

Project title: Tableaux combinatorics for the Abelian sandpile model

Supervisor: Dr Mark Dukes

The Abelian sandpile model is a model of discrete diffusion on a graph. Also known as the ‘chip-firing game’, it has been well-studied from several perspectives by mathematicians, physicists, and computer scientists since its appearance in the 1980s. Some of my recent research has unearthed new and surprising connections between the sandpile model on classes of graphs and other discrete structures from combinatorics and theoretical computer science.

In one instance, the sandpile model on the complete bipartite graph can be cast in terms of 0/1-fillings of rectangular tableaux whose region of ones form what is known as a *staircase polyomino*, a planar generalization of a domino [1]. This correspondence has given rise to a new class of polynomials called *q, t -Narayana polynomials*. These polynomials encode information about both the polyominoes and sandpile configurations on the graph, and are important in proving several symmetries the model exhibits [2]. Another avenue of research [3] has shown that recurrent states of the sandpile model on the Ferrers graph can be encoded as a 0/1-filling of the associated Ferrers diagram, a structure known as an *EW tableaux*.

This project will look at several new and open problems in this area at the interface of discrete mathematics and theoretical computer science. It will look at tableaux representations of several discrete models and the cellular automata that mimic the dynamical aspects of those models. As such it will build on work from my EPSRC project ‘New combinatorial perspectives on the abelian sandpile model’, and the work of my collaborators in Glasgow and Bordeaux.

A background in combinatorics is desirable, but not necessary, for this project. Alternatively, interested students should have a good background in a related field such as algebra, number theory, group theory, or discrete mathematics. Demonstrated research experience in mathematics or theoretical computer science would be highly desirable.

REFERENCES

- [1] Mark Dukes and Yvan Le Borgne. Parallelogram polyominoes, the sandpile model on a complete bipartite graph, and a q, t -Narayana polynomial. *Journal of Combinatorial Theory Series A* 120 (2013), no. 4, 816–842.
- [2] Jean-Christophe Aval, Michele D’Adderio, Mark Dukes, Angela Hicks and Yvan Le Borgne. Statistics on parallelogram polyominoes and a q, t -analogue of the Narayana numbers. *Journal of Combinatorial Theory Series A* 123 (2014), no. 1, 271–286.
- [3] Mark Dukes, Thomas Selig, Jason Smith, and Einar Steingrímsson. The Abelian sandpile model on the Ferrers graph and EW-tableaux. Preprint 2017.