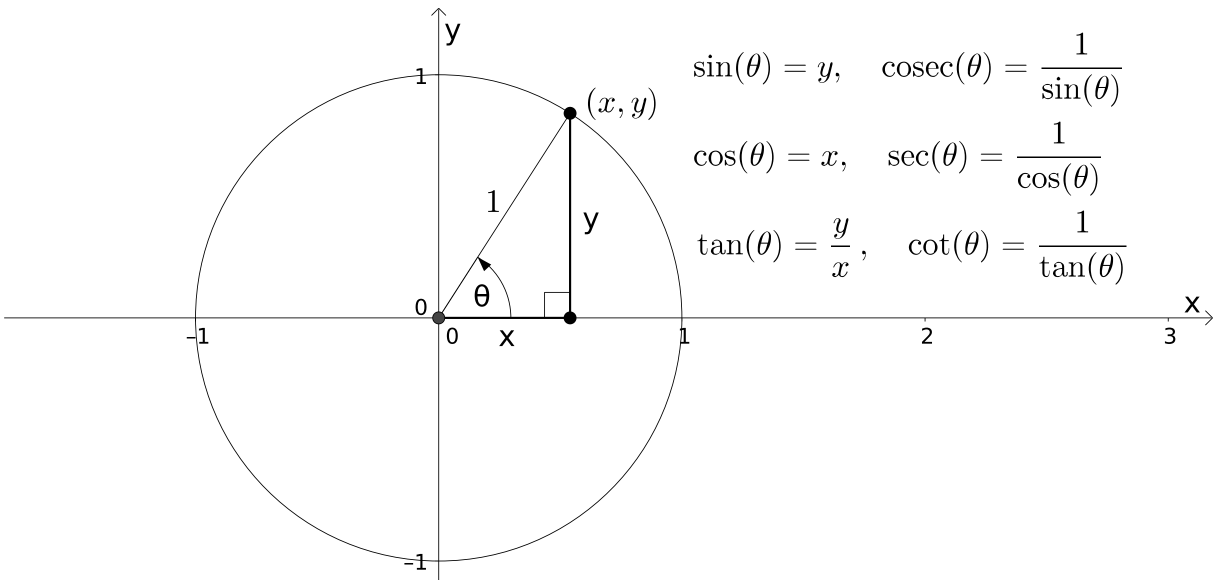
Lines and their Equations

Given two points (x_1, y_1) and (x_2, y_2) on a line, then the slope of the line is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.
The equation of a line is $y = mx + c$, where m is the slope and c is the y-intercept.
The length of a line segment between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
The midpoint of the line seqment between two points (x_1, y_1) and (x_2, y_2) is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

Quadratic Equations

The equation $ax^2 + bx + c = 0$ has solutions $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
The turning point of the graph of $y = ax^2 + bx + c$ lies at $\left(-\dfrac{b}{2a}, -\dfrac{b^2 - 4ac}{4a}\right)$.

Trigonometry



$$\sin(\theta) = y, \quad \operatorname{cosec}(\theta) = \frac{1}{\sin(\theta)}$$

$$\cos(\theta) = x, \quad \sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\tan(\theta) = \frac{y}{x}, \quad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Figure 3: Definition of Trigonometric Functions.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$*$

Table 1: Values of $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$ for important values of θ.

$\sin(-\theta) = -\sin(\theta)$	$\cos(-\theta) = \cos(\theta)$
$\tan(-\theta) = -\tan(\theta)$	$\cot(-\theta) = -\cot(\theta)$
$\operatorname{cosec}(-\theta) = -\operatorname{cosec}(\theta)$	$\sec(-\theta) = \sec(\theta)$

Table 2: Parity Identities.

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$	$\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$
$\tan\left(\frac{\pi}{2} - \theta\right) = \cot(\theta)$	$\cot\left(\frac{\pi}{2} - \theta\right) = \tan(\theta)$
$\operatorname{cosec}\left(\frac{\pi}{2} - \theta\right) = \sec(\theta)$	$\sec\left(\frac{\pi}{2} - \theta\right) = \operatorname{cosec}(\theta)$

Table 3: Co-function Identities.

$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$
$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$
$\tan(A \pm B) = \dfrac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)}$

Table 4: Sum and Difference Formulae.

$\sin^2(\theta) = \dfrac{1-\cos(2\theta)}{2}$
$\cos^2(\theta) = \dfrac{1+\cos(2\theta)}{2}$
$\tan^2(\theta) = \dfrac{1-\cos(2\theta)}{1+\cos(2\theta)}$

Table 5: Half Angle Formulae.

The Sine Rule: $\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)} = \dfrac{c}{\sin(C)}$.
The Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos(A)$.

Differential Calculus

$f(x)$	$f'(x)$	Comments
c	0	Here c is any real number
x^n	nx^{n-1}	
e^{ax}	ae^{ax}	
$\ln(ax)$	$\dfrac{1}{x}$	Here we must have $ax > 0$
$\sin(ax)$	$a\cos(ax)$	
$\cos(ax)$	$-a\sin(ax)$	Note the change of sign

Table 6: Some Common Derivatives

Differentiation using first principles: $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.
The sum rule for differentiation: $(f+g)'(x) = f'(x) + g'(x)$.
The multiple rule for differentiation: $(cf)'(x) = cf'(x)$.

Integral Calculus

$f(x)$	$\int f(x)\,dx$	Comments
k	$kx + c$	Here k is any real number
x^n	$\frac{1}{n+1}x^{n+1} + c$	Here we must have $n \neq -1$
$\frac{1}{x}$	$\ln(x) + c$	Here we must have $x > 0$
e^{ax}	$\frac{1}{a}e^{ax} + c$	
$\sin(ax)$	$-\frac{1}{a}\cos(ax) + c$	Note the change of sign
$\cos(ax)$	$\frac{1}{a}\sin(ax) + c$	

Table 7: Some Common Integrals

Evaluating definite integrals:
If F is an antiderivative of f then $\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$.
The sum rule for integration: $\int_a^b (f+g)(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$.
The multiple rule for integration: $\int_a^b (kf)(x)\,dx = k\int_a^b f(x)\,dx$.

Statistics

The mean is given by $\overline{x} = \frac{1}{n}(x_1 + x_2 + \cdots + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i$.
Given a list numbers $x_1, x_2, \ldots, x_n$ in ascending order, then their median is defined to be $\frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2}$ if n is even and $x_{\frac{n+1}{2}}$ if n is odd.
The variance is given by $\text{Var}(x) = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n}$.
The standard deviation is given by $\sigma = \sqrt{\text{Var}(x)}$.
The line of best fit $y = mx + c$ is found using the formulae:

$$m = \frac{n\left(\sum_{i=1}^{n} x_i y_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} x_i\right)^2} \quad \text{and} \quad c = \overline{y} - m\overline{x}.$$