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the Strategic Use of Buffer Suppliers**

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Optimal Sourcing Orders under Supply Disruptions and the Strategic Use of Buffer Suppliers

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Abstract: This paper analyses procurement from two, risk-averse, suppliers who are responsible for the timely delivery of some inputs. Their production is subject to inherent disruptions. We characterize the optimal contracts when suppliers can invest to lower the risk of delays that are costly to the manufacturer. When investment is contractible, we show that issuing asymmetric contracts, whereby the buyer is more heavily dependent on one supplier, is optimal as the cost associated with supply disruptions increases. When investment is not contractible, we show that large orders can be used as an incentive device. Thus, the strategy consisting of selecting one supplier as a main producer and another as a buffer has further desirable advantages under moral hazard.

Keywords: Investment, Risk, Costly Delays, Order Size and Moral Hazard.

JEL Classifications: D23, D86, L24.

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1. Introduction

Disruptions in the supply chain are responsible for large company losses. According to a report from Achilles, British companies lost a total of £20.4 million in 2013 due to “*suppliers failing to deliver the products of the required quality*”.² Moreover, “*the second most costly disruption was suppliers failing to deliver products on time, which cost £17.2 million.*” Ensuring the timely delivery of intermediate inputs can therefore outweigh the concern of achieving lower production costs in a buyer-supplier relationship. Indeed, Denning (2013) observes that Boeing had lost billions after it increased outsourcing with the intention of reducing production costs and commented that “*Boeing relied on poorly designed contractual arrangements, which created perverse incentives to work at the speed of the slowest supplier, by providing penalties for delay but no rewards for timely delivery.*” While increased globalization has exacerbated concerns over quality and timely deliveries, these have long been important issues. Asanuma (1989) explains that, to qualify as a superior supplier in Japan, a firm must demonstrate a high reliability in quality assurance and a high reliability in keeping up the delivery schedule.

Studies in management science have provided valuable insights into the strategies that help reduce or manage disruption risks and increase suppliers’ reliability.³ These papers are generally descriptive and the results obtained have a more practical emphasis.⁴ Most of the economics papers considering procurement emphasize production cost efficiency related issues as opposed to reliability issues. This paper attempts to fill a gap by considering optimal procurement from suppliers responsible for the timely delivery of required inputs, in the potential presence of moral hazard, when production is subject to disruptions. In doing so, this paper contributes to the literature by (i) proposing a framework for the analysis of contract design in the presence of supply disruptions, and (ii) characterizing an optimal procurement strategy under dual sourcing with contractible and non-contractible investments when accounting for costly orders shortages.

Our study brings to light some circumstances under which an increased reliance on one of the suppliers, while using the other one as a buffer, is an optimal strategy when facing high costs of supply disruptions and moral hazard. Real world evidence shows that such a strategy is adopted in some industries. Boeing for instance, managed 3800 direct suppliers in 1998, but decided to reduce its supply base to 1200 suppliers in 2006. Of these, about 40 to 50 account for two-thirds of their activity (see Bernstein (2006)).

We analyze a situation in which a downstream manufacturer must acquire some inputs, within a given time frame, from two risk-averse suppliers.⁵ The amount each supplier

² See <http://www.achilles.com/en/news/2806-58-million-the-real-cost-of-supply-chain-disruption>

³For instance, Tang (2006) surveys different strategies and states that “*the multi-supplier strategy is the most common approach for reducing supply chain risks*”. Inderfurth and Clemens (2011) pay more attention to the type of contracts issued under multiple sourcing. Assuming that suppliers differ in their costs and reliability they focus at the contracts’ impact on risk sharing under random production yield.

⁴ A notable exception is Li (2013) which adopts, as we do, a more theoretical approach based on principal-agent theory (see, for instance, Laffont and Tirole (1993)).

⁵ The sensitivity to risk allows the manufacturer to incentivize the suppliers to invest by linking their revenue to the amount being delivered (which is uncertain) as opposed to the amount ordered.

delivers is subject to uncertainty due to potential disruptions that are inherent in all production processes (cf. Craighead *et al.* (2007)). We allow for the possibility that orders below a certain level can be guaranteed. To meet the final market demand, the manufacturer must, however, order more than this minimum level from at least one of the suppliers and, thus, always faces a risk of supply shortage. To keep matters simple, we consider that each supplier may either complete his order or only deliver a fraction of the order on time.⁶

As a key feature, we consider that there is a positive correlation between the size of an order and the risk that it cannot be completed on time. Given his assets and the environment in which he operates and, thus, all the potential risks of disruptions, each supplier has access to limited production schemes when trying to achieve a timely order delivery. Larger orders require better organizational skills and are more subject to risk since the targeted production level increases but not the time allocated to complete it. For example, the breakdown of an apparatus may have no impact at all if the order size is small while it can increase the risk of delays for larger orders.

Finally, we also consider that each supplier can undertake some form of investments such as asset acquisition and employee job training in order to prevent possible production failures. Accordingly, the risk of delays is then negatively correlated with the level of investment. To demonstrate the robustness of our results, we shall examine this investment in both discrete and continuous forms. We consider situations where this investment is contractible and situations where it is not.

The manufacturer's objective is to minimize the cost of procurement under the assumption that supply shortages are costly. We purposely assume that suppliers have identical production costs in order to focus on losses that are associated with supply disruptions. Thus, the manufacturer seeks to curtail the risk of shortage in delivery through the strategic arrangements of market orders and the possible inducement of investment.

When investment is contractible and discrete, we show that three types of contracts may be issued in equilibrium: two symmetric contracts whereby either both suppliers invest or none of the suppliers invest and each is ordered the same quantity; and an asymmetric contract requiring investment by only one of the suppliers whose order exceeds that of his competitor's. The result emerges since the expected cost owing to delays is minimized when orders are symmetric and both suppliers invest, yet it is maximized if orders are symmetric and none of the suppliers invest. In addition to this cost, the minimization of the overall procurement cost also involves the cost of investment. As the burden of investment to the supplier increases, minimizing the cost associated with delays becomes less of an issue and the optimal contract shifts from the symmetric contract where both suppliers invest to a symmetric contract where none invest. Interestingly, there always exists an interval over which it is optimal to issue asymmetric contracts and this interval widens as the cost of delays increases. These contracts are associated with an overall lower risk of delays. This result is accentuated when we consider continuous investment levels. We further show, when investment is continuous and the manufacturer is able to fine tune it according to the cost of delay, that as the cost of supply disruptions increases the manufacturer relies more and more on one supplier, requesting greater investment and production from him. When such cost is sufficiently large, one of the suppliers is used as a buffer who does not invest and supplies the

⁶ Note however that in Parlane and Tsai (2013) we considered that production was capped in the event of a disruption and found similar results.

minimum quantity he is able to guarantee. If, however, this quantity is zero, we show that it is not optimal to rely on a single source and the buffer is ordered a small amount which decreases as the cost of delays increases.

The characterization of the optimal contract under moral hazard becomes complex due to discontinuity issues. Nonetheless we show that if the manufacturer requests investment from at least one supplier under contractible investments she then becomes more likely to rely on a buffer supplier under moral hazard either when investment decreases the marginal exposure to risk or when the risk of disruptions decreases faster should a supplier invests. Issuing asymmetric contracts can alleviate the cost associated with moral hazard for the following reasons. First, since risk and size are positively correlated, a risk-averse supplier is more inclined to invest as the size of his order increases. Therefore, submitting larger orders to one supplier can reduce the cost of inducing investment. Second, there is no conflict of interest between the buffer supplier and the manufacturer so long as the buffer supplier is not meant to invest. Thus, an increased reliance on one of the suppliers can lead to lower procurements costs as opposed to issuing two, incentive compatible, symmetric contracts. Finally, while considering continuous investments, we show that the manufacturer may no longer issue symmetric contracts whereby both suppliers invest if she faces moral hazard. Instead, she either foregoes investment or else induces only one supplier to invest.

The paper proceeds as follows. The next section is a short literature review. Section 3 describes the model. Section 4 deals with the optimal contracts when investment is contractible. Section 5 provides an analysis of the moral hazard issue. Finally, section 6 provides a conclusion.

2. Literature review

Issues related to supply chain disruptions have been addressed considering factors such as the supply base design (e.g. single versus dual sourcing), the optimal pricing mechanism and, finally, the suppliers' investment strategies or efforts.⁷

The relationship between the order size (or more specifically single vs. dual sourcing) and the suppliers' investment decisions has been addressed relatively early in economics in settings where the suppliers compete and their cost is private information. For example, Anton and Yao (1989) show that a supplier who has cost disadvantage has an incentive to invest under dual sourcing while he would not do so under single sourcing. They then show that cost the overall procurement costs can be lower under dual sourcing, amid the risk of collusion during the bidding process, since the price paid to the contracted supplier(s) is positively correlated to the highest cost. More recently, Gong et al. (2012) also focus on the implications for investment incentives of using multiple suppliers. They show that the optimality of split-award contracts depends on the socially efficient number of firms at the investment stage. Sole-sourcing is optimal when that number is greater than one, while split-award lowers the buyer procurement cost when the number is one.

Few studies within economics depart from the traditional mechanism design, adverse selection scenarios where reliability is not an issue. An interesting exception is Yehezkel (2014). He considers the problem of a manufacturer who wants his supplier to learn information about the quality of his input and to reveal it truthfully. He shows how these two

⁷ See Li (2013) for an excellent survey of the relevant literature.

distinct information problems lead the manufacturer to distort the contract from the one offered when firms are vertically integrated. And in some instances, the supplier must oversupply low quality and undersupply high quality.

By contrast to the economics literature, the management literature provides more contributions related to reliability failures. Wang et al. (2010), for instance, focus specifically on the suppliers' lack of reliability. These authors analyse two contrasting strategies consisting in either investing in a process improvement that increases a supplier's reliability or extending the supply base in the hope of attracting a more reliable supplier (dual as opposed to single sourcing). They show that the optimal strategy depends on several key variables, including whether the reliability issue is linked to random capacity or random yield, and the suppliers' differences in cost and reliability abilities. In contrast, we assume that suppliers are identical. Hence, in our analysis, sourcing orders are driven by the risk and cost associated with delays as opposed to specific suppliers' characteristics.⁸ Li (2013) considers suppliers' incentives when these are capacity constrained. Her analysis is based on a principal-agent model with both adverse selection and moral hazard. She shows that the manufacturer can strategically use the size of the supply base, some investment in suppliers' capacity and pricing commitment to incentivise the suppliers. She demonstrates that issuing symmetric contracts and offering ex-post negotiating fosters competition. This strategy is optimal when effort is costly to the supplier because greater competition allows the manufacturer to reduce the suppliers' margin while renegotiation helps facilitate the participation of suppliers with high cost realizations. Interestingly, she also shows that issuing asymmetric contracts to ex-ante identical suppliers and price commitment is a strategy that helps stimulate effort because it weakens competition and allows the suppliers to be residual claimants for any cost reduction they achieve. There are important differences between her analysis and the one we provide. First, we do not consider that suppliers are capacity constrained so that orders are not driven by capacities. Second, we consider an investment of a different nature: one that is aimed at reliability improvement. Finally, we assume that the cost of production is common knowledge so we make abstraction of any adverse selection issues.

3. The model

A manufacturer must purchase a quantity Q of a specific input to be delivered at a specific time. For simplicity assume that $Q = 1$. She faces 2 identical risk-averse suppliers (supplier 1 and supplier 2). Each has the capacity produce up to 1 unit. Their production costs are common knowledge. Let c denote the constant marginal cost for each supplier.

Given any order of size x , there are $N(x) < \infty$ different organizations of production that allow the supplier to complete the order on time absent of any disruptions. The more options are available the greater the chances of completing the order on time. We consider that the number of options available $N(x)$ decreases with x since the target moves further away but the time allocated doesn't.

Each production process is subject to disruptions and thus none can guarantee the timely delivery of the order. Each supplier can undertake an investment I , where $I \in \{0,1\}$ when discrete or $I \in [0,1]$ when continuous, aimed at reducing the risk of disruption.

⁸ Wang et al. (2010) also show that when suppliers are identical dual sourcing is optimal when they have low cost and process improvement is optimal when they have a low reliability.

To formalize the aforementioned features, we assume, for any order of size x , that $p(x, I)$ denotes the probability that this order will be completed on time by a supplier who invests I . Technically, this probability is a function of the number of plans available (positive correlation), the risk of disruptions associated with each of these plans (negative correlation) and finally the level of investment undertaken (positive correlation). With probability $(1 - p(x, I))$ the supplier fails to deliver the complete order on time and delivers a fraction of the order αx where $0 < \alpha < 1$. How much is being delivered is verifiable and hence contractible. Thus monetary transfers can differ depending on the quantity delivered.

Figure 1 highlights the timing of the game.

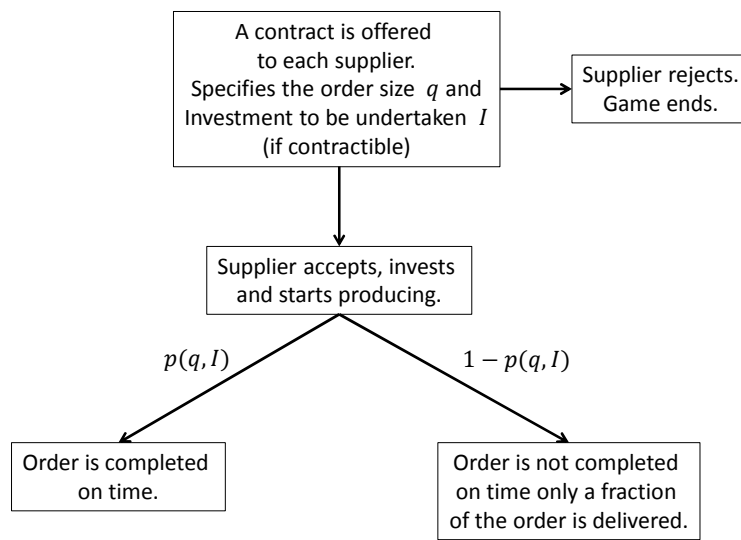


Figure 1: Timing of the game.

We make the following assumptions concerning the function $p(x, I)$.

- i. The function $p(x, I)$ is non-increasing and concave in x .
- ii. Investing decreases the absolute exposure to risk. When $I \in \{0, 1\}$ we have $p(q, 1) \geq p(q, 0)$ for any q . When $I \in [0, 1]$ we assume that the function $p(x, I)$ is non-decreasing in I .
- iii. Finally, let $q_L \in \left[0, \frac{1}{2}\right]$ be such that $p(x, I) = 1$ for any $x \leq q_L$ and any I . This assumption captures the fact that suppliers may be able to guarantee the timely delivery of small size orders.

Assumption (i) reflects the fact that increases in the order size reduces the number of options available to the supplier and therefore increases the risk. The concavity assumption implies that the marginal increase in the risk of delay decreases with the order size. (Orders cannot be infinitely large and suppliers are not capacity constrained. Thus, adding a unit to a large order has less impact on risk than adding a unit to a smaller order.) Assumption (ii) reflects the fact that investment and risk are negatively correlated. Assumption (iii) states that the supplier

may be able to guarantee the timely delivery of small orders. Since $q_L < \frac{1}{2}$ the manufacturer cannot eliminate the risk of supply disruption.

A contribution of this paper is to identify the conditions that affect the ranking of procurement contracts. The assumptions made so far concerning the function $p(x, I)$ concern the *absolute* exposure to risk. Properties 1 (and respectively property 2 introduced in Section 5) relates investment to the *marginal* (relative) exposure to risk.

- **Property 1:** Investing decreases the marginal exposure to risk: for any $x > q_L$ we have

$$\frac{d}{dx}p(x, 0) < \frac{d}{dx}p(x, 1) < 0.$$

Under this property the risk associated with the production of an extra unit is lower for the supplier who invests. In other words, under Property 1, the larger the order size is the greater are the benefits from investing.

Supply shortages are costly to the manufacturer.⁹ The following table highlights the total cost associated with issuing an order q to a supplier.

TOTAL COST	Supplier (order q)	Manufacturer
No delay	cq	0
Delay	$c\alpha q$	$c'(1 - \alpha)q + \tau$

The variable $\tau \geq 0$ is a fixed cost and $c' > c$.

We then solve for the sub-game perfect equilibrium outcome of the game in Sections 4 and 5.

4. Optimal outsourcing strategies with contractible investments

When investment is contractible the contract to supplier i ($i = 1, 2$) specifies the order size (q_i), the monetary transfer conditional on delivering $x \in \{q_i, \alpha q_i\}$ units ($t(x)$), and finally the investment level he has to undertake I_i .

4.1. Discrete investment level: $I \in \{0, 1\}$

The case of discrete investment levels is motivated by the observation that manufacturers often rely on industry accreditation to form a supply base. To be eligible suppliers must comply with certain specific requirements. Such a requirement can take the form of relationship-specific investment in specified technologies to address potential compatibility issues.¹⁰

The manufacturer wishes to minimize the expected cost of outsourcing:

$$C = \sum_{i=1,2} [p(q_i, I_i)t(q_i) + (1 - p(q_i, I_i))(t(\alpha q_i) + c'(1 - \alpha)q_i + \tau)]. \quad (1)$$

⁹ In Parlane and Tsai (2013) we consider that the cost associated with delays could be the responsibility of the manufacturer or the supplier. It does not have any impact on the main results.

¹⁰ For instance, Bakos and Brynjolfsson (1993) report that Boeing asked its suppliers adopt the same computer-aided design system (CATIA) when it manufactured the 777 aircraft.

Subject to the constraints (2) and (3):

$$q_1 + q_2 = 1, \quad (2)$$

$$p(q_i, I_i)\pi(t(q_i) - cq_i) + (1 - p(q_i, I_i))\pi(t(\alpha q_i) - c\alpha q_i) - I_i \geq 0, \quad (3)$$

where the function $\pi(\cdot)$ is increasing and concave to account for risk aversion.

Constraint (2) implicitly assumes that the manufacturer does not want to be stuck with obsolete inventory. Evidence for such a decision can be found in Gans (2007) which gives as an example the strategy of the company Dell. As many other manufacturers it orders upon realization of the demand so as to have the exact number of units it requires. Under this assumption all that matters is the order to supplier 1. We let $q_1 = q$ and $q_2 = 1 - q$. Constraint (3) states that each supplier only accepts contracts generating expected profits greater than a reservation profit taken to be zero.

Lemma 1. *The optimal contract is efficient. The suppliers are fully insured as we have*

$$\begin{aligned} t(q_i) &= cq_i + \pi^{-1}(I_i), \\ t(\alpha q_i) &= c\alpha q_i + \pi^{-1}(I_i). \end{aligned}$$

Proof: The proof is straightforward. The participation constraint, (3), binds and contracts are efficient meaning that $[\pi(t(q_i) - cq_i) - \pi(t(\alpha q_i) - c\alpha q_i)] = 0$.

Given these transfers and the constraint on orders, we can re-write the objective function as:

$$C = c + \sum_{i=1,2} \{\pi^{-1}(I_i) + F(q_i, I_i)\}, \quad (4)$$

where $\Delta c = c' - c > 0$ and where

$$F(x, I) = (1 - p(x, I))[\tau + \Delta c(1 - \alpha)x], \quad (5)$$

is the expected cost associated with supply disruptions when an order of size x is issued to an investor who invests I .

Let $\Delta\pi = \pi^{-1}(1) - \pi^{-1}(0)$. This variable represents the cost or burden of investing to a supplier.

Proposition 1. Optimal investment strategies and orders

For any $0 \leq q_L < \frac{1}{2}$ and $q^* > \frac{1}{2}$, the variables

$$\underline{r} = F(q^*, 1) + F(1 - q^*, 0) - 2F\left(\frac{1}{2}, 1\right) \quad (6)$$

and

$$\bar{r} = 2F\left(\frac{1}{2}, 0\right) - F(q^*, 1) - F(1 - q^*, 0), \quad (7)$$

are such that $0 < \underline{r} \leq \bar{r}$ (where the equality holds only as $q_L \rightarrow \frac{1}{2}$).

The optimal contract is as follows.

- i. When the burden of investment to the supplier is low, $\Delta\pi \in [0, \underline{r}]$, both suppliers invest and they receive identical orders so that $q = \frac{1}{2}$.

- ii. When the burden of investment to the supplier is high, $\Delta\pi \geq \bar{r}$, none of the suppliers invest and they receive identical orders so that $q = \frac{1}{2}$.
- iii. For intermediary investment levels, $\Delta\pi \in [\underline{r}, \bar{r}]$, only one supplier invests and he is ordered a quantity $q^*(q_L) \in \left[\frac{1}{2}, 1 - q_L\right]$.

Proof: See Appendix.

When investment is discrete, there are 3 possible types of contracts issued in equilibrium. Either both suppliers invest or none invests, in which case it is optimal to set $q = \frac{1}{2}$. Or else, only one of the suppliers invests and it is optimal to increase his order so that $q^* > \frac{1}{2}$. When considering the expected cost associated with delays for each of these contracts we establish that $\underline{r} < \bar{r}$ (see proof of Proposition 1), which implies that:

$$2F\left(\frac{1}{2}, 0\right) > F(q^*, 1) + F(1 - q^*, 0) > 2F\left(\frac{1}{2}, 1\right). \quad (8)$$

Thus, the expected cost of supply disruptions is minimized when orders are symmetric and both suppliers invest but it is maximized when orders are symmetric and none of the suppliers invest.

While the above is relevant to the manufacturer, the minimization of the procurement cost, as given by (4), takes into account, in addition to the cost associated with delays, the cost of investing $\pi^{-1}(I)$. As the burden of investment to the supplier increases, minimizing the cost associated with supply disruptions becomes less of an issue and the optimal contract shifts from the symmetric contract where both suppliers invest to a symmetric contract where none of the suppliers invest.

Interestingly, when $\tau \neq 0$ or when $\alpha < 1$, there always exists a non-empty range for the parameter $\Delta\pi$ over which it is optimal for the manufacturer to issue asymmetric orders:

$$\bar{r} - \underline{r} = 2 \left[F\left(\frac{1}{2}, 1\right) - F(q^*, 1) \right] + 2 \left[F\left(\frac{1}{2}, 0\right) - F(1 - q^*, 0) \right]. \quad (9)$$

An interesting question which emerges from the above is: under what circumstances are the asymmetric contracts optimal?

To answer this question we examine how the interval $[\underline{r}, \bar{r}]$ changes as some of the costs associated with delays increase.

Lemma 2. For any given $q_L < \frac{1}{2}$, the interval $\bar{r} - \underline{r}$ widens as τ increases.

Proof: Without any loss in generalities let $q^* = \frac{1}{2} + \sigma$ with $\sigma > 0$. We have

$$\frac{1}{2\sigma} \frac{d(\bar{r} - \underline{r})}{d\tau} = \left[\frac{p\left(\frac{1}{2} + \sigma, 1\right) - p\left(\frac{1}{2}, 1\right)}{\sigma} \right] - \left[\frac{p\left(\frac{1}{2}, 0\right) - p\left(\frac{1}{2} - \sigma, 0\right)}{\sigma} \right] > 0. \quad (10)$$

Given that $p(x, I)$ is concave in x and given property 1, the secant between $\left(\frac{1}{2} - \sigma\right)$ and $\left(\frac{1}{2}\right)$ on the curve $p(x, 0)$ is steeper than the secant between $\left(\frac{1}{2}\right)$ and $\left(\frac{1}{2} + \sigma\right)$ on the curve $p(x, 1)$ and therefore the above expression is positive. ■

What happens as $(1 - \alpha)\Delta c$ increases is less trivial. Indeed, we have:

$$\frac{d(\bar{r} - \underline{r})}{d(1 - \alpha)\Delta c} = \frac{1}{2} \frac{d(\bar{r} - \underline{r})}{d\tau} + 2\sigma \left[p\left(\frac{1}{2} + \sigma, 1\right) - p\left(\frac{1}{2} - \sigma, 0\right) \right]. \quad (11)$$

The last term is also positive provided σ is small enough. However, as σ increases (as $q^* \rightarrow 1 - q_L$) the last term can become negative and the sign of expression (11) becomes undetermined. However, as we show below, we can reach a definitive conclusion when the investment level is continuous.

Given Lemma 2, we obtain the following result as contained in Corollary 1.

Corollary 1: *Asymmetric contracts can be optimal because they reduce the risk of delays associated with outsourcing.*

The intuition is as follows. Changing the orders from symmetric $\left(q = \frac{1}{2}\right)$ to asymmetric orders $\left(q > \frac{1}{2}\right)$ affects transactions costs for two reasons:

1. It affects the cost of supply disruptions measured by $[\tau + \Delta c(1 - \alpha)x]$.
2. It affects the risk delays and therefore the likelihood of paying the above cost. This is measured by $(1 - p(x, I))$.

Both, the cost and the risk, increase with the order size. In other words, large orders are more likely to fail and more expensive when they fail. Thus as the manufacturer issues asymmetric contracts he increases the transaction cost associated with the main supplier's contract. Indeed, since $q^* > \frac{1}{2}$ the first term of expression (9), $\left[F\left(\frac{1}{2}, 1\right) - F(q^*, 1)\right]$, is non-positive. It measures the expected loss arising from issuing a larger order to the main supplier. In contrast, the second term of the same expression, $\left[F\left(\frac{1}{2}, 0\right) - F(1 - q^*, 0)\right]$, is non-negative as it measures the expected gain from lowering the order of the buffer supplier who does not invest. Notice however that the impact on cost associated with delays is nil. Indeed the increase in the cost measured by $\tau + \Delta c(1 - \alpha)\left(q^* - \frac{1}{2}\right)$ matches the savings from issuing a smaller order to the buffer $\tau + \Delta c(1 - \alpha)\left(\frac{1}{2} - (1 - q^*)\right)$. Thus, what makes asymmetric contract an optimal solution is the fact that they are associated with lower risk overall. This result is given even more emphasis when we consider the case of continuous investments.

4.2. Continuous investment level: $I \in [0, 1]$

When investment is continuous, the manufacturer has the ability to adjust it. Greater levels of investment are expected to match greater costs of supply shortages. The question is: does the manufacturer submit symmetric orders and slowly requests greater investments from both suppliers when the cost associated with delays increases?

To answer this query, we solve, for tractability, the case where $\tau = 0$ and we consider that the probability of success is given by

$$p(x, I) = 1 - \frac{1}{2}(x - q_L)(1 - I)^2. \quad (12)$$

Given the optimal transfers (Lemma 1 remains valid) and the probability function, the cost of procuring $Q = 1$ is given by

$$C = c + [1 - p(q, I_1)]\Delta c(1 - \alpha)q + [1 - p(1 - q, I_2)]\Delta c(1 - \alpha)(1 - q) + \pi^{-1}(I_1) + \pi^{-1}(I_2). \quad (13)$$

The optimal investment level (when positive) must satisfy

$$\frac{dC}{dI^i} = 0 \rightarrow \frac{1}{\pi'(\pi^{-1}(I_i))} - \Delta c(1 - \alpha)q_i(q_i - q_L)(1 - I_i) = 0. \quad (14)$$

Proposition 2 below describes the optimal contracts under dual sourcing when $\pi(w) = \sqrt{w}$. In this case we have

$$\frac{2I_i}{1 - I_i} = (1 - \alpha)\Delta c q_i(q_i - q_L). \quad (15)$$

Clearly, provided that $(1 - \alpha)\Delta c > 0$, the optimal investment level increases with the order size and with $(1 - \alpha)\Delta c$.

Let

$$\Delta c_1 = \frac{8}{3(1 - 2q_L) + 4q_L^2} \quad (16)$$

and

$$\Delta c_2 = \frac{2(\sqrt{2 - 3q_L} - \sqrt{q_L})}{(1 - 2q_L)(1 - q_L)\sqrt{q_L}}. \quad (17)$$

One can easily verify, for any $q_L \in]0, \frac{1}{2}[$, that $\Delta c_2 > \Delta c_1$.

Proposition 2. When $\pi(w) = \sqrt{w}$, the optimal investments and orders are as follows. Assume $q_L > 0$.

- When $(1 - \alpha)\Delta c \leq \Delta c_1$ both suppliers receive equal orders and they both invest

$$I = \frac{(1 - 2q_L)(1 - \alpha)\Delta c}{8 + (1 - 2q_L)(1 - \alpha)\Delta c}. \quad (18)$$

- For $(1 - \alpha)\Delta c \in [\Delta c_1, \Delta c_2]$ the manufacturer issues asymmetric orders $(q^*, 1 - q^*)$ with $q_L < q^* < \frac{1}{2}$ and $q^* \rightarrow q_L$ as Δc increases. The supplier with order $q_i \in \{q^*, 1 - q^*\}$ invests

$$I_i = \frac{(1 - \alpha)\Delta c q_i(q_i - q_L)}{2 + (1 - \alpha)\Delta c q_i(q_i - q_L)}. \quad (19)$$

- Finally when $(1 - \alpha)\Delta c \geq \Delta c_2$ the manufacturer uses one supplier as a buffer. The buffer supplier produces $q = q_L$ and does not invest while the main supplier supplies $q = 1 - q_L$ and invests.

$$I_i = \frac{(1 - \alpha)\Delta c(1 - q_L)(1 - 2q_L)}{2 + \Delta c(1 - q_L)(1 - 2q_L)}. \quad (20)$$

Proof: See Appendix.

Clearly, as the cost associated with supply disruptions increases the manufacturer relies more and more on one of the suppliers. For a large cost of disruption, the buffer supplier's order is the smallest quantity he can deliver. This leads to the question as to whether or not single sourcing ever becomes optimal. Corollary 2 below provides an answer.

Corollary 2: *Single sourcing is never optimal. When $q_L = 0$ the manufacturer never sets $q = 0$. (See the proof of Proposition 2.)*

When $(1 - \alpha)\Delta c \leq \frac{8}{3}$ both suppliers receive equal orders and both invest $I = \frac{(1-\alpha)\Delta c}{8+(1-\alpha)\Delta c}$.

For any $(1 - \alpha)\Delta c > \frac{8}{3}$ the manufacturer issues asymmetric orders $(q^*, 1 - q^*)$ with $q^* < \frac{1}{2}$ and $q^* \rightarrow 0$ as Δc increases. The supplier with order $q_i \in \{q^*, 1 - q^*\}$ invests $I(q_i)$ is given by (19).

Figure 2 below shows the optimal investment levels (taking into account the optimal order size) as a function of $(1 - \alpha)\Delta c$.

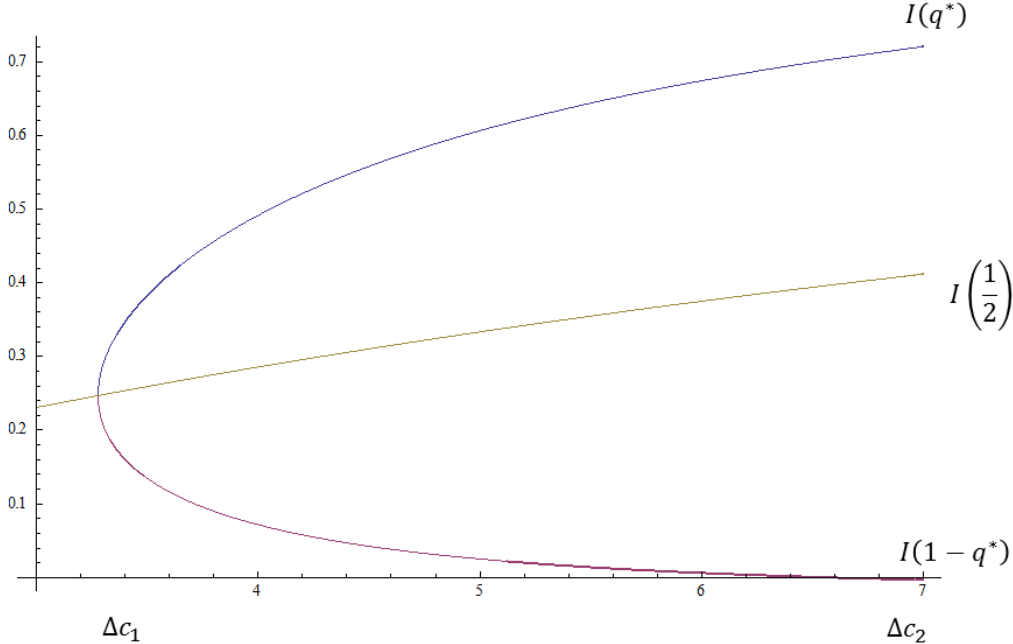


Figure 2: Optimal investment as a function of Δc .

For $\Delta c > \Delta c_1$, the manufacturer relies more on one of the suppliers, requesting that he invests $I(q^*)$ with $q^* > \frac{1}{2}$.

As $(1 - \alpha)\Delta c$ increases beyond Δc_1 the manufacturer starts issuing asymmetric contracts and requests a higher investment from the main supplier while the buffer invests less and less. Overall, the overall investment falls short of what it would be under symmetric contracting: $I(q^*) + I(1 - q^*) < 2I\left(\frac{1}{2}\right)$.

The results established in this section confirm that asymmetric contracts are an optimal strategy when costs associated supply disruptions are high. While the risk associated with the main supplier increases (as well as the cost in the event of a delay) this is compensated by keeping aside a buffer supplier. All in all the risk of delays is lower than what it would be should both suppliers invest.

5. Optimal outsourcing strategies under moral hazard

We now turn to the case where investment is not verifiable and, thus, not contractible. In this case, in addition to constraint (3), the contract must ensure that it is in the supplier's interest to invest the amount that the manufacturer wishes him to invest.

The transfers identified in Lemma 1 such that $t_S^i = cq + \pi^{-1}(I_i)$ and $t_F^i = c\alpha q_i + \pi^{-1}(I_i)$ are incentive compatible provided that $I_i = 0$. In other words, these transfers will ensure that the supplier does not invest.

When the manufacturer wishes to induce $I_i > 0$ the transfers need to satisfy an incentive constraint. This constraint is written differently depending on whether investment is continuous or discrete.

To characterize the optimal contract, we also need to introduce Property 2 which is slightly more restrictive than Property 1.

Property 2: As an order increases the risk of delays increases faster when the supplier does not invest:

$$\frac{d}{dx} \frac{1 - p(x, 0)}{1 - p(x, 1)} > 0 \Leftrightarrow \frac{dp(x, 0)}{dx} \leq \frac{dp(x, 1)}{dx} \frac{1 - p(x, 0)}{1 - p(x, 1)} < \frac{dp(x, 1)}{dx}. \quad (21)$$

As the right hand side of the equivalence sign shows, this property is more restrictive than property 1 however, it is important to note that property 2 is a sufficient but not a necessary condition for our results.

5.1. Discrete investment level: $I \in \{0, 1\}$

Should the manufacturer wish to implement $I_i = 1$, the transfers must satisfy

$$[p(q_i, 1) - p(q_i, 0)][\pi(t(q_i) - cq_i) - \pi(t(\alpha q_i) - c\alpha q_i)] \geq 1. \quad (22)$$

The second term on the left hand side equals zero when investment is contractible. The greater it becomes, the less efficient the contract is, and the more costly moral hazard is to the manufacturer.

Lemma 3. *Under property 1 the efficiency of a contract implementing $I = 1$ increases with the order size.*

Proof: The optimal contracts are such that (22) binds:

$$[p(q_i, 1) - p(q_i, 0)][\pi(t(q_i) - cq_i) - \pi(t(\alpha q_i) - c\alpha q_i)] = 1.$$

Under Property 1 the first term increases as q increases, therefore the second term must decrease to maintain the equality. Thus, the transfers associated with a large order are more efficient than those associated with a small order. ■

Put differently, Lemma 4 suggests that the smaller the order the harder it is to incentivise the supplier. It also implies that larger orders can be used as an incentive devise. Indeed, since an order's risk and size are positively correlated, the larger the order the stronger the incentive of a risk averse supplier to invest.

The optimal transfers that implement investment are such that the supplier may be penalized when it fails to deliver the full order. Thus, the optimal contract depends on the extent to which the supplier can sustain losses (in other words, it rests upon whether or not $\pi(w) < 0$). An alternative interpretation can be provided as follows. A non-negative firm profit condition of the sort $\pi(w) \geq 0$ means that the supplier is protected by limited liability and the profit function must satisfy ex-post voluntary participation.

Lemma 4. *The optimal transfers that implement $I = 1$ are as follows.*

- *If $\pi(w) \in \mathbb{R}$ the transfers are such that the supplier faces some losses in the event of a disruption and we have*

$$\pi(t(q_i) - cq_i) = \frac{1 - p(q_i, 0)}{p(q_i, 1) - p(q_i, 0)} > 0, \quad (23)$$

$$\pi(t(\alpha q_i) - c\alpha q_i) = -\frac{p(q_i, 0)}{p(q_i, 1) - p(q_i, 0)} < 0. \quad (24)$$

- *If $\pi(w) \in \mathbb{R}_+$ the transfers are such that*

$$\pi(t(q_i) - cq_i) = \frac{1}{p(q_i, 1) - p(q_i, 0)}, \quad (25)$$

$$\pi(t(\alpha q_i) - c\alpha q_i) = 0, \quad (26)$$

in which case the supplier extracts some profits.

Proof: The optimal transfers are such that both the participation constraint and the incentive constraint hold with equality. When (3) and (22) hold with equality the profits of the supplier are given by (23) and (24). If however we must have $\pi(w) \geq 0$ (such as when $\pi(w) = \sqrt{w}$) then it is optimal to set profit of the supplier in case of disruption equal to zero and set $\pi(t(q_i) - cq_i)$ such that (22) binds. ■

Let $R(q, 1)$ denote the expected cost of outsourcing, net of production costs, that the manufacturer incurs to implement investment $I = 1$ when issuing an order of size q :

$$R(q, 1) = p_i(q, I)[t(q_i) - cq_i] + (1 - p_i(q, I))[t(\alpha q_i) - c\alpha q_i]. \quad (27)$$

The function $R(q, 1)$ represents the cost moral hazard imposes on the manufacturer. Proposition 3 (below) identifies the conditions under which issuing large orders can lower the cost of inducing investment. Let

$$w_S = \frac{1 - p(q_i, 0)}{p(q_i, 1) - p(q_i, 0)} \quad \text{and} \quad w_F = -\frac{p(q_i, 0)}{p(q_i, 1) - p(q_i, 0)}$$

and let

$$w'_S = \frac{1}{p(q_i, 1) - p(q_i, 0)}.$$

Proposition 3

- Assume $\pi(w) \in \mathbb{R}$ so that

$$R(q, 1) = p_i(q, I)\pi^{-1}(w_S) + (1 - p_i(q, I))\pi^{-1}(w_F).$$

If property 2 holds, then $R(q, 1)$ decreases with q .¹¹

- Assume that $\pi(w) \in \mathbb{R}_+$ we have

$$R(q, 1) = p_i(q, I)\pi^{-1}(w'_S) + (1 - p_i(q, I))\pi^{-1}(0).$$

which decreases with q provided that property 1 holds.

Proof: See Appendix.

Proposition 3 is an immediate result of Lemma 3 (which itself relies on property 1). Larger orders are associated with more efficient contracts. Thus, less distortion in the monetary transfers is required when the order size increases. This brings down the cost of implementing a contract. While it is quite clear that under property 1 expression (25) decreases and thus converges towards (26). The behaviour of $[\pi(t(q_i) - cq_i) - \pi(t(\alpha q_i) - c\alpha q_i)]$ when given by (23) and (24) is less obvious. However, property 2 is a sufficient condition that guarantees that $[\pi(t(q_i) - cq_i) - \pi(t(\alpha q_i) - c\alpha q_i)]$ decreases as q increases.

Intuitively, having both suppliers provide 50% of the inputs and invest a positive amount, requires issuing two contracts, both incentive compatible. When the manufacturer relies on a buffer supplier there is no conflict of interest between the buffer supplier and the manufacturer since the buffer supplier does not invest and does not want to invest even when it is his own private decision. The manufacturer must however modify the contract issued to the main supplier who is responsible for more than 50% of the input. However, the larger the order size the more efficient the contract being issued and thus the less costly the moral hazard issue becomes for the manufacturer. Thus we expect to see more asymmetric contracting when moral hazard is introduced. We now formalize this suggestion.

¹¹ Note that property 2 is more restrictive than what is really required to guarantee that then $R(q, 1)$ decreases with q .

Let $C(q_1, I_1, I_2)$ denote the cost of outsourcing when issuing order q_1 to supplier 1 (and thus $1 - q_1$ to supplier 2) when investments (I_1, I_2) are contractible. In equilibrium costs are given by $C\left(\frac{1}{2}, 1, 1\right)$, $C\left(\frac{1}{2}, 0, 0\right)$ or finally $C(q^*, 1, 0)$ where $q^* > \frac{1}{2}$.

Let $\hat{C}(q_1, I_1, I_2)$ denote the cost associated with each of these 3 contracts under moral hazard. Given the optimal transfers we have

$$\hat{C}\left(\frac{1}{2}, 0, 0\right) = C\left(\frac{1}{2}, 0, 0\right), \quad (28)$$

$$\hat{C}\left(\frac{1}{2}, 1, 1\right) = C\left(\frac{1}{2}, 1, 1\right) + 2R\left(\frac{1}{2}, 1\right) - 2\pi^{-1}(1), \quad (29)$$

$$\hat{C}(q^*, 1, 0) = C(q^*, 1, 0) + R(q^*, 1) - \pi^{-1}(1). \quad (30)$$

Clearly, the manufacturer is generally less likely to induce investment since $R(q, 1) - \pi^{-1}(1) > 0$ for any $q > q_L$. Furthermore, we establish the following.

Corollary 3. *Assume that $p(q, I)$ is such that $R(q, 1)$ decreases with q , then if she was to induce investment from at least one supplier when investment is contractible, the manufacturer is more likely to rely on a buffer supplier under moral hazard as opposed to issuing symmetric contracts and requesting that both suppliers invest.*

Proof: Assume that $\Delta\pi$ is such that $C\left(\frac{1}{2}, 1, 1\right) = C(q^*, 1, 0)$ then we have

$$\hat{C}(q^*, 1, 0) - \hat{C}\left(\frac{1}{2}, 1, 1\right) = \left[R(q^*, 1) - R\left(\frac{1}{2}, 1\right)\right] + \left[\pi^{-1}(1) - R\left(\frac{1}{2}, 1\right)\right] < 0, \quad (31)$$

where the above holds since $R(q, 1)$ decreases with q . ■

Thus, when the manufacturer is indifferent between issuing two symmetric contracts and requesting investment when contractible, she now prefers to use one supplier as the main purveyor.

A question that remains is whether the manufacturer wishes to induce investment at all. To answer this question we refer to the case where investment levels are continuous.

5.2. Continuous investment level: $I \in [0, 1]$

For tractability and to keep in line with section 4.2 we consider the case where $\tau = 0$. To guarantee that a contract intending to induce a positive investment is incentive compatible the monetary transfers and order proposed to supplier i ($i = 1, 2$) must be such that:

$$I_i \in \arg \max_y p(q, y) [\pi(t(q) - cq) - \pi(t(\alpha q) - c\alpha q - \tau)] + \pi(t(\alpha q) - c\alpha q) - y.$$

Therefore, the optimal investment level solves:

$$\frac{\partial p}{\partial I_i} [\pi(t(q) - cq) - \pi(t(\alpha q) - c\alpha q)] - 1 = 0 \quad (32)$$

When both the participation constraint (3) and (32) bind we have:

$$\pi(t(\alpha q) - c\alpha q) = \frac{I_i}{\frac{\partial p}{\partial I_i}} \left[\frac{\partial p}{\partial I_i} - \frac{p(q, I_i)}{I_i} \right]. \quad (33)$$

The first term is positive. The second measures the discrepancy between the marginal and the average increase in the probability of success from augmenting investment. If we use the probability function given by (12) this second term is negative. Thus, when we assume that $\pi(w) = \sqrt{w}$ we cannot have $w < 0$ and the appropriate transfers to induce investment are such that $\pi(t(\alpha q) - c\alpha q) = 0$, while $\pi(t(q) - cq) = 1/\frac{\partial p}{\partial I_i}$ to satisfy (32).

We have

$$R(q, I) = \frac{2 - (q - q_L)(1 - I)^2}{2(q - q_L)^2(1 - I)^2}. \quad (34)$$

The total cost of outsourcing is given by

$$\hat{C}(q, I_1, I_2) = c + F(q, I_1) + F(1 - q, I_2) + R(q, I_1)\mathbb{I}_{I_1} + R(1 - q, I_2)\mathbb{I}_{I_2} \quad (35)$$

where $\mathbb{I}_{I_i} = 1$ if and only if $I_i > 0$ and the function $F(q, I)$ is given by (5) which can be re-written as (recall that $\tau = 0$):

$$F(q, I) = \frac{1}{2}(q - q_L)(1 - I)^2[\Delta c(1 - \alpha)q]. \quad (5^*)$$

Note that the expression (35) takes into account all possible levels of investments.

Lemma 5. *Under moral hazard it is never optimal for the manufacturer to issue symmetric contracts whereby both suppliers produce $q = \frac{1}{2}$ and both invest a positive amount.*

Proof: See Appendix.

The intuition for Lemma 5 is the following. Let us assume that for some values of $(1 - \alpha)\Delta c$ the optimal contract is such that both suppliers invest. If this is so, then the optimal investment must satisfy the first order condition. It is then optimal that a firm who supplies x units invests $I(x)$ such that (see (65) in the Appendix):

$$I(q) = \max \left\{ 0, 1 - \left[\frac{2}{q(q - q_L)^3(1 - \alpha)\Delta c} \right]^{1/4} \right\}. \quad (36)$$

The optimal investment increases with both, the order size and the cost of supply disruptions, just as it was the case under symmetric information. Note, however that, for any given x , when the cost of disruption is low it is optimal to set $I(x) = 0$. In other words, due to moral hazard, investment is only induced provided the cost of disruption is high enough.

From Proposition 2, we know that symmetric contracts with positive investments are optimal when $(1 - \alpha)\Delta c \leq \Delta c_1$. Thus, such contracts are optimal when the cost of disruption is low

enough. What we find is that for such low costs of disruption, the optimal investment level the manufacturer induces is nil.

The complete characterization of the optimal contract under moral hazard is complicated due to the presence of discontinuities. Firstly, there is a discontinuity at $I_i = 0$ ($i = 1, 2$) since the incentive constraint does not come into play when the manufacturer does not wish to induce investment. Secondly, there is a discontinuity at $q = q_L$. Indeed we have $\lim_{q \rightarrow q_L} R(q, I) = +\infty$. Indeed, as the order size converges to q_L suppliers have no incentive to invest since the benefit of doing so becomes negligible.

Let $\hat{C}(q_1, I_1, I_2)$ denote the cost of outsourcing under moral hazard. To get a sense of the optimal contract under moral hazard, we compare $C_1 = \hat{C}\left(\frac{1}{2}, I\left(\frac{1}{2}\right), I\left(\frac{1}{2}\right)\right)$ to $C_2 = \hat{C}\left(\frac{1}{2}, 0, 0\right)$ and $C_3 = \hat{C}(1 - q_L, I(1 - q_L), 0)$. One can easily establish the following:

- For any $(1 - \alpha)\Delta c \leq \frac{2(2\sqrt{1-q_L} + \sqrt{3-2q_L})^2}{(1-2q_L)^3}$ we have $C_2 < C_1$ and $C_2 \leq C_3$. Therefore the manufacturer is better-off issuing symmetric orders but does not induce investment.
- For any $(1 - \alpha)\Delta c > \frac{2(2\sqrt{1-q_L} + \sqrt{3-2q_L})^2}{(1-2q_L)^3}$ we have $C_3 < C_1$ and $C_3 \leq C_2$. Thus requesting that one supplier produces $(1 - q_L)$ and that he invests $I(1 - q_L)$ while the other supplier does not invest leads to the lowest cost.

Hence, when she induces investment, the manufacturer prefers to rely on asymmetric contracts.

6. Conclusion

Supply disruptions are costly to manufacturers. The aim of this paper was twofold. First we provide a model of procurement where suppliers are responsible for the timely delivery of some inputs and where production is subject to inherent disruptions. Second, given this framework we characterize the optimal contracts to two, risk-averse suppliers who can invest to lower the risk of delays.

The main conclusion from the analysis is that issuing asymmetric contracts whereby the manufacturer relies more heavily on one supplier is optimal when dealing with costly disruptions and moral hazard.

When investment is contractible, we show that while the risk associated with the main supplier's contract increases as his order increases, the use of a buffer supplier lowers the overall level of risk and thus the cost of procurement. Under moral hazard relying on a buffer supplier can be optimal when the marginal exposure to risk, defined as the incremental risk from having to deliver an extra unit, decreases with the investment level. In other words, this assumption can be re-phrased as assuming that suppliers who invest are absolutely and marginally more reliable. In this case, we show that large orders can be used as an incentive device and thus help reduce the cost triggered by incentive compatibility issues.

Relying on a buffer supplier gives the manufacturer two advantages. First, there is no conflict of interest between the buffer supplier and the manufacturer thus no cost associated with

moral hazard. Second, because the main manufacturer is responsible for larger orders his contract is more efficient than what it would be if he was in charge of half the production.

In terms of further research, it is interesting to note that while some manufacturers, such as Ford, offer long-term contracts to stimulate investment by suppliers others believe that fierce competition and short term contracting is a better strategy.¹² Keeping Apple's business for instance, is no sure thing -- a lesson learned by suppliers such as PortalPlayer and Audience Inc. PortalPlayer, a chip company that made the media processor for the MP3 and had generated over 90 percent of its sales from Apple's iPod, announced in 2006 that it has not been selected by Apple for use in its new mid-range and high-end flash-based iPods (Lapedus, (2006) and Wasserman (2014)). Audience Inc. - the maker of audio components saw its revenue from Apple plummet to less than 1 percent in 2014 from 82 percent of total sales in 2010 after it was left out of the iPhone 5. These real world examples highlight the important linkages between the design of supply contracts and the strategic use of suppliers in relation to a brand name manufacturer's roadmap of product launches.

¹² See McCracken (2005) for contracts issued by Ford.

7. Appendix

▪ Proof of Proposition 1.

The first step consists in characterizing the optimal order sizes given all possible investment strategies. This is done in Lemma A.

Lemma A: Optimal order sizes.

- For any $q_L > 0$ it is never optimal to rely on a single source ($q_i \geq q_L$ for $i = 1, 2$).
- When both suppliers invest or when none of them invests it is optimal to issue symmetric contracts and we have $q_1 = q_2 = \frac{1}{2}$.
- When only one supplier invests it is optimal to request that he delivers a quantity $q^*(q_L) \in \left[\frac{1}{2}, 1 - q_L\right]$ for any $q_L < \frac{1}{2}$, and such that $q^*\left(\frac{1}{2}\right) = \frac{1}{2}$.

Proof: The cost of procurement is given by (4) re-written here:

$$C = c + \pi^{-1}(I_1) + \pi^{-1}(I_2) + F(q, I_1) + F(1 - q, I_2), \quad (4)$$

where $F(x, I)$ is given by (5).

Assume that $q_1 = q < q_L$ so that supplier 1 can guarantee his order and we have

$$C = c + \pi^{-1}(I_1) + \pi^{-1}(I_2) + F(1 - q, I_2). \quad (4^*)$$

In this case $\frac{dC}{dq} < 0$ so that $q = 0$ is not a solution. Thus, the optimal orders belong to the interval $[q_L, 1 - q_L]$.

For any given I_1 and I_2 we have

$$\begin{aligned} \frac{dC}{dq} = & -\frac{dp(q, I_1)}{dq} [\tau + \Delta c(1 - \alpha)q] + (1 - p(q, I_1))\Delta c(1 - \alpha) \\ & + \frac{dp(1 - q, I_2)}{d(1 - q)} [\tau + \Delta c(1 - \alpha)(1 - q)] \\ & - (1 - p(1 - q, I_2))\Delta c(1 - \alpha). \end{aligned} \quad (37)$$

Moreover we have

$$\begin{aligned} \frac{d^2C}{dq^2} = & -\frac{d^2p(q, I_1)}{dq^2} [\tau + \Delta c(1 - \alpha)q] - 2\frac{dp(q, I_1)}{dq} \Delta c(1 - \alpha) \\ & - \frac{d^2p(1 - q, I_2)}{d(1 - q)^2} [\tau + \Delta c(1 - \alpha)(1 - q)] \\ & - 2\frac{dp(1 - q, I_2)}{d(1 - q)} \Delta c(1 - \alpha). \end{aligned} \quad (38)$$

The above is positive given the fact that $p(q, I)$ is non-increasing and concave in q . Thus the cost function is convex in q .

When $I_1 = I_2$ it is obvious that the only solution is to set $q = \frac{1}{2}$. When $I_1 = 1$ and $I_2 = 0$ we have at $q = \frac{1}{2}$

$$\begin{aligned} \frac{dC}{dq} \Big|_{q=1-q=\frac{1}{2}} = & \left[\tau + \Delta c(1 - \alpha) \frac{1}{2} \right] \left[\frac{dp(x, 0)}{dx} \Big|_{x=\frac{1}{2}} - \frac{dp(x, 1)}{dx} \Big|_{x=\frac{1}{2}} \right] \\ & + \Delta c(1 - \alpha) \left[p\left(\frac{1}{2}, 0\right) - p\left(\frac{1}{2}, 1\right) \right] < 0. \end{aligned} \quad (39)$$

Therefore the optimal order made to the supplier who invested is larger than $\frac{1}{2}$. Finally we have

$$\begin{aligned} \frac{dC}{dq} \Big|_{q=1-q_L} &= \tau \left[\frac{dp(x,0)}{dx} \Big|_{x=q_L} - \frac{dp(x,1)}{dx} \Big|_{x=1-q_L} \right] \\ &+ \Delta c(1-\alpha) \left[q_L \frac{dp(x,0)}{dx} \Big|_{x=q_L} - (1-q_L) \frac{dp(x,1)}{dx} \Big|_{x=1-q_L} \right] \\ &+ [1-p(1-q_L,1)]\Delta c(1-\alpha). \end{aligned} \quad (40)$$

The sign of the above is undetermined and possibly negative in which case $q = 1 - q_L$ is optimal. Note, when $q_L \rightarrow \frac{1}{2}$, that the above simplifies to

$$\frac{dC}{dq} \Big|_{q=1-q_L \rightarrow 1/2} = \left[\tau + \Delta c(1-\alpha) \frac{1}{2} \right] \left[\frac{dp(x,0)}{dx} \Big|_{x=\frac{1}{2}} - \frac{dp(x,1)}{dx} \Big|_{x=\frac{1}{2}} \right] < 0. \quad (41)$$

Thus, in this case, setting $q = \frac{1}{2}$ is optimal. ■

We may now proceed with the proof of proposition 1.

Let $C(I_1, I_2)$ denote the value of cost when the optimal orders are issued:

$$C(0,0) = c + 2\pi^{-1}(0) + 2F\left(\frac{1}{2}, 0\right) \quad (42)$$

$$C(1,1) = c + 2\pi^{-1}(1) + 2F\left(\frac{1}{2}, 1\right) \quad (43)$$

$$C(1,0) = C(0,1) = c + \pi^{-1}(0) + \pi^{-1}(1) + F(q^*, 1) + F(1 - q^*, 0). \quad (44)$$

The function $F(x, I)$ is given by (5). We have $C(1,1) \leq C(0,0)$ provided $\Delta\pi \leq r$ where

$$r = \left(p\left(\frac{1}{2}, 1\right) - p\left(\frac{1}{2}, 0\right) \right) \left[\tau + \Delta c(1-\alpha) \frac{1}{2} \right]. \quad (45)$$

We have $C(1,0) \leq C(0,0)$ provided $\Delta\pi \leq \bar{r}$ where \bar{r} is given by (7) in the text. Finally we have $C(1,1) \leq C(1,0)$ provided $\Delta\pi \leq \underline{r}$ where \underline{r} is given by (6) in the text.

It is finally straightforward to show that

$$\bar{r} - r = r - \underline{r} = \left[F\left(\frac{1}{2}, 1\right) + F\left(\frac{1}{2}, 0\right) \right] - [F(q^*, 1) + F(1 - q^*, 0)] > 0. \quad (46)$$

The above is true because q^* minimizes the cost - and thus it minimizes $F(x, 1) + F(1 - x, 0)$ - when investment levels are (1,0) or (0,1). Thus we necessarily have $\bar{r} > \underline{r}$. ■

▪ **Proof of Proposition 2**

When issuing orders $q_1 = q$ and $q_2 = 1 - q$, the total cost function can be written as

$$C = c + \frac{1}{2}\Delta c q(q - q_L)(1 - \alpha)(1 - I^1)^2 + (I^1)^2 + \frac{1}{2}\Delta c(1 - q)(1 - q - q_L)(1 - \alpha)(1 - I^2)^2 + (I^2)^2. \quad (47)$$

The first order condition with respect to q leads to

$$(2q - q_L)(1 - I^1)^2 - (2(1 - q) - q_L)(1 - I^2)^2 = 0. \quad (48)$$

Moreover, according to (15) in the text the optimal investment associated with order q_i is such that

$$1 - I^i = \frac{2}{2 + (1 - \alpha)(q_i - q_L)q_i\Delta c}. \quad (49)$$

Substituting $(1 - I^1)$ and $(1 - I^2)$ in (48) using the fact that $q_1 = q$ and $q_2 = 1 - q$, the optimal order must satisfy

$$\frac{(2q - q_L)}{[2 + (1 - \alpha)(q - q_L)q\Delta c]^2} - \frac{(2(1 - q) - q_L)}{[2 + (1 - \alpha)(1 - q - q_L)(1 - q)\Delta c]^2} = 0. \quad (50)$$

The above is always satisfied at $q = \frac{1}{2}$. However, by differentiating the above with respect to q and evaluate the outcome at $q = \frac{1}{2}$ one can establish that the second order condition, guaranteeing that $q = \frac{1}{2}$ is a minimum, is only satisfied provided $(1 - \alpha)\Delta c \leq \Delta c_1$ where Δc_1 is given by (16) in the text.¹³

Moreover we have

$$\left. \frac{dC}{dq} \right|_{q=q_L} < 0 \Leftrightarrow \Delta c < \Delta c_2 = \frac{2(\sqrt{2 - 3q_L} - \sqrt{q_L})}{(1 - 2q_L)(1 - q_L)\sqrt{q_L}} \quad (51)$$

Without any loss in generalities let the optimal order be given by $q^* = \frac{1}{2} - \sigma$. Equation (50) can be re-written as

$$2\sigma[A - (1 - \alpha)\Delta c B\sigma^2 - 2((1 - \alpha)\Delta c)^2\sigma^4] = 0, \quad (52)$$

where

$$A = 4(1 - \alpha)\Delta c \left[\frac{1}{2} - q_L(1 - q_L) \right] + ((1 - \alpha)\Delta c)^2 \left(\frac{1}{2} - q_L \right) \left[\frac{3}{4}(1 - 2q_L) + q_L^2 \right] - 8, \quad (53)$$

and

$$B = 8 + (1 - \alpha)\Delta c \left(\frac{1}{2} - q_L \right) > 0. \quad (54)$$

The following results can be easily established:

¹³ Alternatively one can evaluate the Hessian matrix at $q = 1/2$ and show that it is definite positive provided $(1 - \alpha)\Delta c \leq \Delta c_1$.

- When $(1 - \alpha)\Delta c \leq \Delta c_1$ we have $A \leq 0$ and the only solution is $\sigma = 0$ so that $q^* = \frac{1}{2}$.
- When $(1 - \alpha)\Delta c \in [\Delta c_1, \Delta c_2]$ equation (FOC3) has two solutions: $\sigma = 0$ which does not constitute a minimum and

$$\sigma = \left[\frac{-B + \sqrt{B^2 + 8A}}{4(1 - \alpha)\Delta c} \right]^{1/2}.$$

The optimal order $q^* = \frac{1}{2} - \sigma$ decreases as $(1 - \alpha)\Delta c$ increases.

- Finally, when $(1 - \alpha)\Delta c > \Delta c_2$ the solution $\sigma > \frac{1}{2} - q_L$ and it is optimal to set $q^* = q_L$. ■

■ Proof of Proposition 3

Assume that $\pi(w) \in \mathbb{R}$. In this case, and as written in the text,

$$R(x, 1) = p(x, 1)\pi^{-1}(w_S(x)) + [1 - p(x, 1)]\pi^{-1}(w_F(x))$$

where

$$w_S(x) = \frac{1 - p(x, 0)}{p(x, 1) - p(x, 0)} \quad \text{and} \quad w_F(x) = \frac{-p(x, 0)}{p(x, 1) - p(x, 0)}.$$

Using this notation, the participation constraint and the incentive constraint can be written as:

$$p(x, 1)w_S(x) + [1 - p(x, 1)]w_F(x) - 1 = 0, \quad (55)$$

and

$$[p(x, 1) - p(x, 0)][w_S(x) - w_F(x)] = 1. \quad (56)$$

We must prove that $\frac{dR}{dx} < 0$. Taking the derivative of $R(x, 1)$ with respect to x , we have

$$\begin{aligned} \frac{dR}{dx} = & \frac{dp(x, 1)}{dx} [\pi^{-1}(w_S(x)) - \pi^{-1}(w_F(x))] + p(x, 1) \frac{1}{\pi'(\pi^{-1}(w_S(x)))} \frac{dw_S}{dx} \\ & + [1 - p(x, 1)] \frac{1}{\pi'(\pi^{-1}(w_F(x)))} \frac{dw_F}{dx}, \end{aligned} \quad (57)$$

where

$$\frac{dw_S}{dx} = \frac{1}{(p(x, 1) - p(x, 0))^2} \left[\frac{dp(x, 0)}{dx} (1 - p(x, 1)) - \frac{dp(x, 1)}{dx} (1 - p(x, 0)) \right] \quad (58)$$

and

$$\frac{dw_F}{dx} = \frac{1}{(p(x, 1) - p(x, 0))^2} \left[\frac{dp(x, 1)}{dx} p(x, 0) - \frac{dp(x, 0)}{dx} p(x, 1) \right]. \quad (59)$$

Let $r_S = \pi^{-1}(w_S)$ and $r_F = \pi^{-1}(w_F)$. Since the function $\pi(\cdot)$ is concave and since $r_S > r_F$ we have

$$\pi'(r_S) > \frac{\pi(r_S) - \pi(r_F)}{r_S - r_F}. \quad (60)$$

Therefore, the first term of (57) has an upper bound since $\frac{dp(x, 1)}{dx} < 0$:

$$\frac{dp(x, 1)}{dx} [\pi^{-1}(w_S(x)) - \pi^{-1}(w_F(x))] < \frac{dp(x, 1)}{dx} \frac{w_S - w_F}{\pi'(\pi^{-1}(w_F))}. \quad (61)$$

Since the incentive constraint binds we have

$$w_S - w_F = \frac{1}{p(x, 1) - p(x, 0)}. \quad (62)$$

Using the above and, as we substitute in $\frac{dw_S}{dx}$ and $\frac{dw_F}{dx}$ in (57), we can establish (after simplifications) that

$$\frac{dR}{dx} < p(x, 1) \frac{dw_S}{dx} \left[\frac{1}{\pi'(\pi^{-1}(w_S(x)))} - \frac{1}{\pi'(\pi^{-1}(w_F(x)))} \right] \quad (63)$$

Due to the concavity of $\pi(\cdot)$ the third term is positive. Under Property 2 $\frac{dw_S}{dx} < 0$ therefore the above is indeed negative and $R(x, 1)$ decreases with x .

Assume that $\pi(w) \in \mathbb{R}_+$ in which case we have

$$R(q, 1) = p_i(q, I) [\pi^{-1}(w'_S) - \pi^{-1}(0)] + \pi^{-1}(0).$$

Proving that $R(x, 1)$ decreases with x when property 1 holds is trivial. ■

▪ **Proof of Lemma 6.**

The manufacturer solves

$$\min_{q, I_1, I_2} F(q, I_1) + F(1 - q, I_2) + R(q, I_1) \mathbb{I}_{I_1} + R(1 - q, I_2) \mathbb{I}_{I_2}$$

where $\mathbb{I}_{I_i} = 1$ if and only if $I_i > 0$. The function $F(q, I)$ can be re-written as

$$F(q, I) = \frac{1}{2} (q - q_L)(1 - I)^2 [\Delta c(1 - \alpha)q]. \quad (5^{**})$$

and the function $R(q, I)$ is given by

$$R(q, I) = \frac{2 - (q - q_L)(1 - I)^2}{2(q - q_L)^2(1 - I)^2}. \quad (64)$$

Let us assume that there exists an interior solution such that $q = \frac{1}{2}$ where both suppliers invest a positive amount. This solution must satisfy both, the first and second order conditions.

The first order condition with respect to I_i ($i = 1, 2$) is such that

$$\frac{\partial F}{\partial I_i} + \frac{\partial R}{\partial I_i} = 0 \rightarrow I_i = \max \left\{ 0, 1 - \left[\frac{2}{q_i(q_i - q_L)^3(1 - \alpha)\Delta c} \right]^{1/4} \right\}. \quad (65)$$

The first order condition with respect to q is such that

$$\frac{\partial F(q, I_1)}{\partial q} + \frac{\partial R(q, I_1)}{\partial q} - \frac{\partial F(1 - q, I_2)}{\partial (1 - q)} - \frac{\partial R(1 - q, I_2)}{\partial (1 - q)} = 0. \quad (66)$$

Clearly the above is always satisfied at $q = \frac{1}{2}$.

However, as we evaluate the second order condition at $q = \frac{1}{2}$ we find that $q = \frac{1}{2}$ reaches a minimum provided

$$(1 - \alpha)\Delta c > \frac{8}{(1 - 2q_L)(1 + 3q_L - q_L^2)^2}. \quad (67)$$

However, one can show that for any such values for $(1 - \alpha)\Delta c$ the cost of outsourcing is lower if the manufacturer sets $q = 1 - q_L$, $I_1 = I(1 - q_L)$ and $I_2 = 0$. ■

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